Measurement Good Practice Guide No. 80

Fundamental Good Practice in Dimensional Metrology

David Flack
Engineering Measurement Services Team
Engineering Measurement Division

John Hannaford

ABSTRACT
This good practice guide is written for those who need to make dimensional measurements but are not necessarily trained metrologists. On reading this guide you should have gained a basic knowledge of fundamental good practice when making dimensional measurements. An introduction to length units and key issues such as traceability and uncertainty is followed by some examples of typical sources of error in length measurement. Checking to specification, accreditation and measurement techniques are also covered along with an introduction to optical measurement techniques.
Acknowledgements

This document has been produced for the Department for Business, Innovation and Skills; National Measurement System under contract number GBBK/C/08/17. Thanks also to Hexagon Metrology, Romer, Renishaw and Faro UK for providing some of the images and to Dr Richard Leach (NPL), Simon Oldfield (NPL), Dr Anthony Gee (University College London) and Prof Derek Chetwynd (University of Warwick) for suggesting improvements to this guide.
Contents

Introduction ........................................................................................................................................ 1
  Why have you picked up this guide? .................................................................................................. 2

Why bother making dimensional measurements at all? .............................................................. 5
  Open day at the factory ................................................................................................................. 7
  Some history of dimensional measurement .................................................................................. 8
    Manufacture in the early days – the military ............................................................................. 8
    The industrial revolution – the beginnings of the modern age and
    interchangeability ....................................................................................................................... 8
    The 20th century ....................................................................................................................... 10
  So why do we measure things? .................................................................................................... 10
    Why make a dimensional measurement? ............................................................................... 10
    Why do we take so much trouble in making measurements? .............................................. 12
  How do we ensure consistent measurements throughout the world? .................................... 12
  Units of length measurement .................................................................................................... 13
    Who defines the metre? ............................................................................................................. 13
    Metres and inches ..................................................................................................................... 13
    A typical transfer standard – the gauge block ......................................................................... 16
  Units of angle .................................................................................................................................. 18
    Angle gauges ............................................................................................................................. 24
  Resolution, accuracy, tolerance, etc. ........................................................................................... 25
    Accuracy and precision ............................................................................................................. 26
    Resolution, uncertainty, tolerance and error .......................................................................... 27
  International standards ................................................................................................................ 27

Just how well do you have to measure? ......................................................................................... 29
  Car parts ....................................................................................................................................... 30
  Introduction to measurement in manufacturing ........................................................................ 30
  Trend monitoring during production .......................................................................................... 31
  So your measurement has an uncertainty? .................................................................................. 33
    Introduction ............................................................................................................................... 33
    Expression of uncertainty in measurement ............................................................................. 33
    The contributions to the uncertainty budget .......................................................................... 34
    Expanded uncertainty ............................................................................................................... 35
    The statement of uncertainty .................................................................................................. 35
    Confidence level ....................................................................................................................... 36
    Type A and Type B contributions ............................................................................................ 37
    Distributions .............................................................................................................................. 40
  Checking conformance to a specification. How do you know the part meets
  specification? ............................................................................................................................... 42
Verifying articulated arm CMMs ................................................................. 83
Verifying laser trackers ........................................................................... 84
Verifying rotary tables .............................................................................. 85
Formal verification of a machine tool ....................................................... 85
Reversal techniques .................................................................................. 87
  Straightedge reversal .............................................................................. 88
Reorientation and repeat measurement .................................................. 91
Diagnoses ................................................................................................. 91

**Conforming to a quality standard – ’So you are going to be audited?’** ............... 93

I am going to be audited! .......................................................................... 94
A vertical audit .......................................................................................... 97
A horizontal audit ..................................................................................... 98
Closing meeting ....................................................................................... 98
Credibility ................................................................................................. 98
Are your measurements traceable? .......................................................... 99
What is this NPL place? ........................................................................... 101
What is accreditation? ............................................................................. 101
Laboratory considerations ....................................................................... 102
  Calibration labels .................................................................................. 102
  Calibration and calibration intervals ..................................................... 102
  Proper storage of equipment and records .............................................. 106
  Monitoring the environment ................................................................. 106
  Avoiding problems with corrosion ....................................................... 107
  Vibration and draughts - shut the window and stop that banging! ....... 108
  Cleanliness ............................................................................................ 109

**Horribilia – oddball stuff that might be useful** ........................................ 111

Easy mistakes .......................................................................................... 112
How can the results be wrong? ................................................................. 112
Cosine error ............................................................................................. 112
  What is cosine error? ........................................................................... 113
  Cosine error with callipers ................................................................. 113
  Cosine error with dial gauges ............................................................. 113
  Cosine error with a laser interferometer ............................................. 114
  Minimising cosine error ..................................................................... 115
  Introduction to alignment and correcting for cosine error ................ 116
  Cosine error calculations ................................................................. 116
  Allowing for residual cosine error in uncertainty budgets ................. 117
  Magnitude of errors for typical misalignments ................................... 118
Sine error ................................................................................................. 118
Abbe offset ............................................................................................... 119
C.1.1 Sag correction .................................................................................... 190
C.1.2 Tape tension correction ...................................................................... 191
C.1.3 Temperature correction when using a tape ........................................ 191
C.1.4 Slope correction for a tape .................................................................. 192
C.2 Using reversal to determine the error map of a CMM .......................... 192

Appendix D  Some useful physics................................................................. 194
D.1 Mechanics of beams .................................................................................. 194
D.2 Some useful optics ....................................................................................... 197
D.2.1 Reflection .......................................................................................... 198
D.2.2 Anti-reflection coatings ....................................................................... 199
D.2.3 Refraction .......................................................................................... 199
D.2.4 Double refraction ............................................................................... 201
D.2.5 Polarisation ........................................................................................ 201
D.2.6 Diffraction ........................................................................................... 202
D.2.7 Interference ........................................................................................ 202
D.2.8 Optical flats ....................................................................................... 203
D.2.9 Retarder plates ................................................................................... 203
D.2.10 Total internal reflection .................................................................... 204
D.2.11 Beam-splitters .................................................................................. 204
D.2.12 Fibre optic cables ............................................................................. 204
D.2.13 Retro-reflectors ................................................................................ 205
D.2.14 The laser ........................................................................................... 206
D.2.15 Commercial laser interferometer systems ....................................... 208
D.3 Kinematic design ...................................................................................... 209
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>A set of gauge blocks</td>
<td>17</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Circular measure of angle</td>
<td>20</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Serrated type of precision index</td>
<td>22</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Calibration of a precision index</td>
<td>23</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Example of a four-position precision index</td>
<td>23</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Tomlinson angle gauges</td>
<td>24</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Angle gauges can be used additively as well as subtractive</td>
<td>25</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Basic control chart layout</td>
<td>32</td>
</tr>
<tr>
<td>Figure 9</td>
<td>A histogram of the 100 readings</td>
<td>37</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Measured value with measurement uncertainty</td>
<td>39</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Normal or bell shaped curve</td>
<td>40</td>
</tr>
<tr>
<td>Figure 12</td>
<td>A rectangular distribution</td>
<td>41</td>
</tr>
<tr>
<td>Figure 13</td>
<td>The two results and their uncertainties</td>
<td>42</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Uncertainty of measurement: the uncertainty range reduces the conformance</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>and non-conformance zones (Copyright BSI – extract from BS EN ISO 14253-1:1999)</td>
<td></td>
</tr>
<tr>
<td>Figure 15</td>
<td>Conformance or non-conformance</td>
<td>44</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Impact of measurement uncertainty</td>
<td>46</td>
</tr>
<tr>
<td>Figure 17</td>
<td>An articulated arm CMM (Courtesy Faro UK)</td>
<td>53</td>
</tr>
<tr>
<td>Figure 18</td>
<td>An articulated arm CMM courtesy Romer, part Hexagon Metrology</td>
<td>54</td>
</tr>
<tr>
<td>Figure 19</td>
<td>A CMM</td>
<td>54</td>
</tr>
<tr>
<td>Figure 20</td>
<td>An articulated arm CMM (Courtesy Romer, part Hexagon Metrology)</td>
<td>56</td>
</tr>
<tr>
<td>Figure 21</td>
<td>An articulated arm being used to measure the Newport Ship (Courtesy Faro</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>UK and Newport Ship Project)</td>
<td></td>
</tr>
<tr>
<td>Figure 22</td>
<td>A laser tracker (Courtesy Faro)</td>
<td>57</td>
</tr>
<tr>
<td>Figure 23</td>
<td>Multilateration in action</td>
<td>58</td>
</tr>
<tr>
<td>Figure 24</td>
<td>The chart John had in his briefcase</td>
<td>64</td>
</tr>
<tr>
<td>Figure 25</td>
<td>Measurement of the wing sections (exaggerated)</td>
<td>66</td>
</tr>
<tr>
<td>Figure 26</td>
<td>Soaking a plug gauge</td>
<td>70</td>
</tr>
<tr>
<td>Figure 27</td>
<td>An articulated arm like the one Sam was using (Courtesy Faro UK)</td>
<td>76</td>
</tr>
<tr>
<td>Figure 28</td>
<td>ISO 10360 test</td>
<td>79</td>
</tr>
<tr>
<td>Figure 29</td>
<td>In-process gauging (© Renishaw)</td>
<td>85</td>
</tr>
<tr>
<td>Figure 30</td>
<td>In-process gauging (© Renishaw)</td>
<td>85</td>
</tr>
<tr>
<td>Figure 31</td>
<td>Reversal of a straightedge</td>
<td>89</td>
</tr>
<tr>
<td>Figure 32</td>
<td>A hemisphere being measured by the multi-step method</td>
<td>91</td>
</tr>
<tr>
<td>Figure 33</td>
<td>Traceability pyramid</td>
<td>99</td>
</tr>
<tr>
<td>Figure 34</td>
<td>Traceability chain for gauge blocks</td>
<td>100</td>
</tr>
<tr>
<td>Figure 35</td>
<td>An example certificate</td>
<td>103</td>
</tr>
<tr>
<td>Figure 36</td>
<td>Page 1 of the certificate with key areas highlighted</td>
<td>104</td>
</tr>
</tbody>
</table>
Figure 37 Page 2 of the certificate with key areas highlighted .............................................. 105
Figure 38 Put everything away after measurement ............................................................... 106
Figure 39 Shielding the equipment from draughts ............................................................... 109
Figure 40 Cosine error in measuring 3 m .............................................................................. 113
Figure 41 Cosine error of one degree with dial indicator (angle exaggerated) ................. 114
Figure 42 Cosine error with a laser interferometer ............................................................... 115
Figure 43 Cosine error with a laser interferometer (Image © Renishaw plc 2000) .......... 115
Figure 44 Correcting for cosine error .................................................................................. 116
Figure 45 An example of a laser interferometer misaligned .............................................. 116
Figure 46 Misalignment (cosine error) ................................................................................ 118
Figure 47 Sine error ............................................................................................................. 119
Figure 48 A set up with Abbe offset ................................................................................... 121
Figure 49 An improved set up with no Abbe offset ......................................................... 122
Figure 50 Abbe calculation example .................................................................................. 123
Figure 51 Callipers (Tesa Metrology) ................................................................................ 123
Figure 52 A micrometer (Tesa Metrology) ......................................................................... 124
Figure 53 Abbe offset with a laser interferometer (Image © Renishaw plc 2000) .......... 124
Figure 54 Compression corrections for steel spheres between flat steel anvils for various forces .................................................................................................................. 127
Figure 55 Compression corrections for tungsten carbide spheres between flat steel anvils for various forces ........................................................................................................ 128
Figure 56 You can reduce parallax by having the line of measurement of the rule as close as possible to the feature being measured ..................................................... 130
Figure 57 Airy points .......................................................................................................... 132
Figure 58 An autocollimator ............................................................................................... 137
Figure 59 Diagram of an autocollimator ............................................................................ 138
Figure 60 A laser system set up on a machine tool (Image © Renishaw plc 2000) .......... 139
Figure 61 A simple interferometer (Image © Renishaw plc 2000) .................................... 140
Figure 62 Constructive interference (Image © Renishaw plc 2000) ................................ 140
Figure 63 Destructive interference (Image © Renishaw plc 2000) .................................. 141
Figure 64 The beam paths in a linear interferometer (Image © Renishaw plc 2000) ...... 142
Figure 65 The beam paths in an angular interferometer (Image © Renishaw plc 2000) ...... 142
Figure 66 Straightness interferometer (Image © Renishaw plc2000) ............................... 143
Figure 67 Plane mirror configuration (Image © Renishaw plc 2000) ............................... 143
Figure 68 Deadpath error (Image © Renishaw plc 2000) ................................................. 145
Figure 69 Deadpath minimised (Image © Renishaw plc 2000) ........................................ 146
Figure 70 Alignment of a laser interferometer system (Image © Renishaw plc 2000) ...... 146
Figure 71 Angle measurement using an interferometer (Image © Renishaw plc 2000) .... 148
Figure 72 Straightness measurement (Image © Renishaw plc 2000) ............................... 149
Figure 73 A tracking laser interferometer in use at Seat (courtesy Faro UK) ................. 151
Figure 74 An absolute distance meter ............................................................................... 153
Figure 75 An ADM from the front ..................................................................................... 153
List of tables

Table 1 The make up of a Tomlinson set of combination angle gauges………………………………………. 24
Table 2 Impact of measurement uncertainty……………………………………………………………………………….. 46
Table 3 Typical expansion coefficients (ppm/°C) from John’s table …………………………………………………….. 64
Table 4 Expansion (in mm) of a 100 mm bar at various temperatures………………………………………………….. 71
Table 5 The certificate errors…………………………………………………………………………………………………….. 73
Table 6 Compression of steel spheres in the range 10 mm to 50 mm between steel anvils for forces from 250 gf to 50 gf………………………………………………………………………………………………………………….. 126
Table 7 Compression of tungsten carbide spheres in the range 10 mm to 50 mm for forces from 250 gf to 50 gf………………………………………………………………………………………………………………….. 127
Table 8 Typical values for refractive index…………………………………………………………………………………… 200
Fundamental Good Practice in Dimensional Metrology

Preface

The authors hope that after reading this Good Practice Guide you will be able to make better measurements of the size or shape of an object. The content is written at a simpler technical level than many of the standard textbooks on ‘Dimensional Metrology’ so that it can quickly introduce key ideas to a wide audience. We are not trying to replace a whole raft of good textbooks, operator’s manuals, specifications and standards, rather present an overview of good practice and techniques.

‘Metrology is not just a process of measurement that is applied to an end product. It should also be one of the considerations taken into account at the design stage. According to the Geometrical Product Specification (GPS) model, tolerancing and uncertainty issues should be taken into account during all stages of design, manufacture and testing. The most compelling reason is that it is often considerably more expensive to re-engineer a product at a later stage when it is found that it is difficult to measure, compared to designing at the start with the needs of metrology in mind.’ Dr Richard K Leach 2003
GOOD MEASUREMENT PRACTICE

There are six guiding principles to good measurement practice that have been defined by NPL. They are:

The Right Measurements: Measurements should only be made to satisfy agreed and well-specified requirements.

The Right Tools: Measurements should be made using equipment and methods that have been demonstrated to be fit for purpose.

The Right People: Measurement staff should be competent, properly qualified and well informed.

Regular Review: There should be both internal and independent assessment of the technical performance of all measurement facilities and procedures.

Demonstrable Consistency: Measurements made in one location should be consistent with those made elsewhere.

The Right Procedures: Well-defined procedures consistent with national or international standards should be in place for all measurements.

Introduction

IN THIS CHAPTER

- Why have you picked up this guide?
Chapter 1

Why have you picked up this guide?

If you’re interested in weather forecasts, then I’m afraid you should put this book down and go find one on meteorology! Metrology is concerned with the science, and some would say art, of measurement and this guide is concerned with dimensional measurements – that is to say the size and shape of things. The authors hope to spark an interest in precise and accurate measurement among members of the younger generation who will carry on an important but often unrecognised branch of engineering and the sciences. Dimensional metrology is probably not going to make you particularly rich, but it may well give you an interesting and stimulating professional career, and for some of us, that is enough!

Over the last couple of thousand years significant advances in technology can be traced to improved measurements. Whether we are admiring the engineering feat represented by the Egyptian Pyramids, or the fact that in the 20th century men walked on the moon, we should appreciate that this progress is due in no small part to the evolution of measurement. It is sobering to realise that tens of thousands of people were involved in both operations and that these people were working in many different places producing various components that had to be brought together – the technology that enabled this was the measurement techniques and standards that were used.

The Egyptians used a Royal Cubit as the standard of length measurement (it was the distance from Pharaoh’s elbow to his fingertips) while the Apollo space programme ultimately relied on the definition of the metre in terms of the wavelength of krypton 86 radiation.

Regardless of the measurement techniques available to you, better measurements can be made if you apply fundamental good practices, thus the Pyramids have square bases despite their builders using nothing much more sophisticated that a ball of string.

The Egyptians appreciated that provided that all four sides of a square are the same length and the two diagonals are equal, then the interior angles will all be the same – the right angle, or 90 degrees. They were able to compare the two diagonals and look for small differences between the two measurements to determine how square the base of the pyramid was.

Men have walked on the moon because a few brave men were prepared to sit on top of a collection of 10 000 manufactured parts all built and assembled by the lowest bidder and finally stuffed with hundreds of tons of explosive hydrogen and oxygen propellant! A principle reason that it all worked as intended was that the individual components were manufactured to exacting tolerances that permitted final assembly and operation as intended.

The phrase ‘mass production’ these days brings visions of hundreds of cars rolling off a production line every day. From Henry Ford in the 1920s through to the modern car plants such as BMW and Honda, the key to this approach is to have tiers of suppliers and sub-contractors all sending the right stuff to the next higher tier and finally to the assembly line. The whole manufacture and assembly process is enabled by the vital traceable dimensional measurements that take place along the route.
This guide attempts to provide an introduction to good practice in dimensional metrology and highlights the fundamental principles of dimensional metrology that allow experienced metrologists to make precise and accurate measurements.

**NOTE**

**The more difficult sections**

Although this guide has been aimed at the technician you will come across sections that are perhaps aimed at a higher level. Don’t be put off by these sections. In many cases you can just skip them and move on to the next heading.
Why bother making dimensional measurements at all?

IN THIS CHAPTER

- Open day at the factory.
- Some history of dimensional measurement.
- So why do we measure things?
- How do we ensure consistent measurements throughout the world?
- Units of length measurement.
- Units of angle.
- Resolution, accuracy, tolerance, etc.
- International standards.
There shall be standard measures of wine, ale, and corn (the London quarter), throughout the kingdom. There shall also be a standard width of dyed cloth, russett, and haberject, namely two ells within the selvedges. Weights are to be standardised similarly.

Magna Carta Clause 35, issued by King John’s chancery in 1215

(British Library website http://www.bl.uk/treasures/magnacarta/index.html)

This chapter will introduce you to the subject of dimensional metrology. The science of measurement is termed metrology. To accurately define the shape of anything, say the bore of an engine block or the profile of an aerofoil, certain fundamental concepts of geometry, such as straightness and roundness, and of measurement, such as diameter and length, are used. To achieve accuracy in these measurements reference must be made to some standard of length.

What is a dimensional measurement? It is the measurement of geometric features of an artefact. This may involve measuring the size, distance, angle, form or co-ordinate of a feature on an artefact, and the artefact itself may be anything at all - the height of a person, the diameter of a beer barrel, the length of a truck, the radius of a ball and so on.

In our modern metric world we use a unit of length called the metre – originally chosen to be a length that represented 1/10 000 000 of the quadrant of the Earth’s meridian from the Equator to the Pole, it is now defined as the distance travelled by light in a vacuum, in a time of 1/299 792 458 of a second. We will cover this in more detail a little later in this chapter.

This is actually quite a subtle definition because time can be much more accurately measured than any physical lengths can actually be measured, so any improvements in length measuring capability don’t actually change the definition of the metre.

NOTE

The unit of length

The metre is defined by the Conférence Générale des Poids et Mesures. A committee of world-renowned scientists discuss the matter and make recommendations as to the most accurate way of defining the unit. They also define the second as equal to the duration of exactly 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium 133 atom.

Let’s illustrate some points with a short story.

---

2 For an interesting account of the history and evolution of standards of length the reader is referred to A history of engineering metrology by K J Hume. See the section Reference materials and further reading for full reference.
Open day at the factory

Frank was looking forward to the open day; it had been some time since he had visited his son Peter at work. Peter works for a multinational company with manufacturing facilities in several other countries including the USA, Mexico and South Korea. The parts are shipped to the UK for assembly. Peter works in the inspection area.

It was getting near to the end of the day when Peter’s father Frank stuck his head round the door.

‘Hello Son. So this is where you work. What do you do here? I know its something to do with measurement.’

‘Well Dad,’ said Peter. ‘I use this instrument called a height gauge to check the components that we manufacture. I measure all kinds of features and produce a calibration report. Someone else then does further tests and at the end of the day we decide if the component is suitable for use, can be reworked, or if it should be scrapped. My height gauge is very accurate. It can measure to 0.001 mm.’

‘0.001 mm!’ said Frank, ‘who needs to measure that accurately.’

‘Well, the designer has put a tolerance on the component of 0.01 mm so we need to measure ten times better than that to prove it has been manufactured to specification. The specification limits are dictated by what the part needs to do. Overall dimensions may have a coarser tolerance, but this bore, where the shaft fits has a tighter tolerance’.

‘Very good,’ said Frank. ‘But how do you know that the height gauge is that good?’ Peter pointed to the sticker on the side of the height gauge.

‘See that sticker? It shows that the height gauge has been calibrated by a UKAS laboratory and they in turn get their measurements done by NPL, the UK’s national standards laboratory’. Frank thought for a moment.

‘So who are UKAS?’

‘UKAS stands for the United Kingdom Accreditation Service. Laboratories can request to become accredited by UKAS. Once UKAS have checked them out, the labs can then display the UKAS logo on their certificates. UKAS visit at regular intervals to make sure the measurements are still OK.’

‘And what about NPL?’ asked Frank.

‘NPL is the UK National Physical Laboratory. They set the standards for all of the units. Most developed countries have their own equivalent of NPL, such as NIST in the USA’. So your measurements are checked by UKAS but how do you know the bits coming in from the US will fit with your bits?’ asked Frank.

‘We both work to the same procedures and all our measurements are traceable to the definition of the metre. We are traceable via NPL and the New York branch is traceable via NIST. NPL and NIST regularly compare results so they know that a measurement in NIST is equivalent to one at NPL. In fact, the same is true of most national measurement institutes.

NOTE

Short stories

Throughout this guide we will be illustrating some key points through the use of short stories. Look out for further stories later in the guide.
Some history of dimensional measurement

If we look at early dimensional measurements – the Pyramids of Egypt were built using cubits (the distance from the Pharaoh’s elbow to his fingertips) as the unit of measurement. As too was the Ark that Noah was reputed to have built for a particularly demanding customer with a tight schedule due to imminent inclement weather.

The practice of surveying land was established early on in settled civilisations, initially for ownership purposes so that people knew where their boundaries lay and later for tax purposes as the King demanded levies in proportion to the area people owned or farmed. Initially land was measured using rods and chains to lay out a network of triangles whose area could be calculated. Later improvements came to include measuring tapes and theodolites, electronic range finders and distance meters.

Fair trade forced people to look at ‘Weights and Measures’ and standards for mass and volume and length have been found in many cultures going back thousands of years. Thus, ensuring that you paid a fair amount of gold for your barrel of beer or your bolt of cloth.

Manufacture in the early days – the military

A great deal of effort has been exerted over the last few thousand years in the development of better weapons with which to wage war. From the catapults and ballistae of the Romans to the trebuchet of medieval times, handmade pieces of carpentry and the wrought iron of the blacksmiths forge dominated the scene.

But with the coming of gunpowder to Europe a whole new industry started – gun making. The basic process of casting a gun barrel and turning the bore were serious strains on the technology of the day, plus the processes were shrouded in secrecy as one side or the other could gain obvious military advantage.

The siege cannon evolved over the years into the hand cannon and via the mechanisms of the matchlock, the flintlock and the wheel-lock we begin to see recognisable firearms with triggers and sights, etc. All of these weapons were individually hand crafted and fitted by the artisans who built them – if something broke you pretty much had to go back to the maker to get it fixed.

The industrial revolution – the beginnings of the modern age and interchangeability

The trials and tribulations of the American War of Independence in 1776 and the requirement to equip their army, lead the new American Government to place a contract for the supply of 10 000 muskets with Eli Whitney in 1798.

In fact Whitney’s proposal was revolutionary at the time – he had never built a gun, he had no factory and no workers but was proposing to build 10 000 muskets in two years!

It was only his reputation as the inventor of the cotton gin, which automated the removal of seed from cotton fibre and the fact that Thomas Jefferson – the Secretary of State, knew him
that he was awarded the contract. In 1785, Jefferson, while United States Minister to France, had seen the ideas of Honore le Blanc for a system of interchangeable parts and understood the radical nature of Whitney’s proposal.

Thus Whitney had to build a factory from scratch and design and build the machines. He also had to organise the division of labour and train unskilled workers, who were concerned with only one step of production, to enable manufacture of a quantity of parts that could be assembled into a finished musket.

In 1801 it was apparent that Whitney needed money, and an extension on his contract, and he went to Washington to demonstrate to President Adams his system of uniform parts. With the election of Jefferson as President, further problems with extensions and advancements were overcome.

In fact the contract took ten years to complete, with most of the muskets being produced in the last two years. The last of the 10 000 was delivered in 1809 and in 1811 Whitney took on a new contract for 15 000 muskets – they were all delivered within two years – the American Method was established.

Meanwhile in Great Britain the Industrial Revolution was in full swing – steam power was the ‘in thing’ overtaking water power as a motive force and bringing with it a whole new perspective to the concept of precision engineering for that time. Large pistons needed to be made round so that they could run smoothly in the cylinders of the big steam engines – tolerances were getting tighter, so that the engines could be more powerful and efficient. Initially accustomed to working to the nearest eighth of an inch, or perhaps a bare sixteenth, engineers and machine designers were demanding tighter fits for the machines of the Industrial Revolution.

The development of accurate surface plates – flat surfaces manufactured from cast iron or granite – was the foundation of dimensional accuracy in engineering workshops. It was a well-established practice in Henry Maudslay’s day to make the plates in sets of threes, and by lapping one against the other, not only could they be made smooth, but also they were assured to be flat. (If only two plates are lapped together they can form matching surfaces that are actually dished – one concave and the other convex). Joseph Whitworth, who started work as an apprentice in Maudslay’s workshop in London, refined the technique by introducing the practice of scraping the high spots off instead of simply rubbing the two blocks against each other. The result was a superior finish and high accuracy to the finished surface plates as the errors in each plate were not propagated one to the other but eliminated individually by the scraping process.

Whitworth went on to establish his own workshop in 1833 and became interested in unifying screw threads so that a simple task like bolting something together with a nut and bolt did not involve a long search through a box of nuts to find one that fitted the bolt in your hand!

Whitworth had seen Maudslay’s Lord Chancellor micrometer of 1805, which was an application of a precision lead screw and flat reference surfaces – enabling readings to be

---

3 Henry Maudslay (August 22, 1771 - February 14, 1831), born Woolwich, England. He was a machine tool maker and inventor.
taken to 0.0001 of an inch. Whitworth went on to develop a more accurate micrometer-based measuring machine that used a drum mounted on a precision screw to subdivide the screw pitch into smaller increments.

**The 20th century**

The National Physical Laboratory – established in 1901 – became involved in developing improved instruments in the run up to the Great War of 1914 to 1918.

In three years at NPL, from 1915 to 1918, Edgar Mark Eden and Frederick Henry Rolt looked at a number of measuring problems and applied first principles to the measurement of screw thread diameters, pitch error and the comparative measurement of gauge blocks.

Eden developed the floating carriage micrometer that combines a measuring fiducial or indicator and a large drum micrometer on a moving stage to allow the measurement over suitable wires of the effective diameter of a screw thread.\(^4\)

The Eden-Rolt comparator was put together over a weekend and is an elegant design that does not require high precision of manufacture yet enables the high magnification of the differences in length between two gauge blocks to be made. The ability to apply a constant force during the adjustment of the drum greatly improved the repeatability of the measurement and by using it as a comparator together with a set of Johansson’s gauge blocks, accuracy was assured. We will explain what gauge blocks are later in this chapter.

The basic technology of measuring the size and shape of components did not change much during the first half of the 20th century.

**So why do we measure things?**

*Why make a dimensional measurement?*

In manufacturing you should make dimensional measurements basically to ensure that you have confidence that you’ve made the right thing and can demonstrate to a customer that you have taken all reasonable steps to ensure that they get the product they contracted for. In science you may also be making dimensional measurements in the field to characterise the size of wildlife as part of a research project or you may be interested in how heat loss through a material varies with the material thickness. In medicine you may be interested in the change in size of a tumour over a period of time or how the average height of people has increased since the 1800’s. There are all manner of reasons why you may need to make traceable length measurements. Certainly if you are going to publish your results in a refereed journal, measurements need to be traceable.

\(^4\) For further information on screw threads see [http://resource.npl.co.uk/docs/science_technology/dimensional/screw_gauge_booklet.pdf](http://resource.npl.co.uk/docs/science_technology/dimensional/screw_gauge_booklet.pdf)
NOTE

The idea of traceability

Before we progress much further we need to say something about traceability. PD 6461-1:1995 defines traceability as follows:

*property of the result of a measurement whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties*

Traceability is covered in more detail in chapter 7.

In manufacturing, dimensional measurements are vital in monitoring and controlling the variations inherent within any manufacturing process. Simple things like tool wear can be picked up as a drift in size of a turned component within the allowable tolerance band, corrective action can be taken in good time. More complex interactions may require a more detailed measurement process – such as periodic assessment of a whole car body from an assembly line. It is possibly not necessary to measure every car body, but if you measure every feature on every fiftieth body then you have some statistical control of the process.

We also have the requirement for interchangeability of components, it would be irritating to pop down to the garage for a spare part for the car, to bring it home and find it doesn’t fit! During the design process the designer will ensure that a part will have a tolerance band of allowable variation in size associated with it. Provided the part is made within that tolerance band, it will function as intended.

The old rule of thumb used to be the 10:1 toolmakers rule – that you measured to an accuracy ten times better than the tolerance that you were manufacturing to. This makes sense because if you can measure the effect of small changes to the process, you are well on the way to controlling it.

As an example, if you were turning a shaft to a particular diameter on a lathe and using a vernier calliper with a resolution of 0.02 mm to determine the size of the component during the process, you would not expect to be able to hold a tolerance of 0.01 mm. However, using the same lathe and a micrometer with a resolution of 0.001 mm you would be able to measure the size of the component and adjust the tool setting appropriately to make the finishing cut on size – within the tolerance.

The development of measurement technology has been driven by the scientific requirements of the day and these in turn were driven by serious technical and scientific challenges. If we look at the development of clocks as an example – the most demanding requirement was for navigation – to determine the longitude of a position - in other words how far east or west of Greenwich a ship had sailed. An ancillary instrument, the sextant, that measures the angle between the sun and the horizon could be used to determine latitude. Together the clock and the sextant allowed navigators to plot their position with sufficient accuracy for charting purposes. This was a major step forward in allowing Britannia to rule the waves!
**Why do we take so much trouble in making measurements?**

This question is answered in Monty Finniston’s foreword in D M Anthony’s book ‘Engineering Metrology’ by the statement:

‘In an era where science, technology and engineering condition our material standards of living, the effectiveness of practical skills in translating concepts and designs from the imagination or CAD system to commercial reality, is the ultimate test by which an industrial economy succeeds.’

Measurement is an essential part of translating concepts and designs to commercial reality.

As we have seen the need for precision measurement originated in the late 1700s during the industrial revolution and as a consequence of the modern practice of interchangeable global manufacture, where different parts of a machine or product are manufactured in two different countries and often assembled in a third, the importance of dimensional metrology has grown enormously over the last sixty years.

As we contemplate the twenty-first century and worry about world population growth, global warming and the anticipated shortage of oil amongst other things, we expect and require that the application of yet more advanced technology will solve many of our social and environmental problems. Advanced measurement technology and techniques will be required to support these improvements. However, the foundations of good practice in dimensional metrology were laid out long before anyone reading this book was born! It is, therefore, prudent to pay attention to the basic principles of measurement and to get such things right the first time.

We need to make measurements if we want to manufacture things, to control the way other people make things and finally we need to make measurements to describe things.

**How do we ensure consistent measurements throughout the world?**

To ensure interchangeability we need consistent measurement throughout the world. We need to know that a metre in China is the same as a metre in Kenya. The Bureau International des Poids et Mesures (BIPM) – established in 1875 - is the international body responsible for the maintenance and dissemination of measuring standards covering the entire field of scientific endeavour. The BIPM is particularly concerned with ensuring that measurements made in one country agree with measurements made in another. Each country has its own National Metrology Institute (NMI) that is responsible for realising these units. The United Kingdom’s NMI is the National Physical Laboratory in Teddington. Other NMIs include Physikalisch-Technische Bundesanstalt (PTB) in Germany and the National Institute of Standards and Technology (NIST) in the USA.

In October 1999, the directors of thirty-eight NMIs and two international organisations signed the Comité International des Poids et Mesures (CIPM) Mutual Recognition Arrangement (MRA), an historic agreement for the wider acceptance of National

---

5 Anthony D M 1986 *Engineering metrology* (Butterworth-Heinemann)
Measurement standards and of calibration and measurement certificates issued by their institutes.

The MRA is a scheme to give users reliable quantitative information on the comparability of national metrology services and to provide the technical basis for wider agreements negotiated for international trade, commerce and regulatory affairs.

Basically, the MRA consists of a large database that allows anyone to compare the measurement capability of an NMI. It also ensures that a measurement certificate produced in the UK will be accepted in the USA, Germany and in fact in any of the countries signed up to the agreement.

Units of length measurement

Who defines the metre?

Before we give the definition of the metre we will say a little about who is responsible for defining the metre and give a little bit of history.

To quote the International Bureau of Weights and Measures website:

*The Convention of the Metre (Convention du Mètre) is a diplomatic treaty which gives authority to the General Conference on Weights and Measures (Conférence Générale des Poids et Mesures, CGPM), the International Committee for Weights and Measures (Comité International des Poids et Mesures, CIPM) and the International Bureau of Weights and Measures (Bureau International des Poids et Mesures, BIPM) to act in matters of world metrology, particularly concerning the demand for measurement standards of ever increasing accuracy, range and diversity, and the need to demonstrate equivalence between national measurement standards.*

The Convention was signed in Paris in 1875 by representatives of seventeen nations and established a permanent organisational structure for member governments to act in common accord on all matters relating to units of measurement. The Convention, modified slightly in 1921, remains the basis of international agreement on units of measurement. There are now fifty-one Member States, including all the major industrialised countries.

**NOTE**

**SI (The International System of Units)**

The 11th General Conference on Weights and Measures (1960) adopted the name Système International d'Unités (International System of Units), for the recommended practical system of units of measurement. The base SI units are metre, kilogram, second, ampere, kelvin, mole and candela (m, kg, s, A, K, mol, cd).

**Metres and inches**

In the case of length BIPM define the SI unit of length, the metre (m). As with all SI units, multiples and sub-multiples may be formed by recognised prefixes, for example, km, mm.
### SI prefixes

SI prefixes are used to form decimal multiples and submultiples of SI units. They should be used to avoid very large or very small numeric values. The prefix attaches directly to the name of a unit, and a prefix symbol attaches directly to the symbol for a unit.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{24}$</td>
<td>yotta</td>
<td>Y</td>
<td>$10^{-1}$</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>zetta</td>
<td>Z</td>
<td>$10^{-2}$</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>exa</td>
<td>E</td>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta</td>
<td>P</td>
<td>$10^{-6}$</td>
<td>micro</td>
<td>µ</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{9}$</td>
<td>giga</td>
<td>G</td>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>mega</td>
<td>M</td>
<td>$10^{-15}$</td>
<td>femto</td>
<td>f</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>kilo</td>
<td>k</td>
<td>$10^{-18}$</td>
<td>atto</td>
<td>a</td>
</tr>
<tr>
<td>$10^{2}$</td>
<td>hecto</td>
<td>h</td>
<td>$10^{-21}$</td>
<td>zepto</td>
<td>z</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td>deca</td>
<td>da</td>
<td>$10^{-24}$</td>
<td>yocto</td>
<td>y</td>
</tr>
</tbody>
</table>

Since 1983, the metre has been internationally defined as the length of the path travelled by light in a vacuum during a time interval of $1/299\,792\,458$ of a second. This definition is a major improvement on the original one, as it can be realised more simply and accurately using modern techniques. In addition the speed of light is generally regarded to be a universal constant of nature making it ideal as a length standard.

### The unit of length

The modern unit of length is the metre – defined as the distance that light in a vacuum travels in $\frac{1}{c}$ of a second where $c$ is the fixed speed of light in a vacuum, 299 792 458 m s$^{-1}$.

The statement defining the metre, given above has the effect of rendering the definition of the metre dependent on the second – the unit of time. Time itself is very well defined in terms of microwave reference frequencies generated by caesium atomic clocks. However, this definition of the metre is not terribly handy for actually measuring things.
The practical realisation of the metre is, therefore, achieved at National Metrology Institutes (NMIs) by the use of specially prepared iodine vapour cells for use with stabilised helium-neon (He-Ne) lasers. Systems using iodine-stabilised lasers have been built in many of the world’s NMIs, have been compared at the BIPM and shown to be in agreement to a few parts in $10^{11}$. If you are unfamiliar with numbers expressed in this form, see appendix B.1.1.

For many practical measurements in standards laboratories and in industry we rely on laser interferometer systems that have been calibrated against these iodine-stabilised lasers, thus ensuring the traceability of those measurements by an unbroken chain of calibrations of ever increasing accuracy back to the original definition of the metre.

You will, however, still see drawings where lengths are labelled in inches. One (international) inch is EXACTLY 25.4 mm. The (international) inch has been exactly 25.4 mm since July 1959. At this point in time the (international) yard was redefined as 0.9144 metre - until this time the ratio between the US yard and the metre was different to the ratio between the UK yard and the metre. The American inch changed by 2 millionths of an inch and the UK inch by 1.7 millionths of an inch. The international inch falls midway between the old UK and US inch.

The ‘thou’ is a unit sometimes used in engineering and is an abbreviation of one-thousandth of an international inch, *i.e.*, 25.4 μm.

Are inches still legal units? They are, but only in some situations. Certain imperial units are retained for a short period in specific areas such as road traffic signs, liquid measures, *etc.*

*How is the length unit realised*

At NPL the metre is currently realised through the wavelength of the 633 nm radiation from an iodine-stabilised He-Ne laser, with an uncertainty of about 3 parts in $10^{11}$. This is equivalent to measuring the earth’s mean circumference to about 1 mm.

The definition of the metre is rather impractical in terms of making a real measurement and as a consequence much work was done to realise the metre in a useful format. The iodine-stabilised He-Ne laser is the end result of a research programme that started in the early 1970s seeking ways of stabilising the frequency and thus wavelength of a laser so that it could be used to make measurements of length.

The equation $c = f\lambda$ links the frequency ($f$) and wavelength ($\lambda$) of a light source such as a laser. Now as we know the speed of light in a vacuum is constant, if we can arrange for the frequency of the laser radiation to be very stable then we will also have arranged for the wavelength of that light to be very stable. We can then use the stable wavelength of the laser light as a glorified ruler by using the laser in an interferometer. Admittedly the ‘marks on the ruler’ are rather close together – about 633 nm in fact – but by using high-speed digital counters and electronic fringe interpolation we can measure hundreds of metres and resolve to $1/1000^{th}$ of a fringe. Don’t worry too much about this paragraph now. We shall say more about interferometers in chapter 9.

Subsequent research at the BIPM and various NMIs such as the NPL has centred on comparing the iodine-stabilised lasers (with frequencies around 473 THz) *via* a long chain of
intermediate stabilised lasers to the caesium atomic clocks that operate in the microwave region at about 9.2 GHz. The huge span of frequencies from 10 GHz to 470 THz – a factor of 47 000 – has to be bridged by frequency doubling and trebling techniques that allow accurate measurement of the frequencies involved by measuring the beat frequencies – a bit like tuning a guitar. (A factor of 65 536 would require a chain of sixteen doublings, so this is a rather elaborate procedure.)

NOTE

The caesium fountain

The first caesium fountain designed to operate as a frequency standard was constructed at the Primary Laboratory for Time and Frequency (LPTF), Paris in 1990. The accuracy of the LPTF fountain is now assessed to be 2 parts in $10^{15}$, equivalent to a second in 16 million years, it is at present the most accurate of all primary frequency standards.

The stability of the NPL’s iodine-stabilised He-Ne lasers has been assessed at 2 parts in $10^{11}$ and these lasers are used to calibrate commercially available thermally stabilised lasers that are typically stable at the level of 1 part in $10^{8}$ or so. Commercial laser interferometer displacement measuring systems are available from a number of sources.

Dissemination via the shop floor

The definition of the metre can be used or realised in two different ways to measure length practically:

a) Time of flight - a pulse of light is sent over the length to be measured. The time it takes for the light to travel this distance multiplied by the speed of light, gives the length. As light is very fast, this method is easiest to apply over long distances.

b) Interferometry - the technique of interferometry allows a length to be measured in terms of the wavelength of light. By using a light source of known and stable wavelength, lengths up to 100 m can be directly measured, with accuracies approaching 1 part in a thousand million.

Laser interferometer systems are, however, rather expensive, and not everyone can afford to use them; fortunately some hundred year’s old technology can come to our assistance! A Swedish engineer Carl Johansson working at the beginning of the twentieth century produced a very important invention, gauge blocks.

A typical transfer standard – the gauge block

Gauge blocks (Figure 1) are normally simply blocks of steel with two flat and parallel surfaces a known distance apart. Two such blocks very sparingly smeared with a paraffin-based fluid can be pressed together with a sliding and twisting motion so that the flat surfaces adhere to each other in a process called wringing. The force that holds the two blocks together is sufficient to support the weight of the smaller blocks and is thought to be due to the intermolecular forces between atoms and molecules in the adjacent surfaces and the wringing film. The gap between the two surfaces or wringing film is extremely small (estimated to be 0.005 μm), and so the combined length of a pair of wrung gauges can be considered the sum of their individual lengths. By making a set of 102 gauge blocks with 49 blocks ranging in size from 1.01 mm to 1.49 mm in steps of 0.01 mm, 49 blocks ranging from
0.5 mm to 24.5 mm in steps of 0.5 mm and a final 4 blocks from 25 mm to 100 mm in steps of 25 mm, Johansson calculated that he could assemble any stack between 1 mm and 201 mm to the nearest 0.01 mm.

Typically, modern gauge block sets have 112 pieces and include ten extra gauges – nine from 1.001 mm to 1.009 mm in steps of 0.001 mm and one gauge of 1.0005 mm. An increment of 0.001 mm was not necessary in Johansson’s day and it’s really only necessary today, if you are calibrating other gauges or need to measure components with tight tolerances.

![Figure 1 A set of gauge blocks](image)

**NOTE**

**Combining gauge blocks – an example**

If we wish to build up a combination that is 14.656 mm long, we would start with the 1.006 mm gauge block then 1.050 mm, 1.200 mm, 1.400 mm, and a 10.000 mm block and wring them together. The wringing film thickness is typically 0.005 µm and can usually be ignored in most engineering applications.

The addition of gauge blocks to form a single combined block is the basis of Johannson’s system. The distance between the end faces is carefully controlled during the manufacturing process so that the final desired size is reached to within a fraction of a micrometre. During the final lapping process all the gauge blocks in a given batch are wrung together and measured using a high accuracy comparator and a gauge block of known size. Dividing the difference between the measurement and the gauge block by the number of blocks wrung together gives the individual error of each block.
The use of gauge blocks is straightforward and the precision is remarkably high. Johannson’s system works on the principle that by combining a set of gauge blocks with different decade increments he could create any required value to the nearest 0.001 mm.

Many of the world’s NMIs perform the calibration of gauge blocks optically by exact fraction interferometry and this is a slow and expensive technique reserved only for the very best gauge blocks. Once calibrated, however, these ‘laboratory grade’ gauge blocks are used to calibrate lower accuracy gauge blocks by mechanical comparison using much simpler and faster techniques.

**NOTE**

**Gauge block interferometry**

When gauge blocks are measured by interferometry, they are wrung to a platen of similar material. Corrections have to be made for phase changes on reflection at the surface of the gauge blocks – the amount depending on the surface texture and material of the gauge blocks and the platen they are wrung to. Typical values for the phase correction range from 0.02 μm to 0.07 μm.

One important geometric parameter of a gauge block is its flatness. This is important to achieve a good wring and reduce the variation in length across the face. The flatness of something like a gauge block can be assessed, using an optical flat and a monochromatic light source such as a sodium lamp, by examining the optical fringe pattern that is observed when the two surfaces are brought together.

An optical flat is a disk of stress-free glass or quartz that has one or both faces ground, lapped and polished to an optical quality. Lapping and interchanging three flats against one another creates the optical flat in the first place and allows it to be used as an absolute reference for flatness.

While the study of optics does not strictly come within the scope of this guide it is perhaps worth pointing out that many modern instruments utilise optical systems in their construction and operation. Reflection, refraction and interference will be dealt with in a chapter 9, suffice it to say at this stage that a telescope that establishes a line of sight is a very useful item in the metrologists’ toolbox.

**Units of angle**

We have talked about units of length but in dimensional metrology you may also have to measure angle. Angle has its own set of units, as we will now see.

**Practical units of angle**

For practical angular measurements in engineering, the sexagesimal system of units, which dates back to the Babylonian civilisation, is used almost exclusively. An alternative system, the centesimal system introduced by Lagrange towards the end of the eighteenth century, is rarely used. In both systems, the basic unit is the right angle, which may be defined as the
angle between two straight lines which intersect so as to make the adjacent angles equal. In the sexagesimal system the right angle is divided into 90 equal parts called degrees and in the centesimal system it is divided into 100 equal parts called grades.

These parts are further sub-divided as follows:

**Sexagesimal system**

1 right angle = 90 degrees (°)
1 degree = 60 minutes (′)
1 minute = 60 seconds (″)

**Centesimal system**

1 right angle = 100 grades (^
1 grade = 100 minutes (′′)
1 minute = 100 seconds (″″)

The grade is known as the 'gon' in Germany, where the units 'new minute' (1/100 grade) and 'new second' (1/10 000 grade) are included in the laws governing units.

**Radians**

In the theoretical treatment of angle, an alternative method of expressing an angular magnitude is derived from consideration of the angle as part of a circle. In Figure 2 an angle $\theta$ is bounded by two radii OA and OB and subtends an arc AB on a circle of arbitrary radius $r$. The magnitude of the angle is defined by the ratio: length of arc AB divided by $r$. Angles so defined are said to be in circular measure and the unit in this system is the radian.

The radian is the angle subtended by an arc of a circle of length equal to the radius.
Since the ratio of the circumference of a circle to its diameter is $\pi$, the following relationships between the radian, the degree and the grade are established

\[
\begin{align*}
2\pi \text{ radians} & = 360 \text{ degrees} = 400 \text{ grade} \\
1 \text{ radian} & = 57.2958 \text{ degrees} = 63.6620 \text{ grade} \\
1 \text{ degree} & = 0.017453 \text{ radian} \\
1 \text{ grade} & = 0.015708 \text{ radian}
\end{align*}
\]

**Other angular units**

Another system based around the radian is the millieme system in which the unit is the thousandth part of a radian. In a practical adaptation of this system, which has found extensive use in the United States for military purposes, the millieme or mil is defined by dividing the circle into 6400 equal parts instead of the 6283.18... parts obtained when each part corresponds to one thousandth of a radian.

Artillery battalions in many countries use the mil as a unit of angular measure, although there are three different specifications for this unit.

In the North Atlantic Treaty Organisation (NATO), a mil is defined as $1/6400$ of a full circle. This angular measure gets its name from being approximately a milliradian, a unit which is convenient because 1 mil at 1 km is about 1 m (and so 100 mils at 2 km is about 200 m). Or one could say that an object a metre wide one kilometre away from the viewer subtends, to a high degree of accuracy, an angle of one mil.

No conversion to Imperial units is worthwhile here, for all armies use metric maps, even the US Army. There are 1600 mils in 90 degrees, 17.8 mils in one degree. This mil is usually used in artillery discussion. It is also used in long-range precision rifle shooting, where the
crosshairs on riflescopes are often calibrated in mils. This type of riflescope is usually referred to as a mil-dot scope.

The army of the Soviet Union used a mil that was 1/6000 of a full circle, which means that there were 1500 of its mils in a right angle, which would be less accurate though easier to remember.

The military of Sweden during the Cold War desired to demonstrate its independence from both NATO and the Warsaw Pact so they chose a size of greater accuracy. Because a right angle is more nearly 1.5708 radians than 1.600, their mil was 1/6300 of a circle, so that there were 1575 of these mils in a right angle.

The above description of the millieme illustrates the importance of an international definition of a unit!

*Why are angle units special?*

For the measurement of length it is necessary to adopt an ultimate standard, either a material standard such as the former Imperial Standard Yard or a natural standard such as the distance light travels in a certain time. No ultimate standard is required for angular measurement since any angle can be established by appropriate sub-division of the circle. A circle has 360 degrees, no more, no less.

*Closure and self-proving*

The principle of self-proving (circle-closing method) of the division of a circle is fundamental to angle measurement. Imagine you are dividing a circle using only a simple tool like dividers and you want to divide the circle by ten. You would first set your dividers to what you estimate is a tenth part of the circle. If the dividers are now carefully stepped around the circle at the end you will be in error by ten times the error in the original setting. By adjusting the dividers by a tenth of the overall error the circle can then be marked in ten equal sectors.

If we wanted to calibrate a previously divided circle where we know there are errors in each sector we could do so by a similar means. Firstly using one of the sectors (A for instance) as a reference the dividers are set to one tenth of the circle. The accumulated difference between A and each of the other sectors is recorded along with the total accumulated error on closing. The initial setting error is calculated as one tenth of the closing error. This can then be applied to each of the accumulated errors to determine the value of each sector.

A variation on this technique is to use angle A as the master and compare to each of the sectors. Each difference is recorded and the value of A determined algebraically. The example below shows this method applied to a precision index.

The precision index (Figure 3) is calibrated by a self-proving. One index table is mounted upon the other as shown in Figure 4.

Consider an precision index with only four positions A, B, C and D each nominally 90°,
\[ A = 90 + e_1 \]
\[ B = 90 + e_2 \]
\[ etc. \]

We know that

\[ 90 + e_1 + 90 + e_2 + 90 + e_3 + 90 + e_4 = 360, \]

where \( e \) is the error in each individual sector (Figure 5).

If we take a second precision index similar to the first that has angles E, F, G and H we have

\[ 90 + e_5 + 90 + e_6 + 90 + e_7 + 90 + e_8 = 360. \]

During measurement we compare each angle of the first table with each angle of the second table and record a reading on the autocollimator (a sensitive angular measuring device that is described in chapter 9). The reading is effectively the sum of the two errors. If we sum all the autocollimator readings for angle A against in turn E, F, G and H we end up with

\[ e_1 + e_5 + e_1 + e_6 + e_4 + e_7 + e_1 + e_8 = 4e_1 + e_5 + e_6 + e_7 + e_8 \]

but we also know

\[ e_5 + e_6 + e_7 + e_8 = 0. \]

So the mean of the differences recorded gives us the error in angle A alone.

**Serrated Type**

![Figure 3 Serrated type of precision index](image)
Figure 4 Calibration of a precision index

![Calibration of a precision index](image)

Figure 5 Example of a four-position precision index

![Example of a four-position precision index](image)
Figure 4 shows the calibration of a Moore precision index, a practical example of the concept in action. Here every 30 degree interval of the bottom table is compared with every 30 degree interval of the top table.

**Angle gauges**

One practical realisation of angular units takes the form of angle gauges. Angle blocks (Figure 6) are very similar in concept to Johannson’s gauge blocks – they were developed at NPL by Tomlinson in 1939 and also revisited by Chapman at National Research Council laboratory, the Canadian NMI, in the 1970s. The essential difference is of course that instead of the distance between the two faces of the block being important it is now the angle between the faces. The subtlety of angle is that it is not only additive, but also subtractive – thus making the sets smaller. A 1˚ block wrung to a 3˚ block will combine to give 4˚, but reversing one of the blocks will result in a 2˚ block (see Figure 7). The make-up of a Tomlinson set of combination angle gauges is given in Table 1.

<table>
<thead>
<tr>
<th>Degree</th>
<th>90</th>
<th>41</th>
<th>27</th>
<th>9</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minute</td>
<td>27</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td>30</td>
<td>18</td>
<td>6</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The manufacture of the Tomlinson blocks was aided by recognising that determining the size of the gauges from first principle uses the identities $9 + 9 + 9 - 27 = 0$ and that $27 + 27 + 27 + 9 = 90$. Thus the angle blocks were made in sets of three and it was unknown for any of the pieces to deviate by more than 0.6 arc seconds from nominal. Figure 6 shows a photograph of a set, note that the 0.05˚ gauge is not always supplied.
Another concept developed by Chapman was based on the orthogonal design matrices of weighing schemes and angle gauge combinations were chosen so that they could be combined and nulled against each other (wrung in opposition until the faces are nominally parallel).

Chapman’s set consisted of the following sizes

<table>
<thead>
<tr>
<th>Degrees</th>
<th>90</th>
<th>45</th>
<th>27</th>
<th>18</th>
<th>9</th>
<th>6</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seconds</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chapman’s choice of angles meant that a high-resolution angle-measuring instrument of very limited range, such as an autocollimator, could be used to determine the differences between different combinations of the angle gauges. A high accuracy calibration could thus be achieved. Unfortunately the modern practice in calibration laboratories is to preserve the surface texture by not wringing gauges together, negating the benefits of the scheme and it has not been widely adopted.

Starett, and Hilger and Watts manufacture other sets of angle gauges using different combinations of angles but the basic principle is the same.

NOTE

Versatility of angle gauges
With two gauge blocks we can only form three lengths. With two angle gauges we can form eight angles. For a 41˚ and 9˚ gauge these angles are:

41˚, 9˚, 50˚, 32˚, 139˚, 171˚, 130˚ and 148˚.

Resolution, accuracy, tolerance, etc.

You will often come across the terms resolution, accuracy, tolerance, uncertainty and precision. They all mean the same thing don’t they? Unfortunately they don’t and you can be easily misled if you confuse the terms.
Accuracy and precision

The accuracy of an instrument indicates how well it agrees with the (conventional) true value.

The precision of an instrument refers to the dispersion of measurements.

Four different archers each with varying degree of ability illustrate the difference between accuracy and precision. The bull's-eye in the target represents the true value of a measurement.

Inaccurate and imprecise (unrepeatable)

Stone age man missed the bull's-eye and the three attempts were not near each other.

Precise but inaccurate

Robin Hood's Merry Man missed the bull's-eye but the three attempts were near each other.

Accurate but imprecise

Native American’s three attempts are near the bull's-eye, but were not near each other.
Accuracy is a term relating the mean of the measurements to the true value, while precision is representative of the spread of these measurements. We can never achieve perfection in measurement so even a precise and accurate measurement will have some remaining uncertainty. It is best to use the term accuracy only as a qualitative term, or for broad comparisons between methods, and use the term uncertainty as the quantitative assessment. When the uncertainty of a measurement is evaluated and stated, then the fitness of purpose for a particular application can be properly understood.

**Resolution, uncertainty, tolerance and error**

Other terms you will come across in manufacturers’ brochures and calibration certificates are resolution, uncertainty of measurement and error.

The resolution of an instrument is a quantitative expression of the ability of an indicating device to distinguish meaningfully between closely adjacent values of the quantity indicated.

The uncertainty of measurement is a calculation made to describe the bounds within which you have every reason to believe the true value lies.

The tolerance is the difference between the upper and lower tolerance limits. A designer will specify these limits to indicate how well a component needs to be made to meet its specification. The interaction of tolerance and uncertainty is covered later in this guide and also in Flack D R, Bevan K 2005 *Fundamental good practice in the design and interpretation of engineering drawings for measurement processes* Measurement Good Practice Guide No. 79 (NPL).

The error in an instrument is the difference between the indicated value and the known value of some material standard of size (for instance a gauge block).

**International standards**

Sometimes it is important that goods made worldwide are specified in a common way. Equally important is that measurements are made using defined techniques. To ensure that common techniques are used, the International Organization for Standardization (ISO) publishes standards that are used worldwide. More information on ISO can be found in chapter 10.
3

IN THIS CHAPTER

- Car parts.
- Introduction to measurement in manufacturing.
- Trend monitoring during production.
- So your measurement has an uncertainty.
- Checking conformance to a specification. How do you know the part meets specification?
- Determining conformance with a specification - ISO 14253 decision rules.
- Decision-making rules (production 3:1 versus inspection 10:1).
- GO and NOT GO hard gauges (limit gauges).
- Multiple gauging stations and master parts.
- Future developments.
Car parts

John is the jobbing foreman of a medium sized company making parts for the automobile industry. The company has been asked to make a part for a new range of cars and it is John’s job to work out how they are going to measure it.

It was getting dark now and John was alone in the office with only an anglepoise lamp to illuminate the drawing. John took a long hard look at the drawing. It was a complicated part with many features that needed measuring.

His first conclusion was that this component was ideal for that brand spanking new co-ordinate measuring machine (CMM) that the company had just bought. The CMM was ideal for measuring lots of components quickly and could easily tackle all the features on this component.

Getting out the specification for the CMM, he looked up the maximum permissible error. It was quoted as $5 + \frac{L}{100} \mu m$. 

‘Hang on,’ he thought. ‘The tolerance for roundness on that 60 mm diameter hole is 0.002 mm – better than the CMM can manage. We will have to measure that on our roundness-measuring instrument if we want a measurement uncertainty of about a tenth of the tolerance. While we are at it, the overall length has a tolerance of 0.3 mm. We could easily check that with a calibrated micrometer before we put the component on the CMM.’ A plan was coming together.

‘The diameter, right,’ he thought to himself. ‘That has been specified with a tolerance of 0.005 mm, that will require the high accuracy CMM in the inspection laboratory and we can check the surface texture quality with our hand held surface texture meter. Flatness, parallelism and perpendicularity - all those features could be measured on my CMM.’ John made some notes that would form the basis of a procedure for checking the new component. In the morning, he would read the relevant specification, ISO 14253, before he finalised anything.

Introduction to measurement in manufacturing

‘When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind’ Lord Kelvin (1824-1907)

The old adage ‘measure twice and cut once’ is sage advice. Taking action on erroneous information is the worst thing you can do because you end up with something that doesn't work.

You can only make what you can measure.

A manufacturing tolerance describes the range within which the size of a feature on a component can lie and still allow it to function as desired. Thus when manufacturing a component we may set up the tooling, cut the metal, measure the size of the feature and
check whether it lies within the manufacturing tolerance and if not, take a finishing cut to bring the component to the desired nominal size.

If too much material is removed initially, it is not possible to remove more material from the component to bring it within tolerance. A useful term to remember is maximum material condition (MMC)\(^6\) and if one is doubtful about the accuracy of a process, one should initially aim for a size between MMC and nominal – as you can then correct with a finishing cut to the nominal size.

Imagine turning a cylindrical component on a lathe, we take a cut, stop the spindle, measure the diameter and check the size against the drawing. For a turned component the largest external diameter is the MMC and conversely the smallest internal diameter is the MMC. During manual machining it is, therefore, sensible to aim for the upper tolerance on an external diameter and the lower tolerance on the internal diameter, a finishing cut is then applied to bring the component to its nominal size.

The designer determines the nominal size of a feature of the component after consideration of the required functional fit between this component and its mating part and uses tables of limits and fits that have been drawn up and standardised over the years. (The reader is again referred to *Fundamental good practice in the design and interpretation of engineering drawings for measurement processes* for more details of the designer’s role.)

**Trend monitoring during production**

Statistical process control (SPC) is the term used to cover the application of statistics to the control of industrial processes. In its simplest form this may involve measuring the size of every tenth item off the production line and measuring and recording the dimension on a graph that has the upper and lower tolerances marked on it. By taking note of the trends displayed on the graph it is then possible to predict when the process is going to produce components with dimensions that exceed the permitted tolerances, and take corrective actions, such as adjusting the tool setting. Figure 8 shows a basic SPC control chart that plots the process variation with time. The basic aim of SPC is to minimise variation.

---

\(^6\) MMC is described more fully in Flack D R, Bevan K 2005 *Fundamental good practice in the design and interpretation of engineering drawings for measurement processes* Measurement Good Practice Guide No. 79 (NPL)
Statistical quality control (SQC) is a broader topic dealing with the wider use of statistics to control the product from the design stage to the final shipped product. SQC includes such activities as process capability studies, SPC of that process, statistical based sample inspection and the statistical design of experiments.

Inspecting every single component from a production line is very expensive and only really justified for safety critical components such as aircraft parts, for most components a sampling process is used. Standard sampling plans exist that allow one to determine the sample size for a particular batch and to then determine if the whole batch is accepted or rejected based on the number of rejects within the sample. Thus for a general inspection of 450 components a batch of fifty may be selected and inspected. For an acceptable quality level of one percent a single failure in the fifty would be acceptable, but two failures within the fifty would lead to rejection of all 450 components.

Geoff Portas coined Portas' Law: 'Random results are the consequence of random procedures.'

This is illustrated by considering the case of inspecting a batch of 500 components by picking fifty components from the box of 500 and measuring them. The statistics you derive from those measurements – such as the average size and the standard deviation of the sample will allow you to accept or reject the batch depending on the sampling plan that you have selected and the particular criteria used.

However, this sampling plan won’t have told you a great deal about the manufacturing process. If we now change the inspection process to one where we measure every tenth component off the production line, not only can you determine the average size and standard deviation, you can also determine trends. This trend monitoring is a powerful tool in the control of the process and allows the operator to make timely adjustments to the machine so that tolerances are not exceeded.
So your measurement has an uncertainty?

Introduction

Uncertainty of measurement is covered in Bell S A 2001 A beginner's guide to uncertainty in measurement Measurement Good Practice Guide No. 11, Issue 2 (NPL). If the reader is unfamiliar with measurement uncertainty it is advised they read this guide before reading the next section.

The formal definition of uncertainty is:

*parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand*


Putting this statement another way, it is a parameter associated with a specific measurement result that expresses the range of values about that result within which we can confidently expect to find the true value of the measurand.

A complete statement of a measurement result might look like this:

\[ 50.005 \pm 0.005 \text{ mm}, \]

the range of values between which the true value of the measurand might lie is 50.000 mm to 50.010 mm.

Expression of uncertainty in measurement

At its most basic, measurement uncertainty is a statement of how well someone thinks that they have measured something. Measurement uncertainty can, therefore, be considered a guide to the quality of the measurement. There are a number of intimidating and highly mathematical explanations of how to estimate measurement uncertainty – the ISO Guide to the Expression of Uncertainty in Measurement (GUM) being the definitive article (see chapter 10 for the full reference).

We shall present a somewhat simpler explanation here. There is no way to actually know the true value of a particular measurement because, whatever measurement process you use, there will always be some sort of error associated with that process. It follows from this that the result of any measurement is really only an estimate of the true value of that measurand, and so needs to be accompanied by an estimate of the uncertainty of the correctness of the stated result.
Systematic and random errors

Your measurement system can have two types of error. Systematic and random. Systematic errors are errors that are the same from one measurement to the next, for example, an error in the measurement scale. Random errors are those that are different from one measurement to the next.

Even after all the systematic effects have been corrected for, there remains an uncertainty due to both random effects and imperfect correction for the results of systematic effects. The overall uncertainty estimate gives a quantitative (or numerical) assessment of the reliability of the result, and allows us to compare results with one another in a meaningful manner. This can be helpful, as we shall see later.

The contributions to the uncertainty budget

Once we know our contributions to the uncertainty budget they need to be expressed and combined in some manner. If the measurand, $Y$, is a function of several input quantities, $X_i$, we can write

$$Y = f(X_1, X_2, \ldots, X_n).$$

If there were only three input quantities we could write

$$Y = f(X_1, X_2, X_3).$$

For instance, if $Y$ were the volume of a cube, then the three input quantities would be the length, breadth and height of the cube.

Very often, an estimate, $y$, of the measurand $Y$ (in other words, $y$ is the result of a measurement of $Y$) is determined from $n$ other estimates of the input quantities $(x_1, x_2, \ldots, x_n)$ of input quantities $(X_1, X_2, \ldots, X_n)$ through some functional relationship $f$, rather than being measured directly, thus

$$y = f(x_1, x_2, \ldots, x_n).$$

These input estimates $(x_1, x_2, \ldots, x_n)$ may either be quantities whose values and uncertainties are determined directly during the current measurement (for example, ambient temperature, humidity, etc.), or quantities whose values and uncertainties are brought into the process from external sources (for example, data from other certificates, handbooks, etc.)

Using our cube example again the input estimates are the measured values of length, breadth and height.

The combined standard uncertainty of the measurement result, written as $u_c(y)$, is determined from the estimated standard uncertainty of each input estimate $x_i$ and written as $u(x_i)$. Each standard uncertainty associated with the input estimates is categorised according to the method in which its numerical value is obtained. Those uncertainties obtained by using
statistical methods are termed Type A, whilst those obtained by methods other than statistical are termed Type B.

It should be noted that there is no direct correspondence between the Type A and B classifications and the old terms ‘random’ and ‘systematic’. Type A and B classifications refer to the manner in which the uncertainty was estimated, and not to the effect of the uncertainty on the measurement result (in other words an uncertainty previously referred to as ‘systematic’ might be obtained using statistical methods, a method previously thought of as reserved for ‘random’ uncertainties). Furthermore, an uncertainty that is a Type A at one level in the hierarchical national measurement system becomes a Type B at the next level down.

**Expanded uncertainty**

Once we have calculated all the Type A and Type B contributions we have to combine them in to one number called the combined standard uncertainty. Assuming the contributions are not correlated, the combined uncertainty is calculated by squaring each term, adding the squared terms together, and then taking the square root (a process known as quadrature addition).

The combined standard uncertainty is then multiplied by a coverage factor \( k \) to give the expanded uncertainty. The value of \( k \) for a confidence level of 95 % is usually 1.96.

**The statement of uncertainty**

Assuming that we have a certificate with the results stated, for instance, in Table I and the uncertainties in Table II, the uncertainty of measurement will be stated in the following form.

> The uncertainty of each mean value given in Table I has been calculated following the guidance given by ISO document ‘Guide to the expression of uncertainty in measurement’. The expanded uncertainties are given in Table II and each is based on a standard uncertainty multiplied by a coverage factor \( k = 2 \), providing a level of confidence of approximately 95 %.

The above statement is a good model if you need to quote an uncertainty. However, it will need to be modified if the \( k \) value is not 2, for instance when the number of degrees of freedom (see next box) is less than about thirty. An example statement in this case is given below.

> The expanded uncertainty associated with the measurement of uniformity of diameter is based on a standard uncertainty multiplied by a coverage factor \( k = 2.10 \) which, for a \( t \)-distribution with \( v_{\text{eff}} = 27 \) degrees of freedom, provides a level of confidence of approximately 95 %.

### NOTE

**What is a \( t \)-distribution?**

The \( t \)-distribution is another probability distribution similar to the normal distribution. It differs from the normal distribution in that it has an additional parameter, called degrees of freedom, which changes its shape. A \( t \)-distribution with a high number of degrees of freedom (greater than 30) approaches a normal distribution.
**Confidence level**

We have just mentioned something called a confidence level. In the old days before *GUM* metrologists would make a number of measurements, calculate the mean, variance and standard deviation, assume a normal probability distribution and state the result as the mean plus or minus three times the standard deviation.

The old timers were describing the range within which the true value fell. Having made the assumption that the probability distribution is normal we are implying that no more that 3 in 1000 of the results would fall outside the bounds described by \( \pm 3\sigma \). We will describe \( \sigma \) or sigma shortly but basically it is the standard deviation. The total range from \(-3\sigma\) to \(+3\sigma\) is of course \( 6\sigma \) which may ring a bell for those readers with a background in statistical process control.

More recently we have seen competition between manufacturers of measuring instruments who were embarrassed to find that their products accuracy figures were worse than their competition, so they changed the figures and reduced their confidence levels accordingly – thus to the uninitiated their accuracy figures had ‘improved’.

We still see ambiguous statements from manufacturers – so it does pay to make sure that you are comparing like with like.

The expression ‘with 95 % confidence’ comes from the statistics and is actually trying to say that values fall within \( 2\sigma \) or two standard deviations of the mean. If we assume that the results are normally distributed then 5 % or 1 in 20 of the results in any given sample will fall outside those stated uncertainty limits.

If we look at the distribution of 100 readings on an artefact nominally 51 mm in size we might get a histogram plot very similar to that in Figure 9. The red line shows a normal distribution with the same mean and standard deviation. The mean of the distribution is 51.0000 mm with a standard deviation of 0.0012 mm (68 %). The 95 % confidence intervals are 50.9998 mm and 51.0003 mm.
Chapter 3

Figure 9 A histogram of the 100 readings

We can say that 68% of readings are no more than 1σ away from the mean, 95% are no more than 2σ away from the mean and 99.7% no more than 3σ away from the mean. That is to say at 3σ only three readings in one thousand will lie outside the claimed boundaries.

**Type A and Type B contributions**

This next section shows in very basic terms how Type A and Type B uncertainty contributions are determined.

**Type A uncertainty evaluation**

Type A evaluations are normally used to determine the repeatability or randomness of a measurement process. Should \( n \) independent observations of an input quantity be made under the same conditions of measurement, then the best possible estimate of the input quantity is the arithmetic mean or average of all the observations.

The mean can be written as

\[
\bar{x} = \frac{1}{n} \sum_{k=1}^{n} x_k
\]

where \( x_k \) are the individual measurements and \( n \) is the number of measurements. The sigma sign notation (\( \Sigma \)) is mathematical notation for ‘add up all the \( n \) readings’.
A measure of the dispersion of each individual observation around their mean is called the experimental standard deviation and is determined from the square root of the variance of the observations

\[ s = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (x_k - \bar{x})^2} . \]

This standard deviation is for the particular set of \( n \) observations, and would be different to another set of observations of the same measurand. However, an estimate of the standard deviation for the entire population of possible values of the measurand can be made by replacing the \( n \) in the denominator of the equation above by \( n - 1 \)

\[ \sigma_x = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (x_k - \bar{x})^2} . \]

This standard deviation is the uncertainty associated with any one observation taken of the measurand. As would be expected, the variability of the mean value of several observations reduces as the number of observations increases and is given by the experimental standard error of the mean

\[ \sigma_x = \frac{\sigma_x}{\sqrt{n}} . \]

The standard error of the mean is the standard uncertainty associated with the average of several observations. Even though the individual observations might not follow a normal distribution, the average values of sets of observations will generally have a normal probability distribution.

The degrees of freedom of the uncertainty estimation should be reported along with the results of the Type A evaluation. Broadly defined, the degrees of freedom, \( \nu_i \), are the number of terms used to obtain the estimation minus the number of constraints on those terms. For a Type A statistical analysis this results in

\[ \nu_i = n - 1 . \]

In the case of a least squares fit evaluation, two constraints exist (both the slope of the fitted line and the intercept) and the number of degrees of freedom becomes

\[ \nu_i = n - 2 . \]

**Type B uncertainty evaluation**

All uncertainties associated with input estimates that have not been obtained from repeated observations must be evaluated by scientific judgement. Using all possible sources of information, the metrologist needs to draw on his or her experience and general knowledge about the processes to make reliable decisions about Type B uncertainties.
It must be emphasised that Type B uncertainties are usually the most difficult to evaluate due to the rigorous scientific methodology required to substantiate a particular position about an uncertainty. If treated in a trivial manner, Type B components can produce meaningless results in uncertainty estimates. Possible information about Type B contributions can come from:

- Previous measurement data on the same or similar system
- Manufacturer’s specifications
- Figures from calibration certificates
- Uncertainties associated with reference data from handbooks
- Previous experience with the behaviour of certain instrumentation

If the quoted uncertainty, taken from a handbook or calibration certificate, is stated to be a particular multiple of a standard deviation, then the standard uncertainty is obtained by dividing the expanded uncertainty by the multiplier. Similarly if a confidence level is given instead of a multiplier, then a normal distribution can be assumed and the standard uncertainty is obtained by dividing the expanded uncertainty by the appropriate factor for a normal distribution (i.e. 1.64 for 90% confidence, 1.96 for 95% confidence and 2.58 for 99% confidence).

Note that generally the number of degrees of freedom associated with a Type B contribution is infinite. However, there are special cases where it may be necessary to calculate the number of degrees of freedom. In these cases it is necessary to refer to the GUM.

NOTE

A short summary
We introduced the idea of the measurement uncertainty as a figure of merit for how good a measurement is considered to be - by the person who made the measurement. The implication of this statement is that the true value (which we may never exactly know) lies somewhere between the upper and lower bounds defined by the value of the stated uncertainty with a given confidence level. This is shown graphically in Figure 10. The dot shows the measurement results and the tee-ended line (error bar) shows the uncertainty of measurement.
Distributions

There are various ways to specify a random variable. The most visual is the probability density function. The probability density function describes the shape of the characteristic curve derived from numerous repeated measurements. The probability density function represents how likely each value of the random variable is.

The normal distribution, also called the Gaussian distribution, is an extremely important probability distribution. The distributions have the same general form, differing in their location and scale parameters: the mean (or average) and standard deviation (variability), respectively. A plot of the distribution is often called the bell curve because the graph of its probability density resembles a bell (Figure 11).

Some notable qualities of the normal distribution are:

The density function is symmetric about its mean value and as stated earlier:

68.27 % of the area under the curve is within one standard deviation of the mean,
95.45 % of the area is within two standard deviations,
99.73 % of the area is within three standard deviations.

The normal distribution has a symmetrical peaked form, while the rectangular distribution, as its name suggests is flat – having the same probability of a value occurring anywhere between two given values. An example of rectangular distributions is instrument resolution and alignment errors.
NOTE

Rectangular or normal distribution

If you throw a die hundreds of times and plot the results you will see that each number is equally likely. You have a rectangular distribution. If you throw two dice you will find that certain totals are more likely than others. You have a normal distribution.

Figure 12 shows a typical rectangular distribution. In this case it is from an instrument with a resolution of 0.001 mm. The instrument has recorded 50.000 mm but we know the true value could be between 49.9995 and 50.005 mm. The distribution is rectangular as each value is equally likely.

The contribution to the uncertainty, in this case, is found by halving the resolution and dividing by the square root of three (for a rectangular distribution), thus

\[
\frac{0.001}{\frac{2}{\sqrt{3}}} = \frac{0.001}{\sqrt{12}}.
\]

There are even more unusual distributions such as the triangular and inverse triangular. The effect of these assorted distributions is to alter the weighting that is used in the calculation of an overall uncertainty for the total measurement system to which they contribute.
Checking conformance to a specification. How do you know the part meets specification?

*Determining conformance with a specification - ISO 14253 decision rules*

When making a measurement you may think that it is a simple matter of the result falling within the tolerance band to prove conformance. This is not the case as the following example shows.

The designer has specified that a hole should be 50 mm ± 0.005 mm (top and bottom lines in Figure 13). The first operator measures the size with a traceable micrometer as 50.004 mm and states that the hole conforms to the drawing. However, the foreman, looking at this result examines the uncertainty of the micrometer. The measurement uncertainty of the micrometer is 0.003 mm and applying this uncertainty he realises that the actual size could lie between 50.001 mm and 50.007 mm. He gets the hole remeasured on a bore comparator that has a 0.001 mm uncertainty. The measurement comes out at 50.006 mm and conformance is not proven. As a general rule the measurement uncertainty of the equipment should be no greater than ten percent of the tolerance band.

Note that in this case both measurement results agree to within their uncertainties. For operator 1, however, the measured value is less than the uncertainty away from the upper specification limit (USL)\(^7\) and no real information has been obtained about whether the true value is inside or outside the specification limits.

---

\(^7\) Conversely LSL stands for lower specification limit.
ISO 14253 recommends that the following rules be applied for the most important specifications controlling the function of the workpiece or the measuring equipment.

At the design stage the terms ‘in specification’ and ‘out of specification’ refer to the areas separated by the upper and lower tolerance (double sided) or either LSL or USL for a one sided specification (see Figure 14 areas 1 and 2, line C).

When dealing with the manufacturing or measurement stages of the process the LSL and USL are added to by the measurement uncertainty. The conformance or non-conformance ranges are reduced due by the uncertainty (see Figure 14, line D).

These rules are to be applied when no other rules are in existence between supplier and customer. ISO 14253 allows for other rules to be agreed between customer and supplier. These other rules must be fully documented.

Conformance with a specification is proved when the result of measurement, complete statement, falls within the tolerance zone or within the maximum permissible error of the specification for measuring equipment (for example, the maximum permissible error of a CMM). Conformance is also proven when the measurement result falls within the tolerance zone reduced on either side by the expanded uncertainty. The conformance zone is linked to the LSL, USL and actual expanded uncertainty.

---

8 A complete statement of the result of a measurement includes information about the uncertainty of measurement.
Non-conformance with a specification is proved when the result of measurement, complete statement, falls outside the tolerance zone or outside the maximum permissible error of the specification for measuring equipment. Non-conformance is also proven when the measurement result is outside the tolerance zone increased on either side by the expanded uncertainty. The non-conformance zone (4 in Figure 14) is linked to the USL, LSL and expanded uncertainty.

Neither conformance nor non-conformance with a specification can be proven when the result of measurement, complete statement, includes one of the specification limits (for example, measurement 1 in Figure 13 and Figure 15).

It is important that the principle behind these rules is applied to a supplier/customer relationship where the uncertainty of measurement always counts against the party who is providing the proof of conformance or non-conformance, *i.e.* the party making the measurement. That is to say the supplier will reduce the tolerance by their measurement uncertainty to prove conformance. The customer will increase the tolerance by their measurement uncertainty to prove non-conformance.

Referring to Figure 15, three items have been measured. The purple line shows the LSL, the blue line the USL.

- **Measurement of item 1** - neither conformance nor non-conformance with a specification can be proven
- **Measurement of item 2** – non-conformance is proven
- **Measurement of item 3** – conformance is proven
In the case of item 1 the result of measurement, complete statement straddles the USL and neither conformance nor non-conformance with a specification can be proven. In the case of item 2 the result of measurement, complete statement is above the USL and so non-conformance is proven. In the case of item 3 the result of measurement, complete statement is above the LSL and below the USL and so conformance is proven.

**Summary of ISO 14253**

ISO 14253 can be summed up in the following statements:

The supplier shall prove conformance in accordance with clause 5.2\(^9\) of BS EN ISO 14253 using their estimated uncertainty of measurement.

The customer shall prove non-conformance in accordance with clause 5.3\(^10\) of BS EN ISO 14253 using their estimated uncertainty of measurement.

When evaluating the measurement result the uncertainty is always at the disadvantage of the party with onus on proof.

In the past the measurement uncertainty has been ignored when ascertaining conformance with a specification as long as the uncertainty was one tenth of the specification width. This procedure is no longer acceptable.

**Decision-making rules (production 3:1 versus inspection 10:1)**

In a production environment it is common enough to be faced with the apparently simple question ‘Is the item I have just made a good part, or a piece of scrap?’ The obvious way to answer this question is to measure the relevant dimensions of length and diameter and to check that they are within the specified tolerances.

We have already seen that the ISO 14253 standard provides clear guidance about the need to allow for the uncertainty of the measuring device by reducing the size of the acceptance band. Thus there is considerable interest in having access to instruments with smaller measuring uncertainties – unfortunately these are usually more expensive to purchase, and may involve additional expenses such as special air-conditioned rooms or a longer and thus more expensive measuring period.

Figure 16 illustrates the impact of the measuring uncertainty on the acceptable process tolerance and shows that for a nominal 25 µm specification, Machine C which could be a high accuracy coordinate measuring machine with a measuring uncertainty of 2.5 µm, would allow a process tolerance of as much as 20 µm and achieve the traditional 10:1 inspection ratio.

Unfortunately Machine C could easily cost £200,000, and may need to be housed in a temperature controlled environment, thus economic factors could force one to look seriously
at a machine such as Machine A. Machine A could be an articulated arm measuring machine that may cost a mere £20,000, but which can barely meet a 3:1 inspection ratio. The higher production costs associated with producing components within this much reduced process tolerance of only 10 µm may be acceptable if insufficient capital is available to purchase Machine C – it all depends on the actual circumstances of the job in hand, especially the value of the individual components, the number of items to be produced and the duration of the contract.

![Figure 16 Impact of measurement uncertainty](image)

Table 2 Impact of measurement uncertainty

<table>
<thead>
<tr>
<th>Measuring uncertainty</th>
<th>Specified tolerance range</th>
<th>For production</th>
<th>For customer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine A</td>
<td>7.5 µm 25 µm</td>
<td>10 µm</td>
<td>High 40 µm</td>
</tr>
<tr>
<td>Machine B</td>
<td>5 µm 25 µm</td>
<td>15 µm</td>
<td>Average 35 µm</td>
</tr>
<tr>
<td>Machine C</td>
<td>2.5 µm 25 µm</td>
<td>20 µm</td>
<td>Low 30 µm</td>
</tr>
</tbody>
</table>

Figure 16 and Table 2 are based on a diagram in Verlag Moderne Industrie ‘Multisensor Coordinate Metrology’ ©www.sv-corporate-media.de

It should be recognised that many component specifications involve complicated three-dimensional relationships and the fact that the technical language of Geometric Dimensioning and Tolerancing (GD&T) has been developed to describe these relationships is an indication of how complicated they can become.

A simple example of complicated three dimensional relationships could involve the fitting of an alloy wheel to a car stub axle – the design may call for four, five or even six studs to be positioned equally spaced about some nominal pitch circle diameter. Each stud, therefore, will be allocated an \((x, y)\) positional coordinate and nominal diameter and will have a positional tolerance about that nominal location as well as an allowable diameter variation. At
the end of the day the designer is simply trying to ensure that the alloy wheel will fit the stub axle whatever the orientation of the wheel.

During the inspection process it is tempting to measure the central hole in the wheel and to call that the origin of the coordinate system and to measure the position of one stud and use a line between the origin and that stud as an axis. The positions of the other studs can then be measured relative to the origin and the first stud.

However, this approach has the disadvantage that the central hole is not actually a functional parameter – it doesn’t define the centre of rotation of the wheel or the stub axle. In addition, by using one of the studs to define the axis we are throwing away the tolerance associated with the permitted variation in diameter of that stud. In practice the whole wheel would be allowed to rotate about the centre of the pitch circle diameter (PCD) of the studs thus increasing the range of permitted stud diameters and positions that would still allow a wheel to fit the assembly of studs. The functional parameter of the stub axle is, therefore, the least squares fit circle that makes up the pitch circle through all the studs.

For high value components, such as die casting moulds and large forgings for aerospace applications, the use of global three-dimensional optimisation software such as ‘Smartfit’ (Kotem Technology Inc) allows one to reorient the coordinate system to fully utilise the available tolerances and to explore variations that may enable minimal rework or repair, to salvage an otherwise out of specification component. As a general rule of thumb, adding material to a component is difficult, as welding for instance may introduce unacceptable thermal strains or distortions, but making location holes larger (but still within tolerance) or relieving a datum face can be done relatively easily and may allow sufficient movement of the coordinate system to bring everything into tolerance.

We can conclude that the decision-making rules for modern production environments have become more complicated. However, they do bring economic benefits by reducing the cost of production and reducing the amount of scrap.

**GO and NOT GO hard gauges (limit gauges)**

A hard gauge is a mechanical artefact of high precision used for checking a part. The word gauge is often incorrectly used to describe a measuring instrument, for example, pneumatic comparators are often referred to as air gauges.

A system of limits and fits leads to the use of limit gauges with which no attempt is made to determine the size of the workpiece. They are simply used to determine if the component is within the specified limits. They are generally used for inspecting holes or shafts.

Considering the gauging of a hole:

- A GO gauge is a cylinder whose diameter is equal to the minimum hole size.
- A NOT GO gauge is a cylinder whose diameter is equal to the maximum hole size.

That is to say the GO gauge checks the maximum material condition and the NOT GO gauge the minimum metal condition.
A NOT GO gauge is generally shorter than the GO gauge. The length of the GO gauge ensures that the maximum material condition is not exceeded due to geometric errors in the workpiece (for example, straightness).

This is a rather simplistic description as nothing can be made perfect and that includes the limit gauge! Thus the gauge maker requires a tolerance. You should also be aware that the gauge design should be considered in terms of the functional reason for the tolerance and should take into account Taylor’s Principle. (Where more than one dimension is to be checked the GO gauge should check all dimensions at the same time whilst a NOT GO gauge should only check one dimension. This is known as Taylor’s Principle.)

**Multiple gauging stations and master parts**

John was thinking back on some of the work he had been involved in over the years.

He was in particular thinking about some work he did overseas for a sub-contractor to Australian Luxury Vehicles or ALV. The sub-contractor assembled the differentials for the 4 series ALV and this involved some very tight tolerances on the shop floor and the use of banded tolerances and selective assembly together with some controlled deformation of the walls of the differential casing to generate the appropriate preload of the bearings.

The total tolerance was 0 to 30 µm and the bands were 5 µm wide. A CMM was able to establish the dimensions of the casings to about ± 2 µm. This statement was verified by measuring appropriately sized and positioned ring gauges that simulated the drive gear and output shaft bearing seats, together with a selection of length bars that established the scale factor in each axis and ensured traceability of measurement to national standards.

The requirement was to control the degree of engagement of the drive gear with the crown gear so that the differential ran extremely quietly – a desirable feature in a luxury motorcar!

The assembly process involved measuring the distance between the front face of the drive gear and the centreline of the two output shafts. To define the centre line of the output shaft, a dummy shaft was inserted into the differential casing and two tapered plugs centralised the dummy shaft in the output bearing seats. A sensitive dial indicator was then used to determine the distance to the bearing seat of the drive gear prior to its insertion. This information was then used to determine what size of drive gear was appropriate – which band was selected.

The selected gear was then fitted and a check performed on the distance from the front face of the drive gear to the centreline, by again inserting a dummy shaft and tapered plugs, and reading the dial indicator – provided the indicator was in the green, then the part could progress down the line – if in the red, then it was removed for reworking.

The master parts were used to set up the dial indicators at the various inspection stations down the assembly line. By knowing what the measured dimensions actually were, the inspectors could set the dial indicators on each of the dummy shafts to the actual size and know that they were transferring the accuracy of the CMM to the simple indicator.
The importance of this approach is that by using several master parts – in this case surplus differential casings - whose dimensions have been characterised by a high accuracy and very expensive CMM in a good environment, the production engineers were able to use dedicated, robust and relatively inexpensive gauges on the shop floor and still achieve the high accuracies required by their customer.

The master part approach allows one to utilise an expensive centralised facility with experienced personnel to achieve the highest possible accuracy and then disseminate that accuracy directly to the shop floor with the minimum amount of effort and expense. It is not a very flexible method, but it is very useful in a production environment where the same measurements are made over and over again.

**Future developments**

In the past measurement uncertainty was ignored and simple rules were used to determine conformance or non-conformance.

Uncertainty was ignored if in production the uncertainty of the measuring instrument was no greater than a third of the tolerance and in inspection if the measurement uncertainty was no greater than one tenth of the tolerance. Today, however, the ISO 14253 rules must apply for inspection.

While early CMMs offered a marked improvement in speed over the old manual inspection methods, they were used to inspect work that was actually produced on fundamentally accurate machine tools, so often there wasn’t a marked superiority of the accuracy of the inspection machine over the production machine.

Developments in computers and mathematical models led to the implementation of error correction schemes in the 1980s, resulting in highly accurate machine tools such as the Large Optic Diamond Turning Machine (LODTM) at Lawrence Livermore National Laboratory which is about as good as it gets – tricky to inspect products from that with a CMM. LODTM can diamond turn normal and grazing-incidence optics to tolerances of 25 nm rms form and 5 nm to 10 nm rms surface texture. Materials that can be diamond-turned such as copper, nickel, aluminium, and plastics are necessary for these tolerances.

In fact it is likely that the next generation of high accuracy machine tools will involve hexapod configurations (six legs that move a platform) that will require custom built metrology schemes to monitor the position of the machine tool tip. NIST, and several machine tool companies in the USA, Germany and the UK have done a lot of ground breaking work on hexapods over the last ten years or so.

A likely candidate for the metrology scheme for both the advanced machine tool and the next generation CMM will be the multilateration schemes currently being developed at the NPL. A simple system of four trackers (see later for a description of a laser tracker) would provide the \(x\), \(y\) and \(z\) co-ordinates of a point of interest, while a six tracker system may find direct application to the hexapod machines as the laser tracker may be incorporated into each strut. A seven tracker system would provide the metrology for a versatile probing arrangement that could access hidden features and provide not only the \(x\), \(y\) and \(z\) co-ordinates of a point of interest but also the angular orientation of an offset probe relative to that point of interest.
Saving money by spending money

IN THIS CHAPTER

- Car parts again.
- Saving money.
- Modern measurement techniques.
- Spending money.
- What happens if things go wrong?
Car parts again

John, had been measuring the new car components for several months now. The new car had been an immediate success and he was now making and measuring hundreds of components. However, from an efficiency point of view it was a pain having to make 99% of the measurements on his CMM and then taking the component to the inspection room CMM for the final measurement of the hole. Wouldn’t it be nice if they had a higher accuracy CMM in production?

John had looked through all the manufacturers’ literature and found a machine with a specification of $0.9 + L/400$. It was just what he needed but it cost £250,000. But how would he justify it to the notoriously careful finance manager?

John sat down and did some sums.

First of all, removing the component from his CMM and setting it up on the CMM in the inspection room was taking five minutes for each component. At £40 an hour this was costing the company £3.33 per component. Given that they were testing 500 components a week this comes to £86,580 over a year.

Then John thought about some of the other dimensions. They were often manufactured close to top limit. Given the current uncertainty of measurement they were scrapped because John was unable to tell conclusively if the component met the specification or not. It was a safety critical component and the company did not want to be sued if it was proved that an accident was its fault. There was that nasty incident sometime back with the Turbo 800 recall that no one liked to talk about, so their reputation was also at stake.

They were rejecting ninety components a week. Now if they had a smaller measurement uncertainty about 25% of these might actually pass inspection. Let’s see at £75 a component that’s twenty lots of £75, i.e. £1500 per week. That’s £78,000 a year. The machine would pay for itself after two years (allowing for service costs, etc.). Added to this would be the time savings. John had the case he needed for his new CMM.

Saving money

A conventional dimensional inspection department in the 1960’s would have used a wide variety of measuring instruments and tools such as vernier callipers and micrometers, comparators, surface tables and height gauges to check basic physical dimensions.

A large number of GO and NOT GO gauges and gap gauges would have been maintained for dimensional control on the shop floor and these would all have to be periodically examined and, if necessary, adjusted by the inspectors in the quality assurance department using combinations of gauge blocks wrung together. These manual processes were relatively slow and required a large number of qualified inspectors to achieve reasonable work throughput.

In particular it was very difficult and time consuming to inspect complex components with multiple features – imagine for instance a gearbox casing or an engine block. The measurement of the diameter, cylindricity, location and alignment of the bores in a V8 engine
would literally take all day to measure manually. These difficulties and their associated costs in terms of the direct labour and the associated equipment led to the development of CMMs.

You may be in a working environment where traditional measurement techniques are still used – a lot of companies take the view that *if it ain’t broke don’t fix it* and they may be blind to the fact that they can save considerable sums of money by investing in new equipment and technology.

A thought to bear in mind too is that in some circumstances old specialised equipment may be able to do a particular job quite well – the example of the measurement of American Petroleum Institute (API) taper screw threads at NPL comes to mind. A room full of traditional measuring equipment and a set of regional master gauges that have been calibrated can be used to calibrate any number of reference gauges. To replace that equipment with a general purpose CMM is actually quite difficult – we know because we tried to. Also old equipment has a lot of historical calibration information associated with it, which could take a long time to build up again.

For many practical industrial applications, however, there will be a selection of co-ordinate measuring systems that suit your requirements.

The manually operated articulated arm CMM (Figure 17 and Figure 18) offers a relatively low accuracy but enormously flexible approach to measurements within the reach of the arm and is not too expensive.

![Figure 17 An articulated arm CMM (Courtesy Faro UK)](image-url)
Conventional CMMs (Figure 19) range in working volume from perhaps a cube of side 300 mm to upwards of 50 cubic metres or more. They are usually more accurate and much more expensive than alternative techniques, but have the advantage that they are usually computer numerically controlled (CNC) and can operate automatically once programmed.

For those with a medium sized budget and a requirement for high accuracy large measuring volumes up to typically 30 m to 35 m radius, laser trackers are ideal and, by careful use of the equipment and due consideration of the environmental factors, good results are possible. For even larger volumes such as football stadiums or inaccessible objects that you can’t touch, laser radar systems are ideal – they are, however, expensive and because they use time of flight techniques they are generally not as accurate as interferometry based systems. For low cost, but still useful accuracy and range one could consider using an infra-red ranging system mounted on a theodolite.
A wide range of vision based systems exist from single and multiple camera based systems offering off-line analysis to the state of the art real-time systems using high resolution digital cameras.

There are also a number of systems that use structured light, for example, a line of light that is swept over a surface and imaged – the $x$ and $y$ ordinates of the line in the image can be used to solve for the depth data.

**Modern measurement techniques**

Before spending your money it is wise to be aware of some of the modern techniques available to the dimensional metrologist. This section includes some tools that were touched upon in the previous section.

**An overview of co-ordinate metrology**

Co-ordinate metrology represents a significant move away from traditional methods such as single axis comparators to point-by-point methods. The NPL Good Practice Guides No’s 41, 42 and 43 cover CMM Measurement Strategies, CMM Verification and CMM Probing respectively and are strongly recommended to newcomers to the field of co-ordinate metrology.

Co-ordinate metrology can be performed by a number of different instruments and techniques but let us for the moment consider only the three-axis measuring machine or CMM.

In its simplest form the CMM is made up of three linear moving axes that allow a contacting spherical stylus tip to move in three mutually orthogonal directions and touch an object to be measured. The position of each of the three axes is noted during the measurement of a single point, corrections are applied by the computer for stylus tip diameter and probing direction and sometimes for the geometric errors of the machine itself. A numerical best fit is calculated that describes the feature being measured in terms of the limited number of data points actually measured. Finally an associated dimension is calculated (for example, by calculating the distance between the centres of two circular features).

CMMs range in size from small workshop machines that can measure within a 300 mm cube to huge machines used in the aerospace industry that may be 20 m or more along the main axis of the machine and 3 m or 4 m in the other two axes. In the automotive industry there are CMMs that have two or sometimes three measuring arms that allow the CMM to measure both sides of a car body and the floor pan at the same time!

The incorporation of a rotary axis to a conventional CMM provides a convenient way of measuring complex objects such as gears, which have rotational symmetry about at least one axis. Scroll pumps and propellers are other examples of challenging shapes to measure.
Articulated arms

An alternative to the linear axis CMM is the articulated arm measuring machine (Figure 20) – this uses a series of rotary joints and arms to generate the freedom of motion to probe any position within a hemispherical workspace. These machines are generally not as accurate as linear CMMs but they have the advantage of being able to probe around corners and into objects and are particularly useful for anthropomorphic studies and the ergonomics of man and machine interfaces.

![Articulated arm CMM](image1)

Figure 20 An articulated arm CMM (Courtesy Romer, part Hexagon Metrology)

The Newport Ship is the most complete surviving ocean-going 15th century ship excavated in Britain in recent years. Figure 21 shows an articulated arm being used to measure one of its timbers.

![Articulated arm measuring Newport Ship](image2)

Figure 21 An articulated arm being used to measure the Newport Ship (Courtesy Faro UK and Newport Ship Project)
Laser trackers

An extremely popular instrument for large-scale co-ordinate metrology, particularly in the aerospace industry, is the laser tracker (Figure 22). Initially developed at the National Bureau of Standards (now NIST) in the USA in the mid-1980s for the calibration of industrial robots, laser trackers consist of a laser interferometer that measures the radial distance to a spherically mounted retro-reflector\(^{11}\) and two angle encoders that measure the azimuth and elevation of the interferometer beam. The resultant spherical co-ordinate system is used to record the co-ordinates of points of interest and then they are typically transformed into Cartesian co-ordinates for the convenience of the operators and designers by the software.

![Figure 22 A laser tracker (Courtesy Faro)](image)

Laser trackers are very good at measuring the radial distance to the retro-reflector but they are not normally as good at measuring the azimuth and elevation angles. This failure results in an unusual effect in terms of the way the accuracy of the system varies with position within the measuring volume of interest. Overcoming these shortfalls essentially boils down to getting as close as possible to the object of interest and positioning the laser tracker so that critical dimensions can be measured with the minimum of angular motion – letting the system rely on the more accurate interferometer reading.

A subsequent development of the laser tracker includes the incorporation of an absolute distance meter (ADM) within the laser interferometer system. The ADM is able to measure the distance between its internal reference and the external retro-reflector without actually tracking the retro-reflector from a known position. This feature greatly enhances the utility of the systems.

\(^{11}\) A ‘fancy mirror’ in the middle of a ball bearing. Retroreflectors are described in appendix D.2.13.
NOTE

Multilateration

For the very highest accuracy work it is helpful to do away with the necessity for measuring angle and to rely on the length measuring ability of a number of different laser trackers. In the classical arrangement demonstrated by NPL (Figure 23), four tracking laser interferometers are used to monitor the position of a spherical retro-reflector. The fourth tracker provides redundant information that enables the system to be essentially self-calibrating.

This approach is expensive – requiring the use of at least four trackers just to establish the position in space of the retro-reflector – but many of the larger aerospace companies have a stable of trackers that runs into the dozens on one site and perhaps hundreds across the company. Assembling four or more for an experiment or a critical measurement is, therefore, relatively trivial – you just don’t read about it in the popular press or scientific journals because it is deemed a commercial advantage and they pretend not to do it.

Figure 23 Multilateration in action

Spending money

Buying any measurement equipment can seem rather a daunting prospect when faced with the multitude of equipment available. To help you make an informed choice you should ask yourself and your potential supplier the following questions.
• **Training** – Does the company include the cost of training in the purchase price? Do you have suitable staff that could make the most from the training offered?

• **Support** – Is free support included or does technical support cost extra?

• **Accurate for the needs** – Is the equipment accuracy suitable for the tolerances involved? Would less accurate equipment perform the task? As a general rule the uncertainty in the measurement result should be less than 10% of the component tolerance.

• **Calibration, traceability, uncertainty** – Can the supplier offer UKAS calibration? If not is the calibration traceable to national standards? How often will you need to get the equipment calibrated? How much will this service cost?

• **Special requirements** – Does the equipment have special requirements in order to achieve the quoted accuracy, for example, temperature controlled room, clean compressed air, vibration free area, etc.

• **General purpose or specific to task** – Is the best solution a general-purpose measuring instrument, for example, a CMM or would a more specific instrument like a gauging fixture perform the task better?

• **Data storage** – Would the ability to store the measurement data to a computer be an advantage and if so is this facility provided?

• **British or International standards** – Are there any standards relevant to the equipment you plan to purchase (for example, ISO 10360 for coordinate measuring machines)? Does the manufacturer claim equipment conforms to these standards?

• **Hidden costs** – What are the hidden costs of ownership, for example, training, spare parts, consumables, servicing, support and calibration.

Finally get other users’ opinions of the equipment you intend to buy either through membership of a network (such as the National Measurement System’s Measurement Network – visit [http://www.npl.co.uk/measurement-network](http://www.npl.co.uk/measurement-network) for more information) or by talking to local companies that use similar equipment.

### What happens if things go wrong?

As John found out with the Turbo 800 incident, if things go wrong they can be embarrassing at best. If you do not spend a little money on your inspection, two things can basically happen

- You end up rejecting perfectly good components
- You end up passing components that do not meet specification

Rejecting good components will not necessarily do your company’s reputation any harm but all those rejected components cost money. You may start chasing problems in your production that don’t actually exist and waste more money. If you then take into account the effect you are having on the environment and all the energy that goes into creating scrap components, things get even worse. Carry on like that and you may soon be out of a job.

Passing components that are not up to specification is even worse. At best your customer will spot the error and complain. This is not only embarrassing but you won’t get paid. If the customer does not spot the mistake, components will fail early giving either you or your customer a bad reputation. Sales will drop. In the worst case the component may fail in such a way that it causes injury or death. You are now in lawsuit territory.
We can also look at the case of a scientist who makes a deduction based on measurements he has made. If those measurements are not made to sufficient accuracy the deductions he has made could be totally false and he may have to retract his findings. Again this will lead to embarrassment and loss of reputation.
Choosing the right equipment for the job

IN THIS CHAPTER

- Plane parts.
- What bits need measuring.
- Choosing the correct tool.
- Temperature compensation.
Plane parts

Choosing the right equipment for the job is extremely important, as we will see from the next short story based around an aerospace company. Let’s introduce the key characters.

**The Production Manager:**
Bill is the Wing Production Manager at Just Another Aircraft Company (JAAC), an aerospace company that manufactures large wing components for the civilian aircraft market.

**The Technologist:**
Earl is a technical apprentice at JAAC, he has GCSEs in Maths, Physics and English and has been working for the company for twelve months during which time he has been studying for his NVQ level 3.

**The Metrologist:**
John has expertise in large-scale metrology and works for the Contract Inspection Agency (CIA). He has a degree in engineering and approximately fifteen years experience of dimensional measurement.

**Monday morning**

Bill came into his office to find a pile of non-conformance forms on his desk all suggesting that the problems that JAAC have been having with the assembled wings failing to fit the fuselage can be attributed to poor dimensional control during production. This was a serious problem and one that could be very costly for the company.

Bill decided to call in some help and gives his old friend John at the CIA a call.

‘Hi John listen up. I’ve got a problem with our main wing assemblies failing to mate up to the fuselage properly. That bozo in the Fuselage Section says it’s our fault, but he won’t let us use their laser tracker to check out the jigs and assembly process, because he says they need it during fuselage assembly.’

‘Well Bill – sounds like you could do with a bit of help,’ replied John. ‘But how about giving me some background information before I come over? Which components are causing the problem and how big are they?’

‘Well I’m not really sure, most of these wing components are produced on-site and are assembled in my section, but the parts I’m most concerned about are the aluminium alloy wing skins that are up to thirty metres long and have a tolerance of ±1 mm,’ replied Bill. ‘We’ll have to measure these while they’re positioned in a huge steel assembly jig and supported by wooden formers. The assembly jig is located in the old ‘B’ hangar next to the main door and there’s a great deal of auxiliary equipment mounted on it – it’s well-populated you might say – so getting access may be a problem. What I want you to do is come over and have a look around, talk to one of the lads on the assembly team and let him explain what they’re doing. I’ve got a good lad in mind – his name’s Earl’.

‘Ok Bill,’ said John ‘I’ll be over after lunch.’
‘Hi John, I’m Earl,’ said Earl introducing himself. ‘Bill has asked me to show you around and to explain what we’re doing during the assembly process. We can start over here by goods-in if you like.’

‘Sure thing Earl, it’s always good to start at the beginning,’ said John with a smile.

‘Are all these components produced on-site?’ John asked.

‘All the wing ribs and the skin sheets are machined next door in the machine shop. They are then inspected, and once accepted they are stored here until we need them,’ replied Earl. ‘All the fasteners though, they are bought-out components.’

‘Ok, I understand that the components coming into this area have been inspected, what about the assembly jigs – are they checked regularly?’ asked John.

‘They’re checked annually by the tooling inspectors during the Christmas shutdown – they use one of their laser trackers to measure the reference dowel holes to make sure nothing has shifted and then they spot check critical parts of the wing jig against the CAD file,’ replied Earl.

‘Alright then, you’ve got inspected components being assembled on a calibrated jig, so what’s the problem?’ asked John

‘Well the problem is that the assembled wing is not mating up to the fuselage properly, but I think there is something wrong with the components coming out of production because you can see from how they lie in the jig that something is not quite right, so we want to measure the overall length of the wing skins in situ,’

‘You’ll be wanting to use a laser tracker then,’ said John ‘It’ll do the lot, measure the co-ordinates of any point in space that you can get the spherically mounted retro-reflector to. It’s quick and accurate too – about 1 part in 100 000 they claim.’

‘That would be more than good enough – we’re only looking for an accuracy of about 1 mm over that length,’ said Earl. ‘But these trackers are about £75k and the boss says we can’t afford that on our tooling budget and we can’t borrow one either, no one wants to let theirs go for long enough to do the job! So I was wondering about one of those infra-red tapes – they’re only about five hundred quid – what do you think?’

‘Hang on there Earl, you might say it’s only 1 mm in 30 m, but a measuring system that could achieve 1 in 100 000 would have an accuracy of only 0.3 mm over 30 m. Even a laser tracker is only just good enough,’ said John. ‘You really want to be able to measure about ten times better than the tolerance you are inspecting. If you look at the specification for those infra-red tapes, you’ll see that their accuracy isn’t good enough for this application – you’ll get an accuracy of maybe 1.5 mm over 100 m. Now whilst it’s tempting to say that that’s equivalent to 0.5 mm over 30 m it’s not, there is usually a constant and a length dependent term in the uncertainty statement and your tolerance is only 1 mm over 30 m,’ says John.

‘I appreciate that money is tight and the tracker may not be an option,’ continued John. ‘But there are a number of different techniques that we can use to measure the size of large objects – have a look at this chart I brought along,’ (Figure 23).
‘What you can see on the bottom of the chart is a logarithmic scale describing the size of the object and up the left hand side is the measurement uncertainty that you need to achieve. So if we look at a range of 30 m and an uncertainty better than 1 mm you can see that we could be using a bunch of different co-ordinate metrology techniques such as laser trackers, theodolites, photogrammetry and very large CMMs. All of these systems are capable of up to 1 part in 10^5 on a good day, and in some cases can do just a bit better for certain measuring tasks. For instance, budgeting in a laser tracker and using essentially only the laser interferometer to make the measurement avoids the accuracy limitations imposed by the angle encoders.’

‘They are all too expensive for this application – but do you see that Invar tape? That could do the job,’ said John with a smile. ‘However, you have to remember a few basic things – the most important of which is thermal expansion – most materials expand when you heat them up, some more than others. You probably already know that steel has a linear coefficient of thermal expansion of around 11 ppm/°C and aluminium alloys are around 23 ppm/°C. Here are some expansion coefficients of other materials,’ said John, producing another table (Table 3).

**Table 3 Typical expansion coefficients (ppm/°C) from John’s table**

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient (ppm/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invar36</td>
<td>1.2</td>
</tr>
<tr>
<td>Carbon Steel</td>
<td>11</td>
</tr>
<tr>
<td>Aluminium Alloy</td>
<td>23</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>18</td>
</tr>
</tbody>
</table>
'These expansion coefficients are usually represented by the symbol, $\alpha$, pronounced alpha.'

### NOTE

**Uncertainty in expansion coefficients**

It is rare to really know what the coefficient of linear thermal expansion is as it is quite difficult to measure. Therefore, it is common practice to take a value for a given material from an engineering data book and to assume at least a 10% uncertainty in that figure.

‘Ah,’ said Earl, ‘I remember this from college now. The standard reference temperature for reporting the size of an object is agreed by international convention – ISO 1 - to be 20 ºC. Therefore, you must make a correction back to 20 ºC if the object being measured was actually at some other temperature. Here are some equations from my notes!’ Earl wrote the following two equations on the back of an envelope

$\delta l = \alpha(T - 20)l$ 

and 

$l_{20} = l_T - \alpha(T - 20)l_T$

### NOTE

**Help equations!**

Don’t worry about the equations yet. We will cover them at the end of the story. If you are really interested and cannot wait, $l$ stands for length, $T$ for temperature. The $\delta$ or delta refers to a change, in this case a change in length.

‘Yes,’ said John. ‘Now we can apply your equations here. For an aluminium wing skin that is nominally 30 m long at 20 ºC, the expansion would be approximately $30(1 + 0.000\,023)$ per degree Celsius in other words $0.69$ mm of your tolerance band is absorbed by every degree Celsius. On a hot summer day at 30 ºC this would be approximately 7 mm longer while on a cold winter morning at 10 ºC this would be approximately $7$ mm shorter.

‘Now,’ said John. ‘An Invar tape can be calibrated by a suitable accredited laboratory to about 2 parts in 1 000 000, in other words to about $0.06$ mm over 30 m! The trick is to use the tape under the same conditions that it was calibrated at. This is really just supporting it over rollers, hanging in a catenary and with the right tension – you also need a couple of people to operate it!’

‘Now if we measure the length of the wing skin on the leading and trailing edges we can see that they fall within specification. Let’s just check the diagonals too. Oh, that’s interesting! One diagonal is longer than it’s supposed to be and the other is shorter than it’s supposed to be. This is beginning to look like squareness error to me.’

Let’s look at an example of the point John was trying to make (Figure 25). Let’s assume this wing skin is a rectangle 30 m x 6 m. Assuming that the skin is square then both diagonals will be $\sqrt{(30^2 + 6^2)} = 30.594117$ m. If we then imagine shearing one
of the 6 m long sides across 1.45 mm relative to the other short side, we create a parallelogram with diagonals that would be 30.593832 m and 30.594402 m. The difference between the diagonals of 0.57 mm is created by the squareness error of approximately 10 seconds of arc (0.00277777 degrees).

In practice what this means is that if you measure carefully the 30 m leading and trailing edges and the shorter 6 m lengths at either end, the component could pass inspection, but, due to the squareness error, you will build a distorted wing.

If you go a little further into this problem, then the things to look at are the calibration of the machine tool that produced the wing skin – has squareness actually been measured and corrected? Another culprit could be the inspection system used to verify the wing skin. It is not unheard of for a machine shop to manufacture something wrongly, just to get past inspection!

**What bits need measuring**

When faced with an engineering drawing and a component, the first question to ask is ‘What bits need measuring?’. Once you have identified from the drawing the features that need to be measured you need to choose the appropriate tool for each measurement. The following section will explain how the appropriate tool is chosen.

**Choosing the correct tool**

Choosing the correct tool for any measurement job requires the consideration of many factors. These factors include:
The drawing tolerance
Ease of measurement
Speed of measurement required
Multi-purpose or dedicated measurement equipment?
The cost of the proposed solution

As we saw earlier, the drawing tolerance allows us to put a limit on the uncertainty of measurement of the equipment we can use. This will limit the number of tools available for measurement. Do not be tempted to use the most accurate equipment available as this may add to cost, may not be as quick and could therefore be a waste of a valuable resource.

NOTE

As good as you can get
NPL are often asked to make measurements on components. When the person taking the enquiry asks the caller for the desired uncertainty, the reply often comes back ‘as good as you can get’. As good as you can get usually costs ten times more than as good as it needs to be!

For instance it is not worth tying up a £100,000 CMM making measurements that could be just as easily made with a £3,000 height gauge.

Speed of measurement is often important. Will you use manual methods such as height gauges, gauge blocks, etc. (suitable for low volume work) or CMMs and dedicated gauging fixtures (more suitable for high volume work)?

CMMs have the advantage that they can be reprogrammed should the product design change, however, in some circumstances a dedicated gauging fixture may be a quicker and more convenient way of making the measurements.

Knowing the capabilities of different types of equipment will guide you as to which is appropriate for a given measurement. However, this very much depends on you understanding the manufacturer’s specification and the calibration certificate for the instrument. Armed with this information and the drawing tolerances you should be able to make an informed decision.

Instruments can be specified in many ways including:

- International specifications (ISO)
- National specifications (for example, BS, DIN ASME, MOY/SCMI)
- Manufacturers’ specifications
NOTE

MOY/SCMI?
The MOY/SCMI (Metrology/Specification Certification Measuring Instruments) standards are complementary to the standards issued by the British Standards Institute (BSI). The majority relate to measurement equipment of a proprietary kind designed either at NPL or by British manufacturers that, in the ordinary way, would not fall within BSI's terms of reference.

See the resources section within http://www.npl.co.uk/science-technology/dimensional/ for more details.

The first two types of specification are probably the easiest to interpret and are more useful for making direct comparisons. The specifications will outline test methods and if two manufacturers make claims based on the same specifications then a direct comparison can be made.

For some types of equipment, only manufacturers’ specifications will exist. These are more difficult to interpret as different manufacturers will have different test methods and will express the results from these tests in different ways. They will quote terms such as accuracy, resolution, repeatability and error. They may give a maximum value for these parameters over a range or they may quote a length dependent equation that applies over the range. In these circumstances great care needs to be taken in making comparisons.

Another point to take note of is that a temperature range will often be specified over which the error bounds are valid. You cannot rely on the specification if you are operating outside the specified temperature range.

NOTE

Length proportional specification
An item will often be specified in the following manner

Error = 0.5 + L/400 µm (where L is in mm)

The error at any length L is calculated from the formula, for instance at L = 400 mm the error is 1.5 µm. In general terms this is often expressed as

Error = A + L/B µm

Alternatively the error will be expressed as 1 % of reading. If the error of a length measuring transducer is expressed this way and the reading is 0.01 mm, then the error is 0.000 1 mm.
Temperature compensation

One of the largest sources of error in any measurement process is the influence of temperature on both the part being measured and the measuring system itself!

The coefficient of linear thermal expansion describes the tendency of most materials to get longer as they are heated and shorter when they are cooled. Metals in particular are prone to do this.

The amount that a metal expands depends on its internal composition and is expressed in terms of the amount of change, per degree Celsius, per unit length (coefficient of linear thermal expansion). Thus a typical steel bar of nominally 1 m length may expand by 11 µm for every degree Celsius it is heated up. The coefficient of linear thermal expansion may then be expressed as 11 parts per million per degree Celsius, or 11 ppm/°C or more correctly 11 x 10⁻⁶ C⁻¹.

All dimensional measurements should be reported at a standard reference temperature of 20 ºC as defined in ISO 1. ISO 1 Geometrical Product Specifications (GPS) – Standard reference temperature for geometrical product specification and verification defines the standard reference temperature for all dimensional measurement. GPS standard ISO 1 states:

*The standard temperature for geometrical product specification and verification is fixed at 20 ºC.*

There are numerous stories as to why this particular temperature was chosen, but the one we like suggests that as Johansson’s basement workshop in Eskilstuna, Sweden stayed at this temperature all year round, he made and calibrated his gauge blocks at this temperature. As the only supplier of gauge blocks for over twenty years, people got used to working with gauge blocks whose sizes were referenced to 20 ºC and when international standardisation was proposed in 1932 it was reputedly a very short meeting at the BIPM that said lets adopt 20 ºC as the standard and adjourn for lunch!

When making dimensional measurements you will reference the measurements to 20 ºC in one of the following ways:

- By making measurements in a temperature controlled room at 20 ºC;
- by comparison with known artefacts of similar material at a temperature close to 20 ºC;
- by using a measuring machine that measures the component temperature and makes appropriate corrections;
- by making the measurement at some other temperature and making manual corrections.

*Minimising the effects of temperature*

There are many ways to minimise the effects of temperature, these range from expensive solutions such as temperature-controlled rooms to simple solutions like wearing gloves. Two easy ways to minimise the effect of temperature are to take care when handling the artefact and to ‘soak’ the item properly.
Handling

When you handle an item that is of a lower temperature than you, you will invariably heat it up. You can avoid this heating effect by using gloves, tweezers or tongs.

Soaking

If the component that you are going to measure is at a temperature above ambient, either because it has been handled, has recently been delivered or has recently been machined, it needs to be left to ‘soak’ to the ambient temperature. Placing the item on a steel block or surface table is the normal means of soaking (Figure 26).

Typical soaking times could be two hours for small objects in good thermal contact with a flat surface such as a surface table. Anything weighing more than 1 kg should be left at least four hours to soak and preferably overnight. The closer you want the item temperature to be to the laboratory temperature, the longer it will take. If in doubt, affix the probes from an electronic thermometer to both the test item and the soaking plate and wait until the temperatures are the same.

![Figure 26 Soaking a plug gauge](image)

NOTE

Soaking
The key concept is to judge the mass of the object and allow enough time for the exponentially decreasing temperature difference to drive the heat across the thermal boundary from the heat source/sink to the object to be measured. Thermal conductivity is of importance as are the thermal masses of both the object and the heat sink.

Correcting for deviations from the reference temperature

As we have just said in the previous section, how much a material changes size for a given temperature change is known as the coefficient of linear thermal expansion. For a typical material such as steel this is expressed as $11.6 \times 10^{-6} \text{ C}^{-1}$. To correct a length to 20 °C use the following equation:
\[ L_{20} = L_T + (20 - T)\alpha L_T \]

where \( L_{20} \) is the length at a temperature of 20 °C, \( L_T \) is length at temperature \( T \), \( T \) is temperature at which the length was measured and \( \alpha \) is the expansion coefficient.

We will now go through a worked example. We have measured the size of a steel bar to be 300.015 mm at a temperature of 23.4 °C. We will need to look up the expansion coefficient of the steel used in the manufacture of the bar. To calculate the size at 20 °C we enter the data in to the equation above, \( i.e., \)

\[ L_T = 300.015 \text{ mm} \]
\[ T = 23.4 \text{ °C} \]
\[ \alpha = 11.6 \times 10^{-6} \text{ °C}^{-1} \]

\[ L_{20} = 300.015 + (20 - 23.4) \times 11.6 \times 10^{-6} \times 300.015 = 300.003 \text{ mm}. \]

The length of the bar at 20 °C is 30.003 mm. As we will see later, the corrected length has an uncertainty due to the uncertainty of the thermometer used to measure the bar temperature and the uncertainty in the measurement of the expansion coefficient.

**Table of errors for typical materials and temperatures**

Table 4 gives an indication of how a 100 mm long bar made from various materials would expand at temperatures about 20 °C varying from 1 °C to 10 °C.

<table>
<thead>
<tr>
<th>Material</th>
<th>Expansion coefficient</th>
<th>1 °C</th>
<th>2 °C</th>
<th>3 °C</th>
<th>4 °C</th>
<th>5 °C</th>
<th>10 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>23.6 ppm</td>
<td>0.00236</td>
<td>0.00472</td>
<td>0.00708</td>
<td>0.00944</td>
<td>0.01180</td>
<td>0.02360</td>
</tr>
<tr>
<td>Steel</td>
<td>12.0 ppm</td>
<td>0.00120</td>
<td>0.00240</td>
<td>0.00360</td>
<td>0.00480</td>
<td>0.00600</td>
<td>0.01200</td>
</tr>
<tr>
<td>Invar</td>
<td>1.0 ppm</td>
<td>0.00010</td>
<td>0.00020</td>
<td>0.00030</td>
<td>0.00040</td>
<td>0.00050</td>
<td>0.00100</td>
</tr>
<tr>
<td>Tungsten Carbide</td>
<td>4.0 ppm</td>
<td>0.00040</td>
<td>0.00080</td>
<td>0.00120</td>
<td>0.00160</td>
<td>0.00200</td>
<td>0.00400</td>
</tr>
<tr>
<td>Brass</td>
<td>20.0 ppm</td>
<td>0.00200</td>
<td>0.00400</td>
<td>0.00600</td>
<td>0.00800</td>
<td>0.01000</td>
<td>0.02000</td>
</tr>
<tr>
<td>copper</td>
<td>16.0 ppm</td>
<td>0.00160</td>
<td>0.00320</td>
<td>0.00480</td>
<td>0.00640</td>
<td>0.00800</td>
<td>0.01600</td>
</tr>
<tr>
<td>Bronze</td>
<td>18.4 ppm</td>
<td>0.00184</td>
<td>0.00368</td>
<td>0.00552</td>
<td>0.00736</td>
<td>0.00920</td>
<td>0.01840</td>
</tr>
<tr>
<td>Titanium</td>
<td>8.4 ppm</td>
<td>0.00084</td>
<td>0.00168</td>
<td>0.00252</td>
<td>0.00336</td>
<td>0.00420</td>
<td>0.00840</td>
</tr>
</tbody>
</table>

Note that the expansion coefficients given in the above table are approximate only and are used solely to indicate orders of magnitude of expansion for given temperatures.
NOTE

**Temperature units**
The SI unit of temperature is the kelvin, symbol K. You will, therefore, usually see, for instance, $11 \times 10^{-6}$ °C$^{-1}$ written as $11 \times 10^{-6}$ K$^{-1}$. Since industrial temperatures are usually measured in Celsius we have kept to expressing expansion coefficients in degrees Celsius.

**Allowing for temperature effects in uncertainty**

Depending on a number of factors, such as good thermal contact and shading from direct sunlight, a mercury-in-glass thermometer may achieve ± 0.1 °C but can be difficult to read without disturbing. A thermocouple may achieve ± 0.02 °C and a platinum resistance thermometer may achieve ± 0.005 °C. You also have to consider the influence of thermal gradients on a large object – usually the top is hotter than the bottom and typical values are 1 °C m$^{-1}$ to 2 °C m$^{-1}$. In addition there may be lateral thermal gradients across a structure as a result of heat radiating from a window or a steel door. Lastly you have to consider the difference between the surface temperature of an object and the core temperature – think of a large cylinder that has been machined on a lathe – the core is hot, but the surface cools quite quickly, you measure it and find the size correct, you let the job cool and check it only to find that it is undersize. The solution here is to leave the job to stabilise or to put it in a temperature controlled bath or air shower before measuring it. We will cover this later.

As stated earlier, when calculating an uncertainty budget you need to take into account the effect of temperature. Two main sources of uncertainty that need to be taken into account are:

- The uncertainty in the temperature measurement system
- The uncertainty in the knowledge of the expansion coefficient

These two contributions will now be examined in more detail.

**Uncertainty in temperature measurement system**

If you look at the calibration certificate for your temperature measuring system you will see two important values:

- The error in the thermometer at the temperature you are measuring at
- The uncertainty in the determination of this error

You may also be using the thermometer in one of two ways:

- By applying the errors in the thermometer
- By ignoring the errors in the thermometer
Case 1 Applying the errors in the thermometer

Let’s start by taking an example where you have applied the known errors in the temperature measurement system in the measurement of a 100 mm block of steel.

You look at the calibration certificate and see the following values (Table 5).

<table>
<thead>
<tr>
<th>Temperature ºC</th>
<th>Error ºC</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.0</td>
<td>+ 0.10</td>
</tr>
<tr>
<td>19.0</td>
<td>+ 0.15</td>
</tr>
<tr>
<td>20.0</td>
<td>+ 0.13</td>
</tr>
<tr>
<td>21.0</td>
<td>+ 0.08</td>
</tr>
<tr>
<td>22.0</td>
<td>+ 0.03</td>
</tr>
</tbody>
</table>

The uncertainty of measurement \((k = 2)\) is given as 0.03 ºC.

You have made the measurement at 21.0 ºC and made the appropriate correction. However, there is an uncertainty in this length correction due to the uncertainty in the temperature determination.

The uncertainty contribution will be

\[ \frac{0.03}{2} \times 100 \times 12 \times 10^{-6} = 0.000 \text{ 018 mm}, \]

where we have divided by two, the value of the \(k\) factor.

Case 2 Ignoring the errors in the thermometer

If we ignore the calibration of the thermometer when making temperature measurement we can calculate the uncertainty as follows.

Firstly we know that our room is controlled to 20.0 ºC ± 2 ºC. We can, therefore, state from Table 5 that the maximum error in temperature measurement is likely to be 0.15 ºC (the value at 19 ºC). Making the assumption that any given temperature in the range is equally likely we can say that this error has a rectangular distribution. For a rectangular distribution we divide by the square root of three (see Bell S A 2001 A beginner’s guide to uncertainty in measurement Measurement Good Practice Guide No. 11 Issue 2 (NPL)). The contribution to the uncertainty budget this time is

\[ \frac{0.15}{\sqrt{3}} \times 100 \times 12 \times 10^{-6} = 0.000103 \text{ mm}, \]

where we have divided by the square root of three, for a rectangular distribution.
We also still have to include the contribution listed in case 1 above.

*Uncertainty in expansion coefficients*

It is rare to really know what the coefficient of linear thermal expansion with any certainty is as it is quite difficult to measure and varies with batches of material. Therefore, it is common practice to take a value for a given material from an engineering data book and to assume at least a 10% uncertainty in that figure.

To take an example, if we consider the same 100 mm steel bar used in the previous examples. We can assume that the expansion coefficient is known with an uncertainty of $1 \times 10^{-6} \, ^\circ\text{C}^{-1}$. We can also assume that it has a rectangular distribution, *i.e.* the true value is equally likely to lie anywhere in the range $11 \times 10^{-6} \, ^\circ\text{C}^{-1}$ to $13 \times 10^{-6} \, ^\circ\text{C}^{-1}$.

The contribution to the combined standard uncertainty will be,

\[
\frac{1 \times 10^{-6}}{\sqrt{3}} \times 1 \times 100 = 0.000103 \, \text{mm}.
\]
Performance verification - how good is this instrument anyway?

IN THIS CHAPTER

- Car parts yet again.
- How do you check out what’s in your toolbox?
- Formal verification of a CMM.
- What if things are not as they should be?
- Verifying a roundness measuring instrument.
- Verifying articulated arm CMMs.
- Verifying laser trackers.
- Verifying rotary.
- Formal verification of a machine tool.
- Reorientation and repeat measurement.
- Diagnoses.
Car parts yet again

John had spent a couple of years working in the South Korean plant. Now back in the UK, his old inspection area looked very different.

‘Look John,’ said Sam ‘it’s a portable CMM. It’s ideal for all those measurements of larger panels that we find quite difficult at the moment. We can do all kinds of clever stuff with the software and do all sorts of statistical analysis. Look if I press this…’

‘Hold on, hold on,’ said John. ‘But how accurate is it?’

‘Well the manufacturer says it’s good to 0.0005 inch, that’s 0.013 mm in my language.’

‘Hang on Sam,’ said John, ‘how are you going to verify that?’ Sam went slightly red.

‘What did they tell you on your first day?’ Sam thought for a while.

‘I know I can get the step gauge from the CMM and measure that in several orientations, I can also measure a known sphere and do something like the ISO 10360 probe test and I can …’

John walked away confident that he had left the inspection department in good hands.

Figure 27 An articulated arm like the one Sam was using (Courtesy Faro UK)

How do you check out what’s in your toolbox?

Performance verification is what you do to ensure that you have confidence in something. Calibration on the other hand is the process of determining the errors associated with an instrument and is a much more detailed process.

Verification of an instrument usually involves making measurements on artefacts of known size, for example, gauge blocks. However, before making formal verification measurements there are a number of visual verification checks you can make.
**Visual verification**

John was cleaning out his drawers when he came across that old micrometer he used to show the apprentices as an example of how to spot a mistreated micrometer. The paint was all chipped which was an instant giveaway and the graduations were difficult to read. The anvils had gone rusty where they had not been cleaned properly. There was also the steel rule that had a burr on the end. He carefully painted them in a fluorescent paint and put them in labelled bags ready to hand over to Sam.

There are a number of different things you can do to give yourself confidence in an instrument before you use it. The most obvious ones are to look at it - is the instrument clean and does it look well cared for? If it’s rusty or dirty the chances are it has been neglected or abused and should be viewed with suspicion until verified or repaired if that proves necessary.

A sticking dial indicator is another common problem that can be easily detected. A slight push on the anvil should not result in a sluggish response from the indicator.

A vernier calliper can be checked for wear by eye – close the jaws until they are nearly touching and hold the vernier up to a strong light source – you should not be able to see any light shining through the gap.

If using a CMM with a contact probe – is the reference sphere and probe tip clean and free from dust, is there any wear or are there flat spots on the ball? Check using an eyeglass or a microscope; you may be horrified at what you see stuck on the surface of the ball.

Check reference standards such as gauge blocks. Look out for burrs, scratches and corrosion marks. Pay particular attention to the reference surfaces. Minor corrosion on the handle or other non-measuring faces, whilst undesirable, may be acceptable but measuring faces should be undamaged.

<table>
<thead>
<tr>
<th>NOTE</th>
</tr>
</thead>
</table>
| **Have you done your exercises?**
A useful habit to acquire is that of exercising the measurement axis or axes of an instrument before you use it. A vernier calliper can be opened and closed quite easily to ensure that the jaws move smoothly, similarly a dial gauge or electronic transducer can be carefully exercised over its more limited range. A three-axis co-ordinate measuring machine or articulated arm can be exercised over its working volume to ensure that the scales and encoders are working correctly and that the motion is smooth. A stiff, noisy or irregular motion is indicative of a fault that may lead to erroneous readings and poor measurements. |

**Formal verification checks**

Once visual checks are complete you can make more formal checks. Start by checking the zero of the instrument – for a micrometer simply bring the anvils into contact
using the ratchet as normal and observe the reading, it should be zero – if not clean
the anvils again and if necessary adjust the setting. Obviously when using the larger
range of micrometers say 25 mm to 50 mm or 50 mm to 75 mm, etc. you will be using
the calibrated setting piece to check the ‘zero’ of the instrument. See Flack D R 2001
Callipers and micrometers Measurement Good Practice Guide No. 40 (NPL) for more
information.

Now for a really useful verification - it is helpful to select a known object similar in
size to that which you are going to measure. Gauge blocks are very useful – having
accurately known sizes over a wide range – 1 mm to 200 mm. For verifying larger
objects by comparison, long series gauge blocks and length bars are available in sizes
up to 1500 mm in steps of 100 mm. Measure the known object and check that you
agree with the calibrated size.

Bore micrometers measure internal diameters and so should be checked against
similar objects such as ring gauges (plain setting rings) – one is usually provided with
the instrument for set up purposes, but a second ring nearer the size of interest can be
useful.

**NOTE**

**Repair or scrap?**

Some instruments such as micrometers can be adjusted or repaired but others such as vernier callipers should just be scrapped – they are not that expensive to buy new anyway and it is certainly not worth the risk of producing expensive scrap by leaving a faulty instrument lying around to be used by the unwary!

**Formal verification of a CMM**

An international standard exists for the formal verification of CMMs. ISO 10360
details the use of bi-directional length artefacts such as length bars and step gauges to
establish the length measuring capability of a CMM in a number of different positions
and orientations. The basic idea of the specification standard is to allow
manufacturers to have a common means for specifying their machines. It defines a
pass off test and also describes how users can monitor the performance of their own
CMM.

As ISO 10360 states:

The principle of the assessment method for size is to establish whether the CMM is
capable of measuring within the stated maximum permissible error of indication of a
CMM for size measurement, $MPE_E$. The assessment shall be performed by
comparison of the calibrated values with the indicated values of five different material
standards of size, each of a different length. The five different material standards of
size shall be placed in seven different locations or orientations or both, in the
measuring volume of the CMM, and measured three times, for a total of 105
measurements.
An existing NPL Measurement Good Practice Guide, *CMM Verification Measurement Good Practice Guide No. 42* (NPL) covers the performance verification of CMMs and is recommended reading for anyone concerned with the operation of a CMM. The international standard ISO 10360 – *Geometrical product specifications (GPS). Acceptance and reverification tests for co-ordinate measuring machines (CMM)* comes in more than five parts. Part two deals with CMMs used for measuring size.

**Figure 28 ISO 10360 test**

**NOTE**

**Ball bars**

It should be noted that a ball bar is not a recommended artefact because measuring a sphere with a CMM tends to mask any probing errors associated with the probe tip radius and stylus bending. ISO 10360 consists of length measurements on material standards of size.

Ball bars are allowed for making interim checks. A ball bar is a very useful and cheap to make artefact – two precision balls attached to a central bar. A high accuracy CMM or a single axis comparator can calibrate the distance between the two balls. The form error of precision balls is very small and in most industrial situations will be much smaller than the probe repeatability of the CMM you are trying to verify. The disadvantage of the ball bar is that it does not pick up errors related to the probe, stylus or qualification sphere.
ISO 10360 suggested that one of the five test lengths should be less than 30 mm and that another should be at least 66% of the space diagonal. One possible pattern of positions and orientations would be to position the five length bars in the mid volume of the machine, initially aligned with the $x$, $y$ and $z$ axes, and then the four compound diagonals. That would allow you to measure each length bar three times in each position – for a total of 105 length measurements. Figure 28 shows a step gauge being measured in one of the seven locations.

However, users of a CMM should consider what is important to them and may find that performing checks closer to the bed of the machine and in the $xy$ plane of the table may be more representative of what they do on a day-to-day basis.

The CMM operator can perform the ISO 10360-2 test, you don’t need to call in a specialist service firm, unless you discover that the machine is out of specification and want to have the error corrected. The ISO 10360-2 test explicitly calls for the use of material standards of length containing two or more nominally parallel planes, thus forcing you to use either length bars, gauge blocks or step gauges.

A step gauge may cost approximately £5,000 and a calibration might cost around £1,000, a recurring expense that should be undertaken at least annually for a couple of years and then every other year once the stability of the gauge has been proven.

Traditionally the specification for a CMM was usually expressed in terms of its ability to perform three dimensional length measurements. The maximum error was reported as $MPE_E = A + B/L \, \mu m$ where $A$ is a constant usually associated with probing effects and $B$ is a length dependant term that is associated with thermal effects, machine geometry and scale factor errors.

A typical machine specification might therefore be ‘$MPE_E = 2 + L/250 \, \mu m$ where $L$ is in mm.’ This means that for a small object or feature a dimension may be measured with a maximum error of 2 $\mu m$, while for a 1000 mm object, the maximum possible error would be 6 $\mu m$.

To evaluate the probing errors, one measures a high precision sphere, the test sphere, (not the reference sphere mind you) with 25 points in a random order but evenly distributed over a hemisphere. (A suggested point distribution is included in the latest version of the standard).

The CMM software is used to calculate the centre of the measured sphere and then evaluate the largest deviation of the measured points from that centre; this value should be less than the maximum permissible probing error, $MPE_p$, quoted by the manufacturer.

If the measured sizes for all 105 length measurements lie within the limits defined by the maximum permissible error of indication of a CMM for size measurements, $MPE_E$, and the probing error is less than $MPE_p$, then the machine is considered verified.
A maximum of five of the thirty-five measurements may have one of the three replicate values outside the conformance zone (ISO 14253-1). Each such test length shall have the measurement that is out of tolerance repeated ten times at the relevant configuration. If all ten subsequent measurements lie within the conformance zone then the machine has been verified.

### NOTE

**ISO 10360 length measurement results**

It is helpful to check these length measurement results as you conduct the ISO 10360 test so that you have not moved the artefacts before discovering you need to do the ten repeat measurements.

---

**What happens if the verification fails and the CMM is outside specification?**

If the CMM fails the performance verification check, it may not be ‘the end of the world as you know it!’ In fact the phrase ‘fit for purpose’ was coined to describe situations like this. If your high accuracy CMM is not meeting the specification it was built to, but you are only inspecting components with undemanding tolerances then you are at liberty to revise the specification – to one that you have just verified the machine to for instance. Thus a machine specified as a 5 µm machine may only be capable of achieving 8 µm, but if it was being used to inspect components with a tolerance of 25 µm it would be acceptable to relax the specification and say that the machine had been verified to 8 µm. (The impact of such a decision on the acceptance band as described by ISO 14253 is discussed in chapter 3.)

**Interim checks**

The regular performance verification check of a CMM should be a key component of the company quality system and records should be kept of the results obtained. Unfortunately these tests may only be performed once or twice a year – so what do you do to ensure you have confidence in the machine on a daily basis? The answer is to perform interim checks. These checks are also described in the ISO 10360-2, but essentially involve regularly measuring an object that is similar to the regular components inspected by the machine – this may be a special test piece, a calibrated master part, ball plate, hole plate, machine checking gauge, or ball bar. By measuring a series of features on this artefact each day, or perhaps first thing on a Monday morning and recording the sizes on a control chart, one builds up a visible history of the machine’s capability.

Trends may be observed on the control chart indicating that the results are within some specified limit and likely to stay within the bounds for some time, but the real benefits are observed when sudden changes are highlighted. Sudden changes are often indicators of nasty things happening to the machine – damage to an air bearing, a collision with a forklift or crane, or simply dropping a heavy object - sometimes people are embarrassed to admit they have damaged an expensive item and may not
say anything about it. It is up to the operators to ensure that they are confident in the results from the CMM and getting the result they expect from a daily interim check.

If the CMM fails the interim check then the operator is encouraged to undertake a performance verification test to see if the system really is out of specification.

**What if things are not as they should be?**

If things are not as they should be the more complicated instruments, such as CMMs, can be adjusted, serviced, repaired and calibrated.

**Adjustments**

Adjustments typically involve the setting of air bearing pad ride heights and ensuring the squareness of the axes to one another. The manufacturer or his designated agent usually performs these adjustments. Adjustments may also include fine-tuning the error map.

**Servicing**

Servicing may involve changing the air filters and electric motor brushes of the machine and other items that are subject to wear during the normal operation of the machine.

**Repair**

Repair usually indicates that a component such as a probe head or indexing head needs replacing and these are usually factory exchange items.

**Calibration**

Calibration involves attempting to characterise the geometric errors of the machine and is normally only done by the manufacturer. The amount of work involved depends on the level of sophistication of the error correction implemented in the machine. Typically one attempts to measure and, if applicable, correct for all twenty-one parametric errors of a three axis CMM. The pitch, yaw and roll of each axis is measured and corrected, then the straightness errors and scale errors of each axis. Finally the squareness errors of the three orthogonal planes \(xy\), \(xz\) and \(yz\) are determined. The procedures involved vary from manufacturer to manufacturer and often the access to the error correction software is protected by a password known only to the manufacturer or his designated agent. Thus the only way that one can be sure that the error compensation update has been done correctly is to run through a complete performance verification check such as the ISO 10360-2 described earlier.

The instruments used to measure the geometric errors of machine tools and CMMs include laser interferometers (discussed in chapter 9) for measuring displacement, rotations and straightness; autocollimators, and differential electronic levels. Some useful accessories include granite squares, straight edges, dial indicators and LVDTs.
Verifying a roundness measuring instrument

If you are measuring roundness there are several interim checks that you can perform to verify the instrument’s performance. The easiest way to verify the spindle is to measure the hemisphere that came with the instrument to see if your results agree with the calibration certificate. Differences may indicate a problem with the instrument spindle. It is also a good idea to regularly check the gain of the sensor by using a sensitivity artefact such as a flick standard. A flick standard is a cylindrical artefact with a small flat ground into it.

Verifying articulated arm CMMs

As Sam found out at the beginning of this chapter, there is a need to verify the performance of articulating arm CMMs.

Articulated arm CMMs are extremely versatile and portable instruments – they may not be as accurate as some of the other co-ordinate measuring techniques, but they are a very handy addition to the metrologist’s toolbox.

A simple check of an articulated arm CMM would be to place the hard probing ball at the measuring end of the arm in a kinematic socket. The next stage is to measure a series of points as the arms and wrists are manipulated to change the orientation of the arms - without lifting the probing ball out of the socket. Each measured point should of course be in the same place, but one can evaluate the size of a sphere that encapsulates all the points and use that at an indicator of the effect of compounding errors of the angular encoders.

For the verification of articulated arm CMMs the user is referred to B89.4.22 – 2004 *Methods for performance evaluation of articulated arm co-ordinate measuring machines.*
NOTE

**Kinematic socket**
The kinematic socket is a trihedral socket that the ball rests in, allowing three points of contact rather than the less repeatable line contact of a ball sitting in a hole.

Prof Pat McKeown gave an address at the 1998 ASPE meeting in St Louis and he said that if people walk away remembering that a ball sitting in a slightly smaller hole is not a kinematic joint, then he would be a happy man and feel that he had accomplished something that day.

A perfect ball sitting in a perfect hole would form a perfect line contact where the two intersect – it’s not a kinematic constraint, but in a perfect world you could pick up and replace the ball numerous times and the coordinates of the centre of the ball wouldn’t change a bit!

However, we live in a less than perfect world – no ball is perfectly spherical and no hole is perfectly cylindrical and perpendicular to the surface either – thus a real ball placed repeatedly into a real hole will display non-repeatable positioning behaviour – which isn’t very helpful when trying to test the accuracy of something like an articulated arm CMM, or remove and replace a test fixture or part to be machined.

Generally a kinematic socket involves three planes that are mutually orthogonal to each other – the trihedral socket or indent – or can be constructed using three balls fixed in position. (Strictly speaking the planes do not need to be orthogonal, they just need to make decent mutual angles.) The reference sphere is then placed into the socket and touches each plane or ball in only one place. These three points of contact provide a highly repeatable reference position for the centre of the sphere. The accuracy of the centre position is only affected by the sphericity of the sphere (and to some extent by elastic deformation of the contact areas under high load conditions).

---

**Verifying laser trackers**

Verifying rotary tables

The calibration of a rotary table is usually performed using a polygon and an autocollimator. Rotary tables are often made using a worm gear driven by a motor to turn the main gear and it is common to have a whole number of degrees generated by a complete revolution of the worm. Periodic error of the worm is not tested if the polygon has say twelve faces, but if the polygon has thirteen faces then the angle between each facet is 27° 41’ 32’, and by incrementing the worm through such an angle each time, one will build up a knowledge of the worm error.

NOTE

Rotary axes
Rotary axes are used on CMMs to facilitate the measurement of predominantly cylindrical components such as gears. Rotary axes are used on CNC machine tools to facilitate access to five sides of a prismatic part. They also allow the machining of complex compound surfaces and intricate geometries such as centrifugal compressors and pump impellers.

Formal verification of a machine tool

'We have just spent a great deal of money buying a CNC machine to make a specific component, why on earth should we waste time by measuring anything it produces 'cos it's bound to be right isn't it?'

This used to be a common attitude in manufacturing industry; fortunately this attitude is becoming rare as the demands of customers for compliance to basic quality system requirements are being felt throughout the engineering industry.

\[ \text{Figure 29 In-process gauging (© Renishaw)} \]

\[ \text{Figure 30 In-process gauging (© Renishaw)} \]

\[ ^{12}\text{ A prismatic part is a part such as a cube, cone, cylinder, sphere, torus or combination of these geometries. The alternative would be a freeform component, for example, a turbine blade.} \]
The use of in-process gauging (Figure 29 and Figure 30) is part of the modern production line. The use of automatic tool setting features and interchangeable probes on machine centres is a tremendous boon to the manufacturing capabilities of a factory. However, in-process gauging depends on the CNC machine tool remaining geometrically accurate during both the manufacturing and measuring cycles. This may not be true following a tool crash for instance, or during the machine warm up phase on a Monday morning while the heat from the motors establishes equilibrium after the forty-eight hour weekend shutdown.

What should you do to give yourself confidence in the machine tool following a tool crash? The quickest verification test is probably to run a ball bar check using, for instance, a Renishaw QC-10 ball bar system. A double ball-ended transducer is mounted with one ball in a kinematic socket (see appendix D.3) inserted in the tool holder and the other inserted into a kinematic socket attached to the machine bed. Strong magnets are used to hold the balls in place. The CNC machine tool is programmed to move in a full circle in the horizontal plane with a radius that corresponds to the mid-point of the short-range transducer.

Additional testing can be performed in the vertical axes, either by running over limited arcs of 190˚ or so (to ensure that the reversal points are covered), or by using additional fixturing to raise the position of the socket attached to the machine bed.

The transducer measures the variation in radius as the machine tool moves its axes and the data is displayed on a computer that is running the data acquisition software. The shape of the measured circle can be compared to a pre-crash example and to the specification of the machine. Corrective action may be required if the circle is distorted. ISO 230 Part 4 describes the influences of typical machine deviations on circular paths. An ellipse aligned with an axis indicates a scale error while an ellipse inclined at 45˚ suggests an axis perpendicularity problem. Periodic errors in the drive mechanism show up as periodic deviations in the circular trace, while reversal errors are seen as abrupt discontinuities aligned with the axes.

The transducer in the ball bar system is checked using a calibration bar – a slab of Zerodur (a low expansion glass ceramic) – that has a number of magnetic kinematic sockets glued on its top surface. The position of these sockets will have been previously calibrated using a high accuracy CMM to probe the position of steel balls placed in the sockets to simulate the presence of the ball bar.

The ultimate test of a machine tool is of course to make something. This may be a little expensive in terms of materials and machining time, in addition some inspection time is required to confirm the size and shape of the test piece; however, it is a total system check and is strongly recommended for machine acceptance testing.

A number of standard parts have been proposed over the years and the old National Aerospace Standard (NAS) 979 standard describes a ‘Diamond, Circle, Square composite cutting test – 4.3.3.5’ that is suitable for testing CNC milling and boring machines.

NAS 979 was published by the National Standards Association for the Aerospace Industries Association of America in April 1966 and revised in January 1969. Its intention was to:

‘Provide a standard for the selection of cutting tests required to evaluate the performance of conventional and Numerically Controlled machine tools, excluding drilling and turning machines. To provide a standard format for recording and reporting actual performance results.’

The diamond is machined at 45° relative to the machine axes, while the square is aligned with the axes. A number of holes are bored in the test piece to allow the scale errors to be assessed and the edge of the test piece is machined with a ramp to provide a limited assessment of the vertical axis. The machined features are usually assessed using a CMM, but the circle can be assessed using a roundness-measuring instrument and similar conclusions to the ball bar test can be drawn from the results.

There are a number of problems associated with the NAS Cutting Test including:

- It is time consuming.
- After each test, the part has to be measured.
- It is necessary to interpret and decipher the results of the test in order to make corrections, should the machine fail to meet specification.
- Should the machine fail any of the required specifications, the procedure is then repeated, adding to the initial time and cost.
- The end results are not a true reflection of the machines unloaded performance.

The ASME B5.54-1992 standard ‘Methods for performance evaluation of computer numerically controlled machining centres’ has a number of detailed descriptions of the techniques and methods that can be used to evaluate a machining centre. Should the results of the machining test be unsatisfactory then it is wise to cease production on that machine (you will otherwise be producing scrap) and to undertake a service, overhaul and recalibration of the machine.

**Reversal techniques**

We have looked at how it is possible to verify the performance of your equipment. Another related technique allows the complete elimination of certain types of error. That technique is called reversal.

Reversal is a dimensional measurement technique, which separates the errors of the reference from the errors in the measurement artefact. The separation is usually achieved by reversing the position of a reference object along with the sensitive direction of the probe then repeating a sequence of measurements as a function of linear position or angle.

The reversal technique can be used to determine the errors of a CMM (see Appendix C.2). However, let’s first consider a simple case of reversal.
Reversal is a technique that allows the removal of residual machine errors. It relies on measuring the component in two orientations. In one orientation the machine error adds to the result and in the other orientation the machine error subtracts from the result.

For instance, if you measure the straightness of a line on a component what you actually measure is

\[ \text{Workpiece straightness error} + \text{machine straightness error}. \]

If you turn the component over and measure down exactly the same line you measure

\[ \text{Workpiece straightness error} - \text{machine straightness error}. \]

**Straightedge reversal**

Reversal can be illustrated by the use of a straightedge to evaluate axis straightness. In the experimental setup an indicator is mounted on the travelling carriage in such a way that it is aligned in the direction of interest (orthogonal to the axis) and touching a straightedge mounted nearby.

If we assume that the machine slide straightness is given by a function \( M(x) \) and the departure of the straightedge is given by a function \( S(x) \) we can calculate the indicator output for this position by adding the two functions \( M(x) \) and \( S(x) \) together and calling it \( I_1(x) \). Thus

\[ I_1(x) = M(x) + S(x). \]

We then reverse the straightedge by rotating it about its long axis and we also remount the indicator so that it is touching the straightedge but has had its direction or sign reversed. This is an important point to note because now when we calculate the indicator output \( I_2(x) \) we see an apparent reversal of the machine axis and the apparent lack of change in the straightedge output despite the fact that we flipped the straightedge! Thus

\[ I_2(x) = -M(x) + S(x). \]

We can now go on to separate the terms in the two equations by adding them together and taking the differences. Thus

\[ M(x) = \frac{I_1(x) - I_2(x)}{2} \]

and

\[ S(x) = \frac{I_1(x) + I_2(x)}{2}. \]

Figure 31 illustrates the two positions of a straightedge during a reversal exercise.
Using reversal to determine the spindle error of a roundness-measuring instrument

The method described for straightness can also be used to determine the spindle error of a roundness-measuring instrument.

Bryan et al in 1967 and Donaldson in 1972 described the simple reversal method and the method is outlined below. Donaldson's paper describes a simple method of separating the spindle error $S(\theta)$ from the part error $P(\theta)$. The basic assumption in this method is that $S(\theta)$ and $P(\theta)$ repeat themselves in subsequent measurements. Donaldson emphasises the importance of checking repeatability by making successive traces in the first set-up position.

The initial polar record $T_1(\theta)$ is made using normal polarity, i.e., increasing part radius resulting in increasing radius on the polar chart.

For subsequent records the part position and the stylus position are rotated $180^\circ$ about the spindle axis whilst retaining the original shaft and housing positions. Two polar records are then made, $T_{2P}(\theta)$, with normal polarity and $T_{2S}(\theta)$ with reversed polarity.

Donaldson then goes on to show that

$$ T_1(\theta) = P(\theta) + S(\theta), \quad (1) $$

$$ T_{2P}(\theta) = P(\theta) - S(\theta), \quad (2) $$

$$ T_{2S}(\theta) = -P(\theta) + S(\theta). \quad (3) $$
Adding equations (1) and (2) and then equations (1) and (3) yields

\[ P(\theta) = \frac{T_1(\theta) + T_{2P}(\theta)}{2} \]

and

\[ S(\theta) = \frac{T_1(\theta) + T_{2S}(\theta)}{2} . \]

The implications of these equations is that by recording \( T_1(\theta) \) and \( T_{2P}(\theta) \) on the same graph the part roundness error profile is obtained by simply drawing a third average profile halfway between the first two. The same profile averaging procedure can be used to obtain the spindle radial motion error profile \( S(\theta) \) from a polar chart containing \( T_1(\theta) \) and \( T_{2S}(\theta) \).

The component and spindle error values obtained by this method are not influenced by different base-circle radii or different centring errors (limited only by polar distortion) of the various measurements. This method does, however, have disadvantages. When using an instrument with a rotating transducer a difficulty arises in that the stylus does not cross over the centre of the worktable. To overcome this problem Donaldson fabricated an L-shaped stylus arm having a 20 mm offset to allow testing of balls up to 25 mm in diameter.

**An multi-step method to determine the spindle error of a roundness-measuring instrument**

The multi-step method involves placing the workpiece on either an indexing or rotary table (Figure 32). The first measurement is made with the rotary/index table fixed. The table is then indexed through a definite angle \( \theta_i \) \((i = 1, 2, \ldots, m; \ m \) being the number of angular increments) and the error data recorded and stored. The table is then rotated to the next angular position and again error data are recorded and stored. This process continues until the number of indexing steps, \( i = m \).

*The calibration of a roundness standard* by Charles P Reeve of NBS describes a multi-step technique used in the Dimensional Metrology Group of the National Bureau of Standards (now NIST). The technique involves making several measurements (typically ten) with the component rotated between measurements (in this case by 36°). Once the measurements have been completed both the spindle and component errors can be calculated.
A refinement of this method has recently been developed by NPL. See the dimensional pages of the NPL web site for more details.

**Reorientation and repeat measurement**

Measuring a workpiece many times in the same orientation may give you nice repeatable results, but it does not give you much information about the measurement process. For instance, if you are performing measurements with a height gauge and a surface table, try to make repeated measurements on different parts of the surface table.

If you are making a measurement on a CMM it is tempting to use the looping facilities in the software to measure the component in the same location many times. It is far better to move the component to a different location between measurements, or if this is not possible, then remove the workpiece from the machine and realign between measurements.

**Diagnoses**

The results of the measurement can be used to indicate problems with the manufacturing process. For instance, three lobes on a cylindrical component may be an indication that a three-jaw chuck has been over tightened during manufacture of the component. The component will be machined circular in the stressed state. Once the jaws have been released the component will exhibit the three lobed state.
Other numbers of lobes can indicate other machining errors. Three to seven lobes are usually due to the work holding method. A deviation that repeats three times per revolution would be termed a third harmonic. Higher harmonics are usually due to machine instability (for example, chatter) or due to tool/component interactions (for example, poor lubrication).
Conforming to a quality standard – ‘So you are going to be audited?’

IN THIS CHAPTER

- I am going to be audited!
- A vertical audit.
- A horizontal audit.
- Closing meeting.
- Credibility.
- Are your measurements traceable?
- What is this NPL place?
- What is accreditation?
- Laboratory considerations.
I am going to be audited!

First things first, ‘Don’t Panic!’ Find out who is going to audit you? There are three main possibilities.

An ISO 9000 assessment is conducted to ensure that you have procedures for the work that you do and that you are following those procedures.

An ISO 17025 assessment is conducted to ensure that you have procedures for the work that you are doing and that you are following those procedures and that you are getting the right answers!

A technical audit is an official examination of the people, equipment, methods and procedures that are used during the measurement process. A senior colleague or a member of the management team undertakes internal auditing to ensure that the systems and procedures detailed in the company quality manual are being followed and actually work. External auditing is done to convince either an independent accreditation body or a major customer that the measurements you report and hence the products or services that your company supplies to that customer are of good quality and can be relied upon.

The following scenario envisages a visit to a small jobbing shop where a small CMM is used to inspect components before shipping to a major sub-contractor supplying the automotive industry.

The Inspector

The inspector, Sid, has been operating the CMM for the last five years and is responsible for quality assurance within the jobbing shop. He signs the measurement reports and passes off the good components for shipping to the customer. He also scraps those items that fail inspection – which doesn’t always make him popular with the machinists on the shop floor.

The Auditor

The auditor, Irene, is an experienced metrologist working under contract for an independent accreditation body. She has over twenty years measurement experience, has been conducting technical assessments for ten years, and has a reputation as a tenacious assessor.

Sid was waiting in the inspection department when the foreman and works manager walked in and introduced Irene.

‘Irene will be conducting a technical assessment of your inspection department while we go through the company quality manual with the lead assessor,’ said the works manager briskly.

‘Right,’ said Sid cautiously. ‘Where would you like to start Irene?’

‘I see that this room has air conditioning – do you have an environmental logger to record the air temperature and such like?’ enquired Irene.
‘Yes, we have a chart recorder on the wall behind the CMM, it records the air temperature and relative humidity on a circular chart and we change the chart once a week and file the old ones away in this filing cabinet,’ replied Sid confidently.

‘That’s a good start, now tell me, do you have a calibration certificate for the logger?’ asked Irene.

‘Certainly have – it’s over here in the filing cabinet with the old charts’ replied Sid strolling over to the cabinet and pulling out the file. ‘Here you go, a certificate from a UKAS laboratory for the calibration of the temperature and humidity sensors that was issued four months ago.’

‘I see on the chart here that there was a dip in the temperature over the weekend – the temperature dropped below 15 °C and didn’t rise to 20 °C until Monday afternoon. What happened here?’ asked Irene.

‘We had a problem with the air conditioning plant when the power tripped over the weekend. The air re-heaters didn’t reset properly and when the chiller and air handler came back on line they over cooled the room. When I came in on Monday morning I reset the circuit breaker and as you can see the room temperature was back to normal by the afternoon’ explained Sid.

‘Did you perform any measurements on Monday morning?’ asked Irene expectantly.

‘Oh no, I couldn’t – not with everything as cold as that – we can only do accurate measurements when we’re working between 19 °C and 21 °C as the components are made of aluminium alloys and their thermal expansion coefficients are around 23 ppm. It wasn’t until after lunch that I could even check out the CMM,’ said Sid.

‘What do you mean by ‘check out’ the CMM?’ asked Irene.

‘Well every Monday I run a quick verification of the CMM performance by measuring this master component – that one with the red label in the corner. It’s actually a scrap part that I saved as a representative part. I put it in the same place on the CMM and run the same measurement programme and record a few of the key dimensions on that statistical process control chart over on the wall so I get a bit of confidence that everything is alright with the machine.’

‘And did it check out alright on Monday afternoon?’ asked Irene.

‘Funnily enough it didn’t initially, so I ran the measurement programme again and got the same results – way out of the tramlines on the SPC chart. Something was wrong with the y axis of the machine – it was measuring longer than it should have by about 0.020 mm. So I moved the master part and put a good ring gauge in the same place and measured the roundness – sure enough it came out as an oval, so I knew I had a scale factor problem. I checked the scale temperatures for the x, y and z axes and got a hint of the problem – the y and z axes which are both on the top of the CMM were at about 21 °C while the x-axis which is down here under the bed of the CMM was sitting down at 16 °C.’

‘It looked like the x axis was still cold from the weekend while the y and z axes being less massive and closer to the air vents had warmed up more quickly. At the end of the day I told the Foreman that I couldn’t make any measurements at all on Monday and it would be better to wait for the machine to stabilise overnight and to try again on Tuesday morning – which is what we did. I’ve noted the air conditioning failure in my notebook and recorded the testing and results that I got that day. I sent a memo to the
Works Manager with the suggestion that we replace the re-heater circuit breakers with something that will recover properly from a power failure.’

‘So you are confident in the results from this CMM are you?’ enquires Irene.
‘Actually I am’ said Sid ‘we’ve had this machine for five years now and it was installed and calibrated by the manufacturer. Then we did a performance verification test to the ISO 10360–2 standard using a calibrated step gauge that we measured in seven different orientations within the working volume of the machine. That test was successful and verified the performance of the CMM as being within the manufacturer’s specification so we were happy and they got paid! We’ve had the machine serviced annually since then and each time the service engineer brings in a step gauge and we check out the performance, if it’s not in specification he has to adjust the machine and redo the check.’

‘Do you have any traceable length standards of your own, or do you rely on the service engineer’s step gauge?’ asked Irene.
‘Well the service engineer’s step gauge is calibrated annually by the NPL so that is traceable. We’ve got a fairly new set of grade 0 gauge blocks that came with a UKAS accredited laboratory certificate and I use those to calibrate my working sets of grade 1 gauge blocks by mechanical comparison and I used those working sets for checking some of the critical jigs and fixtures and calibrating our micrometers and verniers. We’ve also got an old set of length bars that I’ve got a lot of calibration history on over the last twenty years. We don’t use them that often so there’s not much wear and tear on them, so I’ve extended the calibration interval. I only get them calibrated every five years now but they are handy when I want to be sure about my measurements on a big job,’ explained Sid at length.

‘Well you seem to have the equipment and the standards sorted out, perhaps we should look at a recent measurement report. Tell me about this job’ said Irene walking over to a stack of components with their inspection reports waiting to return to production and pointing at one of them.
‘Right you are then,’ said Sid. ‘This is a batch of differential casings and that one you pointed at has a serial number stamped on the side which ties up with the measurement report that it’s sitting on.’

Sid went through the measurement report with Irene. The report refers to a measurement procedure QA CMM 0003 GEN that covers:

- the verification checks
- probe qualification, check for dirt too
- clamping fixtures
- part alignment
- measurement program name and location of results files
- comparison to known artefacts, etc.
- measurement results, statistical analysis and uncertainty budget,
- name of operator, date of measurement, etc.

Irene went through all the documentation checking things such as authorisation and document control.
Later that day, Sid had an appointment in his diary for the closing meeting. He stepped through the door to the boardroom. He could see Irene at the head of the table alongside the Company Director, his Team Leader and the Company Quality Manager.

‘Let’s start,’ she said. Irene then went through all she had seen. She mentioned a small number of minor issues that needed addressing and agreed timescales for their resolution. ‘And to close,’ she said, ‘can I say how professional, helpful and organised all the staff I met were. I look forward to my next visit in a year’s time.’

A vertical audit

During a vertical audit a technical assessor will typically select at random a measurement report or a calibration certificate produced by the facility being assessed and they then go through the whole process that was undertaken to produce that report.

The assessor would look at the written measurement procedure that described the measurements that were done and make sure that the procedure was up to date and had been complied with.

The assessor may ask the inspector or metrologist to demonstrate a particular measurement procedure and may ask questions to ascertain if the person understood what they were doing. You are encouraged to explain what you are doing and why, so for instance if you had taken the object to be measured and cleaned it with a degreasing agent you could explain that normally you would leave it for a period of time to stabilise before actually calibrating it, but that to save time and for the purposes of the assessment only you would carry on and measure the object immediately.

The data sheets or metrologist notebook would be examined to see that the raw data and any calculations that were made had been accurately transferred to the measurement report or certificate. The date the measurements were made would be noted and a check of the environmental records would be made to ensure that the facility had been operating within the specified limits at the time of the measurements.

The equipment used to perform the measurements would be checked to ensure that each item used was itself within calibration at the time of the measurements, that the calibration certificates were available and that any applicable correction had been applied.

A detailed look at the relevant uncertainty calculations would also be carried out.

The technical assessor would also be looking at the general level of tidiness and cleanliness within the facility, checking that the appropriate calibration stickers were applied to the measuring equipment and standards. It is helpful, therefore, to get into the habit of cleaning down the horizontal surfaces and washing the floor regularly as this helps make a good impression and keeps the level of dust in the laboratory down to acceptable levels.
A horizontal audit

A horizontal audit involves tracking a particular process from one end to the other and it will usually cross a number of boundaries between areas, functions or departments. Thus an auditor may decide that they want to see, for example, how a purchase order for a piece of equipment is raised and processed within the company or how calibration certificates are produced. They will be looking to see if procedures exist to tell people what they are supposed to be doing and they will be checking to see that these procedures are complied with.

Closing meeting

At the closing meeting, any non-compliances or findings are reported to the lead assessor and these will then be discussed with the head of laboratory or quality manager. A schedule of corrective actions is drawn up and agreed. Typically minor findings may be resolved over the course of a few weeks or even months, but a major finding will have to be resolved urgently.

Minor findings or non-compliances could include such observations as typographic errors in a certificate, an out of date organisational chart or a missing calibration label. However, if similar minor non-compliances are found to be common across an organisation this can add up to a more unacceptable quality risk.

Major findings could include using calibration standards that are out of calibration date – the actions arising from this would include a suspension of that activity until the standards were recalibrated and a recall of those certificates or reports that had been issued since the standard’s certification expired. The implications are quite serious because you may have passed off as acceptable some items that were actually out of specification – in other words you have sold faulty items to your customer. This can be a very expensive mistake!

Another example of a major finding could be discovering that the calibration had been carried out by an untrained person – that is a person who has not had evidence of their training included in their training file and whose name is not of the list of approved service providers. This is a major finding because, once again, it casts doubts on the reliability of the measurements that have been reported.

Credibility

A metrologist or quality inspector must be quite clear on the subject of credibility – if you have gone to the trouble of making a measurement, then you must be confident that the result you give is correct within the uncertainty that you state. If you have any doubts at all go back and check it – re-measure if necessary and if you are still not happy get someone else to measure it as well! You will make hundreds if not thousands of measurements during a career and it takes only one incident where an error or mistake slips through to blow your reputation or credibility as a competent metrologist.
It should go without saying that falsifying measurement reports and omitting data because your boss told you to is not only immoral and wrong but counter productive in the long run. Imagine that a critical and expensive component for an aircraft that is in fact faulty is passed off as fit for use. Any subsequent investigation as a result of the crash would lead eventually to you the person who signed off the inspection report – the investigators would not be very sympathetic as to why you signed if off, only that you did knowingly approve a faulty component. Far better to say ‘No I will not falsify data’ to the manager applying the pressure, report the matter to your boss and ultimately resign rather than compromise your personal and professional integrity.

It is one of the reasons that, sometimes, quality assurance departments are normally a separate department from production areas – so that a manager being pressed to deliver a certain quantity by a certain date is not able to exert any undue influence on the people checking the quality of that product.

Are your measurements traceable?

Sid hinted at the idea of traceability when talking to Irene. You probably realise that your equipment gets calibrated against some higher standard. You may even realise that these higher standards in turn are calibrated by some other organisation. But where does it all stop? You have discovered the idea of traceability.

During the audit the auditor will be making sure that all your equipment is traceable. But what does the word traceability mean in terms of calibration. PD 6461-1:1995 defines traceability as follows:

> property of the results of a measurement whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties

Traceability is illustrated graphically in Figure 33. Traceability is usually assured by having your equipment calibrated at a UKAS accredited laboratory, at a national laboratory or in-house against traceable standards.

Traceability is important to ensure that components made in different organisations assemble correctly.

![Figure 33 Traceability pyramid](image-url)
A typical traceability chain may look like this:

- Micrometer checked in–house against gauge blocks calibrated at a UKAS accredited laboratory.

- UKAS\textsuperscript{15} accredited laboratory calibrates the customer’s gauge blocks by comparison against a set calibrated at NPL by interferometry.

- The gauge block laboratory at NPL has the lasers used in interferometry, thermometers, pressure measuring system, \textit{etc.} calibrated against the appropriate primary standard.

Figure 34 shows the traceability chain for gauge blocks in diagrammatic form.

At the head of the traceability chain is the definition of the metre (see Chapter 2). No uncertainties have been quoted in the lower branches of the traceability tree, as these will depend on local working practices. If you are interested, typical uncertainties can be found on the UKAS website at www.ukas.org.

\textsuperscript{15} United Kingdom Accreditation Service.
What is this NPL place?

We have mentioned NPL, but what is NPL? NPL is the United Kingdom's national standards laboratory, an internationally respected and independent centre of excellence in research, development and knowledge transfer in measurement and materials science. For more than a century NPL have developed and maintained the UK's primary measurement standards - the heart of an infrastructure designed to ensure accuracy, consistency and innovation in physical measurement.

In terms of the traceability tree NPL lives above the bottom two layers in Figure 34. More is said about NPL in the appendix A.1.1.

What is accreditation?

The United Kingdom Accreditation Service (UKAS) or its overseas equivalents will have accredited some organisations offering measurement services. But what does accreditation mean?

The definition of the term accreditation is given in ISO Guide 2 (BS EN 45020:1998 Standardization and related activities. General vocabulary):

*Procedure by which an authoritative body gives formal recognition that a body or person is competent to carry out specific tasks*

Accreditation by UKAS means that testing and calibration laboratories have been assessed against internationally recognised standards and can demonstrate their competence, impartiality and performance capability. Accreditation is a worldwide concept. In most developed countries there is an accreditation body similar to UKAS in the UK.

The standard that accredited calibration laboratories need to adhere to is ISO 17025:2005 *General requirements for the competence of testing and calibration laboratories*.

ISO 17025 specifies the general requirements for the competence to carry out calibrations. It covers calibrations performed using standard methods, non-standard methods and laboratory-developed methods. The standard is applicable to all organisations performing calibrations. ISO 17025 is applicable to all laboratories regardless of the number of personnel or the extent of the scope of calibration activities.
Laboratory considerations

If you are going to be audited there are a few laboratory considerations you should take account of.

Calibration labels

All equipment should carry a label to indicate its calibration status. This label should show the date and reference of the calibration and the date of the next calibration. Before any measurement it is wise to check that the equipment you will be using is still in calibration.

Calibration and calibration intervals

When making dimensional measurements you will need to make reference to the calibration certificate for the instrument you are using or the standard you are comparing against. In this section we will look at a typical calibration certificate and point out the key features you should be looking for.

Interpreting the calibration certificate

A typical calibration certificate, as issued by a UKAS accredited laboratory, is shown in Figure 35. The first thing to notice about this certificate is the UKAS logo. This shows that the laboratory in question has been UKAS accredited. The number underneath the logo is the laboratory number. The certificate also has a date of issue and a unique serial number.
An approved signatory, who is one of two people in the company approved to do this, has signed the certificate. The certificate then has a short description of the basis of test and the environmental conditions during the test. The results of the calibration then followed by a statement of the measurement uncertainty. Note that the statement on the first page indicates that the uncertainty of measurement is quoted at $k = 2$.

The results are split in to two sections. The first section (Figure 36) describes some general tests on the micrometer geometry and basic operation. The second page gives the errors in the micrometer screw at the tested positions (Figure 37). The ‘Actuals’ are the measured values and the ‘Permissibles’ are the specified limits. The ‘Permissibles’ will be taken from a specification of accuracy for the instrument. The ‘Actuals’ are the values measured by the laboratory.
**CERTIFICATE OF CALIBRATION**

**NANSON ENGINEERING & METROLOGY LTD.**

**ISSUED BY**

BLACKHORSE ROAD
LOUGHBOROUGH
LEICESTER

**DATE OF ISSUE** 21 May 2003

**SERIAL NUMBER** N160710

**TELEPHONE** COVENTRY (024) 7666 6888

**FAX** 0447666 4450

**PAGE OF 3 PAGES**

Submitted by:

J.C. Morris
A.J. Brooks

Description:

Micrometer Head

**Description**

Calibrated by Nanson Procuracy CP 024 to the schedule BS.1744 : 1951

This Micrometer Head has been examined at 20°C centigrade &1" using calibrated equipment and certified standards. The measured results which were not within the specified tolerances of BS.1744 : 1951 are as follows:

<table>
<thead>
<tr>
<th>General / Description</th>
<th>Permissible</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flatness of Measuring Face</td>
<td>0.00003&quot;</td>
<td>0.00001&quot;</td>
</tr>
<tr>
<td>Squareness of Face to Spindle &amp; Bearing Shank Axis</td>
<td>0.00003&quot;</td>
<td>0.00002&quot;</td>
</tr>
<tr>
<td>Straightness/Uniformity in Dia. of Bearing Shank</td>
<td>0.00020&quot;</td>
<td>0.00005&quot;</td>
</tr>
<tr>
<td>Action to be Smooth and Even throughout its Range</td>
<td>Found Satisfactory</td>
<td></td>
</tr>
</tbody>
</table>

Continued ....

---

The above expanded comments is based on a standard uncertainty multiplied by a coverage factor k=2, providing a level of confidence of approximately 95%.

The uncertainty evaluation has been carried out in accordance with ISO 17025 requirements.

This certificate is issued in accordance with the laboratory accreditation requirements of the United Kingdom Accreditation Service. It provides traceability of measurement to recognised national standards, and is made of measurement facility in the National Physical Laboratory or other recognised national standards laboratories. This certificate may not be reproduced other than in full, except with the prior written approval of the issuing laboratory.

---

**Figure 36 Page 1 of the certificate with key areas highlighted**
Calibration intervals

How frequently a calibration should be carried out is an important, if sometimes difficult, question to answer. There is no fixed period for the validity of a calibration certificate - it is not like your car’s MOT certificate. However, measurement results stated on certificates are usually ‘on the day’ values and no allowance is made for subsequent drift. There are two main considerations when determining re-calibration intervals. Firstly, all measuring devices change characteristics with time; the question is how much do they change? New devices should be calibrated relatively often in order to establish their reproducibility - essentially their metrological stability or the change in their measuring ability between calibrations. Initial estimates of reproducibility are sometimes made using type-test data from earlier calibration results of similar instruments but the resultant uncertainty of measurement has to be cautiously higher, until real data is available. Secondly, the required uncertainty of measurement should be assessed. If the instrument’s reproducibility is shown, by successive calibrations, to be substantially better than the uncertainty required, then the interval between calibrations can be extended - perhaps even up to five years or so. At the other extreme - where the instrument's reproducibility approaches the
uncertainty needed - the calibration intervals should be much shorter, perhaps even daily in some cases.

**Proper storage of equipment and records**

When being audited it is important that you can find everything quickly. All equipment should be stored properly when finished with and the paperwork should be filed where you can find it easily. Have a system and stick to it!

**NOTE**

**Completion**

On completion of the measurement process, the measuring equipment needs to be stored away in its correct location. Care must be taken when locating each instrument into its own specified box or container. Each instrument must be cleaned and positioned (Figure 38) as required to minimise any damage and be available for everyone involved in the process.

![Figure 38 Put everything away after measurement](image)

**Monitoring the environment**

As we saw in an earlier short story, when making dimensional measurements it is important to record the temperature at the time of measurement and make appropriate corrections as necessary.
What might not be so obvious is the need to know the temperature when you are not making measurements. This is where some kind of temperature monitoring or logging device can be useful.

For instance, did the temperature control in your room fail briefly overnight and the temperature drop to 15 °C? If it did, some of your equipment may not be at 20 °C. Large pieces of equipment, for instance CMMs, can take a long time to recover from large changes in temperature.

Some equipment may run automatically. If these systems do not have any built in logging, how do you know if the temperature was within specification for the duration of the measurement run?

Simple equipment, such as a thermohygrograph, can give you a quick visual check that nothing untoward has been going on with the temperature. If you need to know in more detail how the room is performing you may wish to use an electronic temperature-logging device.

**Avoiding problems with corrosion**

Most material standards of size used in length metrology will be made of steel, for instance gauge blocks, plain setting rings, plug gauges, screw ring gauges, etc.

An unfortunate property of steel is that it rusts fairly easily. Steel will rust in the following circumstances:

- If it gets wet.
- If the air humidity rises above about 60 % relative humidity.
- If it comes into contact with human skin.

We can take precautions to prevent rust occurring in each of these circumstances.

Steel will rust if it gets wet so the first piece of advice is not to clean steel gauges using water. Gauges should be cleaned using a suitable organic solvent.

To stop rusting due to high humidity make sure that the gauges are used and stored in a room where humidity is controlled to stay below 60 % relative humidity. In addition, gauges should be greased or lightly oiled when not in use or wrapped in rust preventative paper – making sure that the correct side of the paper is in contact with the gauge.

To stop gauges rusting due to contact with the moisture and perspiration in human skin, gauges should be cleaned thoroughly with a suitable solvent immediately after use. A further precaution you can take is to only handle gauges whilst wearing gloves. The wearing of gloves will also help prevent you from warming the gauges up. The use of tongs will also prevent the direct contact between you and the gauge.
Vibration and draughts - shut the window and stop that banging!

Measurements of length can be badly affected by vibration. Vibration can come from many sources, for example:

- Road traffic
- Railways
- Aircraft
- Demolition work
- Human activity
- Activities within the workplace (for example, hammering or machine tools)

Vibration can be of low or high frequency. High frequency vibration may be averaged out during measurement. Many problems of vibration are caused by structural resonances of the measurement apparatus.

Vibration can be minimised using various passive and active anti-vibration devices, such as large seismic masses resting on compliant supports, while pneumatic isolators are used to reduce the effects of vibration on, for instance, optical tables. For example, audio manufacturers supply expensive anti-vibration mounts for high-end audio equipment, however, a board mounted on squash balls makes a relatively good anti-vibration mount.

Draughts from doors, windows, etc. can have an effect on your measurements. A door being opened suddenly will cause a rapid change in temperature. A constant draught on one side of your equipment may cause a thermal gradient with the result that your equipment distorts.

The easiest solution is to stop the draught at source. If this is not possible then putting the equipment in a suitable enclosure can help.

Optical instruments are particularly sensitive to draughts as the draught results in a change in the refractive index of the air. In the case shown in Figure 39 the optical path of the autocollimator has been shielded from draughts from the air-conditioning by the use of a specially designed Perspex enclosure.
Cleanliness

A piece of air-borne dust can range in size from 0.1 µm to 100 µm and so could easily affect your measurements. Dust is most noticeable when a light measuring force is used such as when making roundness measurements.

It is important when making dimensional measurements to keep everything scrupulously clean. It goes without saying that you should remove all traces of oil and grease from the component before measurement.

Equipment should be cleaned before use and dusted thoroughly with, for instance, a fine brush.

However, it is amazing how much dust can be deposited from the air on to measurement surfaces. The source of this dust is fibres from clothes, dirt from shoes and particles of skin. The dust from these sources can be minimised with some simple precautions:

- Install tacky mats – simple floor mats that are slightly tacky so as to remove dirt from the soles of your shoes.
- Always wear a laboratory coat. In addition you may wish to consider covering your hair and wearing overshoes in situations where dust must be kept to a minimum.
- Install a dust filtration system to your air-conditioning plant.
- Install an air-lock entrance to your room so that outside air that has not been filtered cannot enter your laboratory.
- Make sure the air pressure in the room is higher than ambient. Clean air can leave the room, but dirty air then cannot enter.
Horribilia – oddball stuff that might be useful

IN THIS CHAPTER

- Easy mistakes.
- How can the results be wrong?
- Cosine error.
- Sine error.
- Abbe offset.
- Elastic compression.
- Parallax errors.
- Other errors.
Easy mistakes

John was in the pub after work reminiscing with some of his colleagues about some of the measurement howlers they had seen over the years.

‘I’ve seen some things in my time,’ said Sue the Quality Manager as she sipped her drink. ‘You know we had this one guy who decided to measure everything he made on the lathe. On his first day he discovers that all the one inch spindles were coming out at 1.001 inch, bang on upper limit. He tried to be clever and set the machine to make the spindles at 0.999 inch to correct for this.’ ‘I know what’s coming next,’ said John. ‘Anyway the spindles were sent to inspection and low and behold the spindles all measured 0.999 inch. The idiot had been measuring them whilst they were still hot.’ They all had a laugh. Ron added that he had seen a similar thing when a machine operator had forgot to zero his micrometer.

‘I can beat that,’ said John. ‘The worst I have seen was in my previous job. We had this scientist who was too clever for his own good. He wanted to measure the position of an antenna that stuck about two metres up in the air. He had a translation stage with a built in scale that resolved 0.001 mm. He was very proud of his invention until I told him to look up Abbe offset. He wasn’t so chuffed when he realised that the antenna was so far away from the scale he’d be lucky to achieve 0.05 mm on the actual position of the antenna!’

Bill then said, ‘compression is the one that gets most people. They have some super-duper instrument that resolves 0.01 micrometre but that’s operating with a force of 1 newton. They then measure their thread measuring cylinders with it and wonder why they are getting the wrong answer.’

How can the results be wrong?

There are a number of ways in which your measurements may be in error as John and his invisible friends were discussing. This chapter will explain in detail some of these errors and indicate how they can be corrected for or minimised. The types of error we will discuss include:

- Cosine error
- Sine error
- Abbe offset
- Temperature compensation
- Elastic compression
- Parallax errors

Cosine error

One source of error you may come across is cosine error and it can affect all kinds of measuring instruments.
**What is cosine error?**

If you were asked to measure the length of a wall with a tape measure, you would, without thinking align the tape roughly parallel with the floor and measure from one end of the wall to the other. You would not dream of holding the tape in the top corner and measuring to the opposite bottom corner. Without realising it, you have minimised cosine error by measuring parallel to the floor.

![Figure 40 Cosine error in measuring 3 m](image)

Figure 40 shows an extreme case of alignment error (the aim was to measure the 3 m side wall). Normally the tape measure would be mis-aligned by only a few degrees. In this example we can see that the length of the room across the corners is longer than the 3 m side wall. In practice cosine error is a lot smaller, as we shall see in the next section.

**Cosine error with callipers**

Cosine error can occur with any measuring instrument. With callipers, cosine error can occur when the axis of the callipers is not square to the edge of the block you are measuring. There will also be a sine error, as we shall see later.

**Cosine error with dial gauges**

Cosine error can also occur when using dial gauges. In Figure 41 the axis of the dial indicator is at an angle of one degree to the Y-axis. The calculation below shows the error introduced by the one degree misalignment.
The $Y$-axis is the adjacent side of a right-angled triangle. The length of the hypotenuse is 0.050 mm. Therefore:

$$Y = 0.050 \times \cos 1^\circ = 0.04999 \text{ mm},$$

$$\text{cosine error} = 0.050 \text{ mm} - 0.04999 \text{ mm} = 0.0001 \text{ mm}.$$  

We can see that for a one degree misalignment the error is only 0.01 $\mu$m. This would not be significant for a dial indicator but may be more significant for a high-resolution measuring instrument.

**Cosine error with a laser interferometer**

Figure 42 shows an example of cosine error with a laser interferometer system (laser interferometers are discussed in chapter 9). In the first case there is no cosine error. In the second case the moving stage is at an angle to the laser beam (the scale) and the measurements will have a cosine error.
Figure 42 Cosine error with a laser interferometer

Figure 43 shows how a misaligned laser interferometer system will always measure short. The true distance is the hypotenuse of a right-angled triangle.

Minimising cosine error

The obvious way to minimise cosine error is to take great care in aligning your system whether it be a dial indicator on a stand or a laser interferometer system.
Introduction to alignment and correcting for cosine error

As we have seen, cosine error with a laser interferometer always causes you to measure short. Any misalignment in a measurement system will result in a cosine error. We can calculate the effect that this misalignment has on the measurements, as we will see in the next section.

![Figure 44 Correcting for cosine error](image)

In the example shown in Figure 44 the scale length is 10 cm (0.1 m), however, the true length is 0.12 m.

Cosine error calculations

If we take the example of a laser interferometer misaligned by 1 mm in 1000 mm, i.e., the return spot moves 1 mm over a 1000 mm movement of the carriage, we have a right-angled triangle where the length of the opposite side is 1 mm and the length of the adjacent side is 1000 mm (Figure 45).

![Figure 45 An example of a laser interferometer misaligned](image)
We can, therefore, calculate the angle as follows

$$\tan^{-1}\left(\frac{1}{1000}\right) = 0.057296^\circ.$$ 

From this angle we can calculate the cosine of the angle

$$\cos(0.057296) = 0.9999995.$$ 

This is in effect the error per millimetre. If we multiply this value by 1000 and subtract the result from 1000 we have the error over 1000 mm. Thus the error over 1000 mm is

$$1000.000 - 999.9995 = 0.0005 \text{ mm}.$$ 

If you are sure about the movement of the beam, *i.e.* you have measured it, you can correct for the alignment error mathematically.

**Allowing for residual cosine error in uncertainty budgets**

The above has assumed that you know the magnitude of the cosine error and can correct for it. In practice you align your system as best you can. However, when aligning an interferometer, there still may be a residual cosine error. Generally you look at the return spot and you can be sure that it moves less than say 0.5 mm but you cannot be sure of the exact magnitude of the movement. All you can say is that it is somewhere between 0 and 0.5 mm.

For instance you may align your laser interferometer system but know that you cannot detect movements of the beam less than 0.5 mm in 250 mm. By using the above formulae you can then calculate the cosine error. In this case

$$\tan^{-1}\left(\frac{0.5}{250}\right) = 0.11459$$

$$\cos(0.11459) = 0.999998.$$ 

This cosine error, $\delta L$, becomes a length dependent term, where the error is

$$\delta L = L - 0.999998L,$$

*i.e.* $\delta L = L(1 - \cos(\theta)).$

When calculating an uncertainty budget you would probably assume a rectangular distribution for this contribution and divide by the square root of three to get the value at one standard deviation, thus

$$\delta L = \frac{L(1 - \cos(\theta))}{\sqrt{3}}.$$ 

Using the above example the uncertainty contribution would be
\[ \delta L = 0.000 \, 289L . \]

**Magnitude of errors for typical misalignments**

We have shown that a misalignment of 1 mm in 1000 mm is equivalent to 0.0573°, *i.e.*, an error of 0.5 \( \mu \)m. Figure 46 shows the effect of various misalignments as shown in the graph legend (*i.e.* 0.5 mm, 1 mm, 2 mm etc.). The \( y \) axis shows the error in the measurement over the lengths listed on the \( x \) axis.

![Cosine Error Graph](image)

**Figure 46 Misalignment (cosine error)**

As you can see from the graph the larger the distance you align over the less significant a given misalignment is. A 5 mm misalignment over 100 mm results in a larger error than a 5 mm misalignment over 1000 mm (brown line). For a given length (\( x \) axis) the error reduces with better alignment.

**Sine error**

The sine error caused by misalignment of a flat contact is a more serious error than the cosine error. A misaligned micrometer would exhibit this error as shown in the Figure 47.
Taking the example of a misalignment of one degree and an anvil width 5 mm and considering the right hand triangle in Figure 47 we have,

\[ x = 2.5 \times \sin(a) \]
\[ x = 2.5 \times 0.01745 \]

From this we can calculate that \( x \) is 0.044 mm.

As you can see a sine error can be quite considerable in magnitude.

**Abbe offset**

**Abbe principle 1890**

The Abbe principle was named after Dr Ernst Abbe. To quote Hale, 1999:

*The measuring instrument is always to be so constructed that the distance being measured is a straight-line extension of the graduations on the scale that serves as the reference. ... Should the measuring axis and that of the scale belong to two different axes, which are separated by a certain distance, then ... the length being read off will be identical to the length being measured in general only when the moving system undergoes pure parallel motion, with no rotation. If the system undergoes a rotation between the initial and final settings, then the scale reading and the measured length are different.*
The Abbe principle can be paraphrased as ‘maximum accuracy may be obtained only when the standard is in line with the axis of the part being measured.’ The vernier calliper violates the Abbe principle because the measuring jaws are offset from the scale (Figure 51). The micrometer, however, complies with the Abbe principle and has the measuring scale in line with the anvil.

It is clearly not possible to comply with the Abbe requirement when building a three axis measuring machine – thus James Bryan of the Lawrence Livermore National Laboratory (LLNL) extended the Abbe principle to assist in the design of the ultra high precision machine tools and measuring machines that he was involved with during the 1960’s and 1970’s.

**NOTE**

**Bryan principle 1979**

Jim Bryan made the following point in 1979 with regard to the Abbe principle. Abbe would probably have had the implications of this statement in mind but it is worth stating the principle here.

‘The displacement measuring system should be in line with the functional point whose displacement is to be measured. If this is not possible, either the slideways that transfer the displacement must be free of angular motion or angular motion data must be used to calculate the consequences of the offset.’

‘Similarly the straightness measuring system should be in line with the functional point whose displacement is to be measured. If this is not possible, either the slideways that transfer the displacement must be free of angular motion or angular motion data must be used to calculate the consequences of the offset’

**What is Abbe offset?**

In an ideal world the scale of a linear measuring system should be colinear with the displacement to be measured or else the measurement must be corrected for the associated Abbe error. The Abbe error is an error in measuring a feature's spatial dimension (such as diameter or length) or linear displacement which results from a changing angular orientation between object and measurement reference component of the instrument. This effect is observed when the measured feature or reference point of motion does not lie along the same line of the measurement reference. The spatial separation between measured point and reference line is known as the Abbe offset.

The Abbe offset is the distance between the desired point of measurement and the reference line of the measuring system. In Figure 48 the Abbe offset is 9.25 mm.
Figure 48 A set up with Abbe offset

NOTE

A perfect world
In a perfect world our slideways would be free from error. In this case we could position our measurement system wherever we liked. The Abbe principle is only important because we live in a real world where slideways have error.

Minimising Abbe offset by good design

In Figure 49 the designer has modified the set up shown in Figure 48 so that the axis of the laser is in line with the scale being tested. In this case it is possible to reduce the Abbe offset to zero. This should always be the aim but may not always be possible.
Allowing for Abbe offset in uncertainty budgets

Allowing for Abbe offset in an uncertainty budget relies on knowing the magnitude of the Abbe offset and also the magnitude of the errors in motion (for example, straightness).

If we know that our motion is straight to 0.005 μm in 30 mm then from the bottom triangle in Figure 50 we can deduce that the angle of the normal is

$$\tan^{-1}\left(\frac{0.005}{30}\right) = 34.38 \text{ secs}.$$  

The error over an offset of say 60 mm is

$$x = 60 \times \tan(34.38\text{o}) = 60 \times \frac{0.005}{30} = 0.010 \text{ mm}.$$  

This could be quite a significant contribution to the uncertainty budget. To reduce its effect you either need to reduce the offset or use a slideway with a smaller straightness error.

In an uncertainty budget you would probably estimate that this value was the bounds of a rectangular or triangular distribution. Assuming a rectangular distribution the contribution would be

$$\frac{0.010}{\sqrt{3}} \text{ mm}.$$
Examples of Abbe with different measuring equipment

Callipers are a prime example of a piece of equipment that exhibits a large Abbe offset. The dashed line in Figure 51 shows the line of measurement, which as you can see is offset from the measurement scale. Any errors due to straightness errors in the beam will be magnified at the point of measurement.
Conversely the micrometer (Figure 52) has a scale in line with the measurement axis. The Abbe offset in this case is zero. Figure 53 shows an example of Abbe offset when using a laser interferometer. Here a yaw angle of 5 seconds of arc is seen to cause an error of 0.002 4 mm in measurement of the position of the moving optic.

As you can see with any piece of measuring equipment there is the possibility of Abbe error. It is important, when you come to design your own equipment, to minimise the Abbe error as much as possible.

**Elastic compression**

*Background*

When you measure any item mechanically you invariably apply a measuring force to it. The item under test will, therefore, tend to compress. How much the item compresses depends on:

- The measurement force
- The geometry of the component (for example, sphere, cylinder)
- The component and anvil materials
- The type of contact (point, line)
The length of contact

The formulae for calculating the amount of compression can be found in Puttock M J and Thwaite E G 1969 Elastic compression of spheres and cylinders at point and line contact National Standards Laboratory Technical Paper No. 25 (CSIRO: Melbourne). This document can be difficult to get hold of now but many engineering textbooks give the formula for compression although not necessarily in a metrological context.

### NOTE

**Elastic compression calculator**

An elastic compression calculator can be found at: [http://emtoolbox.nist.gov/Main/Main.asp](http://emtoolbox.nist.gov/Main/Main.asp)

---

### A simple worked example (compression of sphere between flats)

We will now look at a simple worked example of the compression of a sphere between two flats (for example, a sphere between micrometer anvils).

### NOTE

**Newtons and grams force**

The SI unit of force is the newton (N). However, most dimensional metrologists still use gram-force (gf). You can use either unit in these equations as long as you are consistent.

Note that 1 N is approximately 100 gf.

Case 3 of Elastic compression of spheres and cylinders at point and line contact shows how to calculate the compression of a sphere between two parallel planes. For a sphere in contact with a single plane the formula is

\[
\alpha = \frac{(3\pi)^{\frac{2}{3}}}{2} P^{\frac{2}{3}} (V_1 + V_2)^{\frac{2}{3}} \left( \frac{1}{D} \right)^{\frac{1}{3}}
\]

where \(D\) is the diameter of the sphere, \(P\) is total applied force and \(V\) is defined as

\[
V = \frac{(1 - \sigma^2)}{\pi E},
\]

where \(E\) is Young’s modulus for the material and \(\sigma\) is its Poisson’s ratio.

For a sphere between two parallel planes the above formula becomes
\[ \alpha = (3\pi)^{\frac{2}{3}} P^{\frac{2}{3}} (V_1 + V_2)^{\frac{2}{3}} \left( \frac{1}{D} \right)^{\frac{1}{3}}. \]

Let’s say we have a steel ball of 25 mm diameter between steel anvils and the measuring force is 250 gf (2.5 N).

From the table of \( V \) values in Puttock and Thwaite, the \( V \) value for steel in terms of gf mm\(^{-2}\) is \( 1.36 \times 10^{-8} \), therefore,

\[ \alpha = 0.000\ 040 \times P^{\frac{2}{3}} \left( \frac{1}{D} \right)^{\frac{1}{3}}. \]

Substituting \( P = 250 \text{ gf} \) and \( D = 25 \text{ mm} \) into this equation gives a compression of 0.000 52 mm.

**Table of compression of spheres between steel anvils under typical measurement forces**

Table 6 shows the typical compression of steel spheres in the range 10 mm to 50 mm under measurement forces ranging from 250 gf (2.5 N) to 50 gf (0.5 N) measured between flat anvils. This is also shown graphically in Figure 54.

**Table 6 Compression of steel spheres in the range 10 mm to 50 mm between steel anvils for forces from 250 gf to 50 gf**

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>250 gf</th>
<th>200 gf</th>
<th>100 gf</th>
<th>50 gf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.000743</td>
<td>0.000641</td>
<td>0.000404</td>
<td>0.000254</td>
</tr>
<tr>
<td>20</td>
<td>0.000590</td>
<td>0.000508</td>
<td>0.000320</td>
<td>0.000202</td>
</tr>
<tr>
<td>30</td>
<td>0.000515</td>
<td>0.000444</td>
<td>0.000280</td>
<td>0.000176</td>
</tr>
<tr>
<td>40</td>
<td>0.000468</td>
<td>0.000404</td>
<td>0.000254</td>
<td>0.000160</td>
</tr>
<tr>
<td>50</td>
<td>0.000435</td>
<td>0.000375</td>
<td>0.000236</td>
<td>0.000149</td>
</tr>
</tbody>
</table>
Figure 54 Compression corrections for steel spheres between flat steel anvils for various forces

For comparison Table 7 and Figure 55 gives the compression for a tungsten carbide sphere between steel anvils.

Table 7 Compression of tungsten carbide spheres in the range 10 mm to 50 mm for forces from 250 gf to 50 gf

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>250 gf</th>
<th>200 gf</th>
<th>100 gf</th>
<th>50 gf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.000577</td>
<td>0.000497</td>
<td>0.000313</td>
<td>0.000197</td>
</tr>
<tr>
<td>20</td>
<td>0.000458</td>
<td>0.000395</td>
<td>0.000249</td>
<td>0.000157</td>
</tr>
<tr>
<td>30</td>
<td>0.000400</td>
<td>0.000345</td>
<td>0.000217</td>
<td>0.000137</td>
</tr>
<tr>
<td>40</td>
<td>0.000363</td>
<td>0.000313</td>
<td>0.000197</td>
<td>0.000124</td>
</tr>
<tr>
<td>50</td>
<td>0.000337</td>
<td>0.000291</td>
<td>0.000183</td>
<td>0.000115</td>
</tr>
</tbody>
</table>
Of course other geometries and other materials will give different values. We have of course assumed that only Hertzian compression is occurring, usually a safe assumption for most cases. However, beware if you are calculating the compression of, for example, very small diameter wires. Other cases where care may be needed include thin walled cylinders where bending may also occur.

**Compression – another case**

While we cannot go through every geometry mentioned in Puttock and Thwaite we will briefly give an indication of how you would calculate the compression of a cylinder between parallel planes.

The formula for the compression of a cylinder in contact with a plane is

\[
\alpha = \bar{P}(V_1 + V_2) \left[ 1 + \ln \left( \frac{8a^2}{(V_1 + V_2)\overline{PD}} \right) \right].
\]

The symbols are as before except that the force per unit length is given by

\[
\bar{P} = \frac{P}{2a}
\]

where \(2a\) is the length of cylinder in contact with the anvil.

The normal case encountered would be the compression of a cylinder between two parallel planes. In this case the total compression is the sum of the compression for each contact. If the two anvils are identical in size and of the same material it is simply a matter of doubling the compression for a single contact.
Let’s say we have a steel cylinder of 25 mm diameter between steel anvils of 2 mm diameter and the measuring force is 250gf (2.5N)

\[
\bar{P} = \frac{250}{2} = 125 \text{ gf/mm}
\]

\[
\alpha = 125(2.72 \times 10^{-8}) \left[ 1 + \ln \left( \frac{8}{(2.72 \times 10^{-8}) \times 125 \times 25} \right) \right] = 0.000042 \text{ mm.}
\]

We have two anvils of the same size and material so the total compression is 0.000 08 mm. You can see that the compression is much less than for a steel sphere of the same diameter.

**Examples of compression error in dimensional measurement**

Compression can occur during many situations found in dimensional measurement. These include:

- Measuring a gauge block with spherical tipped anvils
- Measuring a cylinder between flat and parallel anvils
- Measuring a sphere between flat and parallel anvils
- Measuring a screw thread using thread measuring cylinders or ball ended feelers

In each case there is a compression equation that you can use to calculate the appropriate correction. If you can minimise the measurement force this will help to reduce the amount of compression. However, if the measurement force is too low then you may have inadequate contact between the anvils and the measurement surface. Higher measurement forces tend to crush any dust between anvils and workpiece, therefore, when operating at lower measurement forces your measurements will be more susceptible to the effects of dust particles.

**Parallax errors**

*The principle of parallax*

Parallax is a form of observational error where the object appears to shift when you change your position. This can be observed next time you are in the passenger seat of a car. Compare your reading of the speed on the speedometer with that of the driver. There is sure to be a difference.

Parallax can be a problem when you are reading a meter. Since the pointer is slightly above the scale (to allow the pointer to move freely), you must look straight at the pointer to have a correct meter reading. In other words, you must be in line with the pointer and the scale.
How to avoid parallax

Parallax errors can be minimised by having the line of measurement of, for example a rule, as close as possible to the feature being measured. Some dial indicators avoid parallax error by having a mirror behind the pointer. This ensures that you are always looking at right angles to the scale. The mirror on the scale of a meter helps get rid of parallax error. If there is any parallax, the image of the pointer in the mirror will be visible. If you are looking at the meter correctly (no parallax error) the image of the pointer in the mirror will not be visible because the image will be directly behind the pointer.

An example of parallax error

We will now look at a typical example of parallax error and how to avoid it.

Parallax when using a rule

Figure 56 shows how parallax can be avoided when using a rule. By standing the rule on its edge the effect of parallax can be avoided. If the scale was facing upwards when the measurement was made then there is a chance of parallax error. If the scale markings were on the top surface then the possibility of parallax error is greatly increased.

Figure 56 You can reduce parallax by having the line of measurement of the rule as close as possible to the feature being measured

Other errors

The moving tag on the end of the tape measure – what’s it for?

John Hannaford, one of the authors of this guide relates a tale.

Yes indeed folks as a young man I was confused by the moving tag on my Dad’s tape measure and thought it wrong that it should rattle about while I was using it – it wasn’t as accurate as my rule surely? So I positioned it in the middle of the slot and hit it with a hammer and riveted it in place! Little did I know at the time that the slots in the tag allowed it to move just enough to accommodate the thickness of the tag and
thus allow the tape measure to measure internal and external dimensions accurately! From then on the tape measure would always read inaccurately – half the width of the tag short on external measurements and half the width of the tag long on internal measurements.

This is an example of a systematic error.

**Clamping**

Clamping can be a source of measurement error. It is tempting to place as many clamps as possible on your workpiece to stop it moving. It is also tempting to do these clamps up as tight as possible. Both of these practices are wrong as they may lead to distortion of the workpiece.

As we have seen earlier a workpiece will expand as the temperature increases. If the workpiece is clamped in such a way as to prevent this expansion then the workpiece will distort. You should, therefore, use as few clamps as possible and use a pliable material such as cork between the workpiece and the clamp.

Over tightening will also cause distortion and this again can be alleviated by placing cork between the workpiece and the clamp and not tightening too much.

**Sag under self-weight**

When using items such as combination end bars and straightedges it is important that they are supported correctly. The calibrated length of an end bar, for example, is only valid when it is supported in the correct way. Bands engraved on the cylindrical surface of the bar will indicate the support point for an end bar. However, if the bar is used in combination with other bars then the correct support points have to be calculated.

**Horizontal supports**

All items will sag under their own weight. Horizontally supported standards are, therefore, supported at the Airy points. The Airy points are equal distance from the ends and separated by $0.5774 L$ where $L$ is the length of the standard. Sir George Airy while Astronomer Royal (1835 to 1881) derived this relationship.

The result of supporting at the Airy points is that an end standard, for example a gauge block, will have the tangents to the curves at the extremities of the standard horizontal. As a result, the defining ends of the standard are vertical and parallel.
Considering the example of a 175 mm standard $0.577L$ is equivalent to 100.975 mm (Figure 57). Practically, the support points are 101.0 mm apart and equi-distant from the centre of the standard.

An alternative set of support points are the Bessel points ($0.5594L$ apart) that are the supports employed when checking line standards. These supports ensure the minimum shortening of overall length.

The formula, $d = 0.5537L$, gives the centres of supports that ensure minimum deflection ($d$ is the centre support distance). These supports are not normally used in metrology but are of value in machine tool design where, for example, it is necessary to minimise the curvature of a slideway.

The formula, $d = 0.5505L$, will give the supports so that the tangents to the curves immediately above the support points are horizontal which may be useful in some applications.

**Vertical support**

An end standard supported vertically will foreshorten. The equation for the foreshortenings is

$$X = \frac{SL^2}{2E},$$
where $X$ is the amount of foreshortening, $S$ is weight density ($\text{Nm}^{-3}$), $L$ is length and $E$ is Young’s modulus of elasticity.

NOTE

**What can possibly go wrong using a rule?**

The zero end of a rule is often subject to wear and abuse – the most extreme case we’ve heard of relating to a deliberate attempt to deceive an external inspector. A guillotine operator had made a mistake setting the fence which controls how far the metal can enter the throat of the guillotine and had cut a batch of expensive armour plate 10 mm too short for the intended application.

When the foreman realised this he was horrified at the thought of having to buy in replacement material once the external inspector had rejected the parts as not conforming to the required size. So the foreman took a 1 m steel rule and cut the first 10 mm off the end using the guillotine.

When the inspector arrived to check the batch of plates, he used the offered ruler to check the cut plates and found them all to be within specification, he signed off the acceptance sheet and went away happy.

He was not so happy a few days later when the plates were welded in place and found to be too small by 10 mm – and there was nothing he could do because he had signed off the plates as acceptable!

The moral of this little tale is that you should indeed ‘measure twice and cut once’ – but that your first measurement should be a verification that the equipment you are using is working correctly, and hasn’t been tampered with!
Optical methods

IN THIS CHAPTER

Further car parts.
- The autocollimator.
- Laser interferometers.
- Optical CMMs.
- Laser scanners.
- Optical comparators (profile projectors).
- Theodolites.
- Photogrammetry.
- Aircraft parts.
- Car parts – John’s return.
We will cover basic optical theory and introduce a number of useful optical elements in the appendix to this guide. We can devote this chapter to looking at various optical instruments and their use in practical measurements.\textsuperscript{16}

**Further car parts**

John was paying another visit to the inspection area when he saw Sam working with a new piece of equipment.

‘The company must be doing well if they are buying you all these new toys,’ said John.

‘Oh, hello John!’ said Sam. ‘Yes, this is one of those laser interferometers that I have always wanted to get hold of.’

‘Well Sam, I hope you have fun with it, but just remember what I told you about old Mr Abbe.’

‘No John, I hadn’t forgotten,’ said Sam. ‘These interferometers give you a whole new series of problems though. They operate in air so the system has to calculate the refractive index of the air. I also have to be mindful of deadpath error and make sure that it is aligned correctly. The laser itself will be regularly calibrated so that we know that it is reading correctly.’

‘Sorry,’ said John. ‘Old habits die hard. I am quite interested in this refractive index correction. How does that work?’

‘Well, you know how the lines on a glass scale get further apart as you increase the temperature. Well, it’s the same with the laser beam scale, its wavelength. Only with the laser beam scale it’s also affected by pressure and humidity amongst other things. For some measurements, the refractive index variations in this room don’t pose a significant uncertainty. This interferometer system has air sensors built in and it automatically calculates the refractive index of the air so I can use it for demanding measurements. To further reduce the uncertainty, I could control the temperature and draughts in the room better and use a wavelength tracker or even a refractometer to measure the actual refractive index of the air.’

\textsuperscript{16} A particularly useful book for further reading is Williams D C (Ed.) 1993 *Optical methods in engineering metrology* (Chapman and Hall: London/New York).
NOTE

Refractive index of air
The refractive index of air is calculated using the Edlén equation. If you need to use the Edlén equation make sure you are using the version as modified by Birch and Downs. (Birch K P, Downs M J 1994 Letter to the Editor Correction to the Updated Edlén Equation for the Refractive Index of Air Metrologia 31 315-316)

The autocollimator

The measurement of small angular motion is important in a number of applications such as setting up and checking machine tools. Measurement of small angle is made using an instrument called an autocollimator (Figure 58). An autocollimator is used with a reflecting surface.

Figure 58 An autocollimator

The optical arrangement of an autocollimator is shown in Figure 59 below.
Figure 59 Diagram of an autocollimator

A light source illuminates a target graticule (A) located at the focal point of an objective lens. The light from the target falls onto a partial (typically 50%) beam-splitter and half of the light is reflected towards the objective lens (B) (the other half is lost internally within the instrument). The light emerges as a collimated beam and falls on a mirror (C) placed nominally at 90˚ to the optical axis and is reflected back towards the objective lens. The return beam is focused by the objective lens through the beam-splitter cube and some of the light falls on the eyepiece graticule where the image of the target can be seen via the eyepiece. The reading on the graticule is noted and then the mirror is turned through an angle $\theta$. The image of the target is seen to move a certain distance $d$ along the graticule. The law of reflection says that the angle of incidence equals the angle of reflection, so by turning the mirror by an angle $\theta$ we have actually caused the return beam to change direction by $2\theta$. The relationship between the mirror rotation and the observed displacement is governed by the following relationship

$$\sin 2\theta = \frac{d}{f}$$

where $f$ is the focal length of the lens.

Fortunately the small angle approximation comes into play, the sine of an angle is the same as the angle in radians (within practical limits) for angles less than about 1˚ or 0.01745 rad, and the above formula simplifies to

$$\theta = \frac{d}{2f}.$$  

Thus by having a long focal length lens and the ability to read the displacement, $d$, with a fine resolution, measurements of small rotations of the mirror may be made.

In early autocollimators the displacement of the image was read by eye against the scale of the graticule and a careful operator could achieve a resolution of perhaps 0.5˚. Later models of autocollimators were photoelectric and used a vibrating slit, which partially obscured a photodetector, mounted on a micrometer to form a repeatable null detector. This greatly reduced operator fatigue and also improved the sensitivity of the autocollimator allowing a resolution of typically 0.1˚. Modern autocollimators use
electronic detectors such as linear arrays and position sensitive detectors to monitor the change of position of the return image in one or two axes and can achieve resolutions of 0.01’ or better.

**Laser interferometers**

Laser interferometer systems have taken over from gauge blocks as the most useful dimensional metrology devices in many applications. Lasers are now used in a wide variety of measuring instruments and systems from range finders that can measure the distance to the moon to a fraction of a millimetre to xy wafer steppers for the semiconductor industry that control the position of a silicon wafer to a fraction of a nanometre. A laser can be considered to be a very bright light source of single wavelength (colour) whose waves are all coherent (in step).

A typical laser interferometer system set up on a machine tool is shown in Figure 60. The interferometer is set up for linear measurement, but this is only one of a number of possible applications of laser interferometer systems, as we will see later.

![Figure 60 A laser system set up on a machine tool (Image © Renishaw plc 2000)](Image © Renishaw plc 2000)
Figure 61 shows a very simple interferometer system. The beam of light from a monochromatic (single colour) light source hits a half silvered mirror (also known as a beam-splitter). Half the light reflects from the moveable mirror and half reflects from the fixed mirror. The beams combine at the half silvered mirror and reach the eye of the observer. If the mirrors are perfectly square to one another, the observer will see a series of light and dark patches as the moveable mirror is very slowly moved. If one of the mirrors is now very slightly tilted the observer will see a series of light and dark bands. These bands are called fringes. If we consider that the two beams are made up of waves the bright bands of light can be explained by looking at how the beams combine in Figure 62. Here the two beams are in phase, *i.e.*, their peaks and troughs coincide. This will happen if the two paths are equal, *i.e.*, each mirror is the same distance from the beam-splitter. It will also happen if the paths differ by an exact number of wavelengths of the incident light.

If the moveable mirror is moved back so that its position is shifted by one quarter wavelength, then the beam will return to the observer 180° out of phase and destructive interference will occur, resulting in darkness (Figure 63). The observer, counting the flashes of light (fringes) as the mirror moves, could measure the distance moved by the moveable mirror.
The interferometers described in the rest of this chapter are a variation on this very simple arrangement. The main difference is the use of a cube-corner as a reflector. The use of a cube-corner decreases the sensitivity to angular misalignments and offsets the return beam which simplifies the design of a commercial instrument.

**NOTE**

**Beam-splitters**

The semi-silvered mirror is sometimes referred to as a beam-splitter. It is basically a device that reflects 50% of incident light and transmits the other 50%. However, the proportions of transmitted and reflected light are not always 50%-50% and can be for instance 67%-33%.

**Optical configurations for laser interferometers**

This section will go through some of the various optical configurations for laser interferometer systems.

*Conventional displacement.*

Figure 64 shows in diagrammatic form a laser interferometer set up for distance measurements. The linear reflector is the component whose position is being measured. The linear interferometer consists of a beam-splitter and a second reflector.

The beam from the laser is incident on the beam-splitter and 50% of the light goes to the fixed linear reflector and 50% goes to the movable linear reflector. The two beams recombine at the beam-splitter and carry on to a detector. The detector determines the distance moved by counting fringes.
Angle interferometers

An angular interferometer is shown in Figure 65. In this case both reflectors are mounted in a single block. The measurement beam is $A_2$ and the reference beam $A_1$. As the block rotates the difference in length ($A_1 - A_2$) changes and it is this length that is measured. Knowing the separation of the mirrors it is possible, with simple trigonometry, to convert the distance ($A_1 - A_2$) into an angle.

Straightness interferometers

When measuring straightness the outgoing beam from the laser passes through the straightness interferometer which splits it into two beams which diverge at a small angle and are directed to the straightness reflector. The beams are then reflected from the straightness reflector and return along a new path to the straightness interferometer as shown in Figure 66. At the straightness interferometer the two beams are converged and a single beam is returned to the laser head.
The straightness is measured by detecting the optical path change from a relative lateral displacement between the interferometer and the reflector.

Multiple pass interferometry

When making displacement measurements it is possible to double the resolution of the interferometer by using it in a double pass configuration. Figure 67 shows one such configuration. The linear interferometer has an additional reflector and a quarter wave plate attached. The moving reflector has been replaced with a plane mirror.

The interferometer shown in Figure 67 is a polarising interferometer. That is to say that the light used is circularly polarised. The beam-splitter is a polarising beam-splitter so that one polarisation reflects from the moving mirror, whilst the other polarisation forms the reference path. The two polarisations may also be of different frequency. The quarter wave plate is a device to rotate the plane of polarisation.
The input beam is split at the beam-splitter. Half the light is reflected from the upper cube-corner and returns to the detector. This is the reference path. The other half is transmitted to the mirror and reflected back towards the beam-splitter. Because it has been through the quarter wave plate it is then reflected from the beam-splitter and on to the bottom cube-corner. The light is then again reflected from the beam-splitter, through the quarter wave plate, and back to the mirror where it is reflected back to the beam-splitter where it is transmitted back to the detector.

**Some limitations of laser interferometers – length errors**

Laser interferometers are like any measurement instrument. They are subject to error if not used correctly and have limitations when used in the real world. This section will highlight some of the possible problems and show how they can be minimised.

**Refractive index of air**

The fundamental limitation of laser interferometers are that they are susceptible to apparent variations in the velocity of light due to variations in the refractive index of the air along the line of sight that the beam travels through. They are excellent in a vacuum so using them in outer space or vacuum chambers is ideal. Unfortunately the metrologists using the system do like to breathe occasionally so operating in air is a reality of laser interferometer operation.

**NOTE**

**Velocity of light**

The velocity of light is actually constant. Light travels at constant speed through a vacuum, but its propagation through a media can be quite complex. The wave theory of light gives a simple explanation that refractive index is the ratio of the wave velocities in vacuum and the medium.

\[
\text{refractive index} = \frac{\text{speed in vacuum}}{\text{speed in medium}}
\]

The air temperature, air pressure, relative humidity and concentration of carbon dioxide all influence the refractive index, hence the speed of light and the number of waves, and hence the apparent displacement of the retro-reflector or mirror.

The influence of these factors are usually expressed in terms of what will cause an error of 1 µm over a metre – a part per million or ppm. Thus a 1 °C change in temperature, a 2.5 mmHg (note the approximate relationship, 1 mmHg = 133.322 Pa) change in air pressure or an 80% change in relative humidity will cause a 1 ppm error.
NOTE

Compensation factor
You may be more familiar with the compensation factor that represents the ratio of the wavelength of light in air to the wavelength of light in a vacuum that is used by some manufacturers. This number has the format 0.999abcd where ‘abcd’ is 7288 for standard conditions of 20 °C, 760 mmHg and 50 % relative humidity. A series of look up tables is often provided for the digits ‘abcd’ for different atmospheric conditions.

Limitations of lasers – direction errors

A commonly overlooked source of error when using lasers, and in fact any form of optical alignment instrument is that of refraction – the bending of the line of sight by variations in the refractive index across the line of sight.

Abbe error

An example of Abbe error with a laser interferometer system was given in the previous chapter (Figure 53).

Deadpath error

Another source of error when using interferometers is the deadpath. The deadpath is the distance between the interferometer and the moving optic when the counters are zero. The deadpath is distance L1 in Figure 68. Deadpath can be physically reduced as in Figure 69 or can be minimised in software by setting the interferometer count to L1 at the datum position.

Figure 68 Deadpath error (Image © Renishaw plc 2000)
The deadpath is an uncompensated distance so errors will occur due to changes in pressure, temperature, humidity and carbon dioxide content of the air during the measurement.

**Alignment**

It is important the laser interferometer system is aligned with the direction of motion. If not, a cosine error will be present as discussed in the previous chapter.

Figure 70 shows how the laser beam paths should be aligned with the direction of motion.
**Other error sources when using a laser interferometer**

The above errors are those that the user has control of during set-up. You should, however, be aware that there are some other sources of error that are inherent in most laser interferometer systems. These errors include the stability of the laser wavelength, non-linearity in the optics and thermal drift in the optics.

The laser source of any interferometer will have some type of frequency stabilisation to maintain its wavelength accuracy. The interferometer’s accuracy is based on the laser wavelength accuracy and its repeatability on the short-term stability of the laser wavelength. These errors are a function of the distance measured. The manufacturer’s specification or calibration certificate for the interferometer will usually give these values.

Non-linearity of the optics can be a problem in polarising interferometers and is due to the optical components not perfectly splitting the two components of the polarised beam. This error is periodic and has a spatial frequency related to the wavelength. It can also be caused when the light emitted from the laser head is not perfectly linearly polarised i.e., it is slightly elliptical.

Thermal drift in the optics is caused as changes in temperature alter the optical paths in the optics. This occurs mainly when the two beams do not pass through the same amount of glass. The change in temperature causes both the physical size of the optic and its refractive index to change. This error is a function of the change in interferometer temperature over the period of measurement.

The Agilent Product Note – *Achieving Maximum Accuracy and Repeatability* covers these errors in more detail. It is available from [www.agilent.com](http://www.agilent.com).

**Applications - checking a machine tool**

Laser interferometers can be used to assess the scale error in a machine tool directly (Figure 60) by comparing the reading obtained from the interferometer with that from the scale. The usual arrangement is to have a retro-reflector mounted in the quill of the machine and to have the interferometer block mounted on the bed of the machine. Movement of either will result in a measurable displacement that can be compared to the scale.

As we have stated in the previous section, compensation must be applied for the variation in the refractive index of air as the velocity of light in air is slightly different from the velocity of light in a vacuum. The refractive index of air is typically 1.00027127 and variations at the 1 part in 10^6 level are observed when the temperature varies by 1 °C, the air pressure by 2.5 mmHg and the relative humidity by 80 %.

As we have seen (Figure 71), angle measurements can be made interferometrically using special optics. It should be noted that the pitch and yaw angles of a carriage or axis can be measured by this technique, but the interferometer is insensitive to roll. Roll errors are usually determined using differential levels (which work well for horizontal axes as they are referenced to gravity but this is not the case for a vertical
Axis straightness measurements can be made interferometrically using another special set of optics (Figure 66 and Figure 72). In this case a Wollaston prism causes the two orthogonal polarisations of the laser to diverge at a slight angle (about 1.5° for the short range prism that allows measurements up to 3 m and 0.15° for the long range prism that allows measurements up to 30 m) and a special double mirror that reflects the divergent beams back along their outgoing path. The prism is mounted where the tool normally goes and the mirror is mounted where the workpiece normally goes. Lateral displacement of the prism causes a change in the displacement output of the interferometer, but small pitch, yaw or roll motions of the prism do not affect the measurement accuracy.
**NOTE**

**Wollaston prism**
A Wollaston prism is a prism that splits a light beam into two plane-polarized components. They rely on the principle of double refraction (see Appendix D.2.4).

![Figure 72 Straightness measurement (Image © Renishaw plc 2000)](image)

The sensitivity of the laser straightness measurement is actually rather limited due to the very shallow angle of the Wollaston prism and the matching mirror. Thus a straightness deviation or lateral displacement of $x \ \mu m$ actually generates a change in the interferometer output of only $2\sin(\theta/2)$ of $x$. As the short range optics often use a Wollaston divergence angle of 1.5° this means that the sensitivity is reduced by a factor of 1/38, while for the long range optics, that use an angle of only 0.15°, the sensitivity is reduced by 1/382.

An example of bad practice would be to have a large residual slope on the straightness measurement. If the laser is not perfectly aligned with the machine axis – which is difficult to arrange – then the measuring beams will travel across the surface of the mirrors. This means that the flatness of these mirrors is a significant contribution to the measured straightness error. It can be removed, however, by rotating the mirror assembly through 180° and performing a reversal – the average of the results from the two positions being the true measure of straightness of the axis.
Good practice is to take the extra effort to reduce the residual slope to less than say 10 µm. This can be done fairly quickly using the guidelines laid out in the operator’s manual, provided you remember to adjust the angular orientation of the laser head in proportion to the ratio of the actual travel of the axis divided by the distance from the laser head to the reference mirrors. (Sometimes the laser head on its tripod is quite some distance from the near end of the axis travel.)

For ultra-high precision machines, with travels of say between 1 m and 2 m, physical straightedges are preferred because of the high accuracy that can be achieved particularly when using straightedge reversal techniques. Unfortunately straightedges do not scale particularly well due to their self-weight deformation, cost of manufacture and long-term stability.

For very large machines with axis travels in excess of 30 m, one may use a taut wire and either a microscope or some form of proximity sensor to perform horizontal straightness measurements. For vertical straightness measurements one should apply a correction for the catenary sag, but one should note that this is not a particularly accurate technique.

**Alignment lasers**

An alignment laser projects a laser beam along the axis of travel of a machine and an electronic sensor, such as a quadrant photodiode, senses the position of the beam. The method is subject to beam wander due to thermal distortion of the laser cavity, plasma instability in the laser cavity, and currents, convection cells and refractive index gradients in the air.

**Tracking laser interferometers**

The first tracking laser interferometer or laser tracker was demonstrated in 1986 by Lau, Hocken and Haight who were trying to determine the positioning accuracy of industrial robots (Lau K, Hocken R H and Haight W C 1986 Automatic laser tracking interferometer system for robot metrology *Precision Engineering* 8 3-8).

Essentially a laser tracker is a servomechanism that is capable of pointing the measurement beam of a displacement measuring laser interferometer at a target retro-reflector. A portion of the return beam is picked off using a beam-splitter and directed onto a two-axis optical position detector, such as a quadrant photodiode, to provide the error signal for the feedback loop that controls the tracking function. By providing the ability to rotate in both the horizontal and vertical axes the servomechanism provides the tracker with a spherical co-ordinate measuring system based on measuring the radial distance and the two pointing angles to a given point.
Figure 73 shows a tracking laser interferometer (the blue piece of equipment in the photograph). The retroreflector is in the hand of the operator.

Various optical arrangements have been used to accomplish the beam steering – the gimballed mirror type being one method used commercially. A laser is mounted vertically and the measurement beam falls on the surface of a mirror mounted at 45°; the reflected beam emerges horizontally. By rotating the mirror assembly about the vertical axis a full 360° can be swept out. Vertical motion of the emerging beam can be achieved by tilting the mirror about its horizontal axis – a range of typically - 45° to + 60° being common. As the reflected beam will be deflected by twice the angle of tilt of the mirror, an unfortunate consequence of using a tilting mirror is that the angle sensor must be more accurate.

It is in general very difficult to accurately align the laser beam on the mirror coincident with the centre of rotation of both axes and this leads to the use of an error correction scheme based on a kinematic model of the tracker geometry to correct for these inaccuracies.

A novel approach adopted by the NPL involves the use of a high precision spherical air-bearing that both defines the mechanical centre of rotation and the optical reference surface for the interferometer. This coincidence between the optical and the mechanical centres means that none of the parametric error testing prior to use that is common to the commercial trackers needs to be done. The operational simplicity and accuracy comes at a price, however, the air-bearings are expensive! An alternative approach is use conventional bearings and to place a precision sphere nominally at the centre of rotation to act as the measurement datum. This method is cheaper to implement but maintains the accuracy inherent in the more expensive air-bearing design.
Other manufacturers use an optical fibre to transfer the light from the laser to the rotating optical head of the tracker or they mount the laser directly on the moving head.

The most common target for a laser tracker is the spherically mounted retro-reflector (SMR). A SMR is a precision steel sphere that has been hollowed out, with a cube corner mounted within it, so that the apex of the prism coincides with the centre of the sphere.

The commercially available SMRs are suitable for a number of applications and have sphericity errors of 1 µm and apex centring to within 2.5 µm. The hollow corner-cube is preferred over the solid glass design because it avoids the refraction errors. The acceptance angle of these SMRs is typically ± 40°, which can be restrictive – particularly in multi-tracker arrangements. Cat’s-eye retro-reflectors offer larger acceptance angles of typically ± 60°, but at the expense of being larger and more expensive to make.

An attractive target for multiple trackers is a sphere made of glass with a refractive index of two at the laser wavelength. It is an attractive target because it can be viewed from any angle and is essentially a universal cat’s-eye. Its disadvantage is that it has a much lower reflection efficiency (typically 10 % to 11 %) that can cause tracking problems with commercial systems. This glass is known as TaFD44 or ‘unobtanium’ within the metrology community as it is very difficult to find a manufacturer prepared to make it in blanks thick enough for decent sized spheres, however some manufacturers are now starting to offer spheres of alternative materials of moderate size.

The limitations of accuracy of laser trackers come down to the inability to measure angle accurately and variations in the refractive index of air.

A comprehensive survey of techniques is given in a best practice guide Best practice for non-contacting CMMs written by University College London, National Physical Laboratory and Leica UK is available from the UCL website17.

The laser tracker instrument has evolved extensively over the last twenty years or so and currently three commercial organisations offer laser trackers. The specifications are reasonably similar – essentially offering a measuring accuracy of 1 part in 10^5 over ranges up to 40 m, with operating ranges up to 160 m.

**Absolute distance meters**

The latest developments in laser trackers have seen the incorporation of absolute distance meters (ADM) into the optical system so that the displacement reading can be reset following a beam break – a distinct operational advantage over a simple tracking interferometer that must be physically reset at a known location.

The distance measurement with an ADM (Figure 74 and Figure 75) is based on the phase measurement principle. A laser diode emits light pulses with a defined

---

17 The web site address is: [www.ucl.ac.uk](http://www.ucl.ac.uk)
wavelength and pulse repetition frequency. A time difference exists between the internal reference path and the external measurement path, the light pulses, reflected from a target and received by the instrument, have experienced a phase shift in relation to the light pulses received through the internal reference path. That phase difference between those two signals is proportional to the distance between instrument and target.

The optical signals are converted into analogue electronic signals and then into digital signals. The phase difference between the reference signal and the measurement signal is then calculated. If the phase difference were higher than 360°, for example 410°, then the instrument would calculate a distance corresponding to 50°. To avoid such an error the pulse repetition frequency of the laser light pulses is reduced and a second measurement is carried out.

**Six degrees of freedom probe**

The addition of the six degrees of freedom probe is a significant boost to the usability of laser trackers as it is now possible to probe directly a number of features that would have been difficult to access with an SMR alone. A CCD camera mounted above the laser tracker mirror uses a zoom lens to provide an optimised view of all the LEDs in the probe. By applying photogrammetric principles (see later) to the images it is possible to calculate the orientation of the probe. With prior knowledge of the distance between the retro-reflector (whose co-ordinates are known in the tracker co-ordinate system) and the centre of the contact probe, one can then calculate the position of the centre of the contact probe in the tracker co-ordinate system. The Leica T-probe is an example of this type of system.

**Hybrid systems**

The combination of an articulated arm CMM and a laser tracker forms a very versatile hybrid system. The arm provides good access to hidden features, while the laser tracker links the multiple arm set-ups together in a single global co-ordinate system.
Optical CMMs

Optical CMMs are similar to conventional CMMs. The only difference is that the probing system is normally a microscope and associated camera. They are used mainly to examine components that are either too delicate or too small for conventional probing.

Laser scanners

The subject of laser scanners is of increasing importance and could be a guide on its own. Therefore, we will only give a brief introduction to their use here.

This type of sensor often works by projecting a laser stripe across the object to be scanned. Figure 76 shows a laser scanner in use with an articulated arm CMM. A camera senses the projected stripe and height variations in the object are seen as changes in the shape of the line. Each viewed stripe forms a profile that is built up from several hundred measured points.

From the profiles a three dimensional point cloud representation of the surface can be produced. Laser scanners are a relatively new technology and work is ongoing to demonstrate their traceability.

They have uses in, for instance, scanning antiquities or scanning handmade models of prototype products.

Figure 76 A laser scanner in use with an articulated arm (courtesy Faro)
Optical comparators (profile projectors)

Optical projectors cast on a screen an image of the object being tested. They have advantages over microscopes in that they have a larger field of view and lead to less eyestrain. Traditionally they also had the added advantage that several people could observe the image at once and measurements could be made directly on the screen. The major disadvantages of the optical projector are its bulk and its high cost.

The major problem when using a profile projector is in checking that you have a sharp image. The reflection from the workpiece can also cause problems with contrast.

Theodolites

A theodolite consists of a sighting telescope that can be pointed at the object to be measured. Two encoders then measure the angle from the (normally gravity referenced) vertical direction and from a (arbitrary and adjustable) horizontal direction to the line of sight. If the position and orientation of the theodolite is known, then the two angles define a line in space, joining the theodolite to the object. If a second observation is made, from a second known location, then a second line in space is generated and the position of the object is calculated as the intersection (or, more strictly, the point of closest approach) of the two lines – further observations (lines) from other locations can be added, if required, to strengthen the solution. Note that if the distance between the theodolites is not known – merely the direction (perhaps measured by sighting one theodolite at the other) then the survey will produce shape, but not size, information – a scale factor will have to be determined.

Traditionally, theodolites were slow to use, having to be manually pointed and the scales manually read. However, recent developments have included electronic encoders linked to a computer, motor drives and image recognition systems so that it is now possible to perform fully automatic surveys. A typical accuracy for a survey would be 1 part in 10^5 of the object dimension.

The advent of cheap, powerful, computing has also meant that ‘real-time’ systems now exist that will display the coordinates of points of interest as they are measured – allowing setting of jigs and fixtures to be performed with ease. The same computing power also means that it is now practicable to perform a, so-called, ‘bundle adjustment’ as a post-processing exercise to optimise the measurement results.

The chief advantages of theodolites for industrial measurement are that the systems have a high accuracy and reasonable cost and are very flexible – any object from a metre to many kilometres can be measured. The technique is also non-contacting, so that delicate and inaccessible objects can be measured with ease. Whilst prepared ‘targets’ are required for automatic recognition systems the very flexibility of the human operator means that ‘natural features’ of many types (for example, bolt holes, tooling balls, component intersections, etc.) can be measured in the same survey. The chief disadvantages are (except for automatic target recognition systems) that skilled operators are required and the measurement process is rather slow.
NOTE

Bundle adjustment?

Bundle adjustment is a technique initially developed for photogrammetry, but now used also for theodolite survey, where instrument parameters (position, orientation, calibration etc.) are included in the analysis (suitably weighted) to produce the optimum solution for the object co-ordinates.

Good measurement practice when using theodolites is to ensure that the intersection angle is as close to 90˚ as possible. Figure 77 gives an indication of the relative size of the error ellipse associated with an angular error of known magnitude affecting both theodolites. The extreme cases show that a little planning is usually necessary before embarking on a measurement. T indicates a theodolite and P a measurement point. It is also good practice to take ‘face left’ and ‘face right’ readings of the same point – i.e. turn the telescope end-for-end and point back to the same place. The results should then be averaged, as this eliminates many of the manufacturing errors of the theodolite, particularly at close range.

Figure 77 A diagram of the intersection of several theodolite lines of sight: in each case the point of interest can be expected to lie within the shaded ellipse.

Photogrammetry

Photogrammetry is the technique of obtaining three-dimensional data from two-dimensional measurements made on two (or more) photographs of the object. It was originally developed to produce contour maps from aerial photography – a pair of
photographs was placed in a ‘stereo-plotter’ (a combination of stereoscopic viewer and mechanical computer, about the size of an upright piano) and an operator viewed the photographs and laboriously ‘flew’ a pointer within the stereoscopic image. The advent of powerful computers and better mathematical analysis techniques, however, has meant that measurements can now be made from multiple photographs taken from arbitrary locations of any object whatsoever (the same techniques have been used from electron-micrographs to aerial photography!).

Essentially a camera can be thought of as a recording theodolite – rays of light from the object pass in straight lines through the camera lens and onto the image plane. If the position and orientation of the camera is known, then the position of a point on the image plane defines a line in space. A second photograph of the same object, taken from a second location, provides a second line in space, and the intersection (or point of closest approach) of the two lines corresponds to the point of interest. In general, however, it is not possible to know the position and orientation of the camera accurately and so a bundle adjustment, or other similar mathematical technique, is used that calculates the camera parameters as well as the object co-ordinates.

For the highest accuracies – used, for instance, to measure the shape of large radio antennae, in ship-building, etc. – large format film (or glass-plate) cameras are used, together with special ‘targets’ to designate the points to be measured and the resulting photographs are analysed on a two-axis travelling microscope. Using these methods accuracies up to 1 part in $10^6$ of the object dimension can be achieved. For lower accuracy work, however, the modern development of the digital camera has meant that photographs can be taken very rapidly and analysed quickly (and, sometimes, automatically) to produce results at the 1 part in $10^4$ level.

It is also possible (at a lower accuracy) to use two or more video cameras, with image analysis taking place automatically in the time between the individual frames to perform ‘real-time’ photogrammetry to track moving objects – this has been used very successfully to monitor athletes in motion to provide data to improve their performance and to monitor relative alignment between two moving objects (the articulated arm of the NASA space shuttle, for instance, is guided by real-time photogrammetry).

The advantages of photogrammetry include the very rapid acquisition of the basic data (two or more photographs can be taken simultaneously), allowing the measurement of dynamic events, the non-contacting nature of the measurement (allowing operation in inaccessible and hostile environments – including nuclear reactors and under-water), the flexibility of the systems (any size or shape of object can be measured) and the high accuracy that can be obtained. Analysis can also be done rapidly (from digital photographs) and systems can be very cheap (from a few hundred pounds for a low accuracy system using a domestic digital camera and a home PC).

The disadvantages include the high cost and slow analysis time for the high accuracy film based systems and the relatively poor accuracy of the digital systems.
Aircraft parts

Counter trade agreements are common in decisions to purchase arms and aircraft from a particular company, due to the very large sums of money involved. Often as part of the counter trade agreement, the selling company agrees to have some of the components manufactured in the purchaser’s country. This involves transferring portions of an existing production line to the other country. The production jigs are disassembled to form a flat pack kit of parts and are despatched by sea, due to the weight of the steel structure. On arrival at the foreign company the jig has to be put back together and inspected by experienced fitters and inspectors from the parent company.

Bill had just received one such jig from overseas. He selected a suitable location on the factory floor where the flow of material to and from the jig was optimised. Earl then checked that the concrete floor had no cracks or gaps that would allow movement of the base. The floor was drilled and rawl-bolts inserted to bolt the base to the floor. The use of location dowels and shims to position the vertical portions of the assembly jig would make the rest of the assembly process fairly straightforward.

The location of the centre box is a critical feature of the tail plane jig as it determines the position of the horizontal stabiliser pivot point and actuator arm.

The shipping factory had measured the co-ordinates for certain key features prior to the jig being disassembled and shipped. Earl had the job of measuring these reference features and positioning the centre box so that its critical features are within the tolerances allowed.

The jig is a frame made up of 100 mm square sections, approximately 4 m long, 2 m high and 2 m deep. The reference features are dowel holes in machined faces in the corners of each of the frames. The assembly process is greatly assisted by the use of a laser tracker that is used in conjunction with a SMR and a precision tooling insert. The tooling insert has a magnetically preloaded kinematic socket machined into a thin cylindrical disk. The centre of the SMR is thus a calibrated distance above the flat surface at the back of the insert. Protruding from the middle of the rear of the disk is a short precision ground cylinder that is a snug-fit in the dowel-hole and coaxial with the SMR. When the SMR is mounted in the tooling insert and the insert placed into the dowel-hole, the recorded co-ordinate of the SMR will be coaxial with the dowel hole but offset a known distance.

Having established a reference co-ordinate system based on three dowel holes near the bottom, top and far corners of the jig, it was then possible for Earl to measure the position of the centre box reference points and determine the orientation and location of the centre box relative to the jig co-ordinate system. The fitters can then use shims to adjust the position of the centre box, while monitoring the position of the SMR in real-time. This greatly increases the speed and ease with which adjustments can be made.

Having measured and adjusted the reference points to be within the tolerances of the pre-strip configuration, all that remained was to measure the individual details of the
sub-assemblies that bolt onto the jig to ensure that they too will meet the positional tolerances of the drawing.

**Car parts – John’s return**

It had been some time since John had worked in the inspection area so he was not surprised to see further changes to the equipment. He was also not surprised that he did not recognise what it was.

‘Hello Sam,’ said John. ‘What’s all this then?’

‘Well,’ said Sam, ‘we are now measuring some very tiny components to very high accuracy. This is the Nano Absolute Measuring Machine or NAMM. I can place a component on here and the NAMM will rapidly measure the component, compare it to the CAD drawing (we’re still using that old technology in the design office), compute the uncertainty and let me know if the component passes or fails. And, before you ask, it has been verified, complies with good engineering design, is traceable and, as I said, automatically calculates uncertainties. The assessors are going to carry out an e-assessment on it tomorrow. I have all my documentation on this little gizmo here,’ she said holding up her i-library.

‘That’s great’, said John ‘and it’s nice to see you too. Soon you will be telling me that you are teleporting the items from the production line to the measuring machine.’

‘No, we couldn’t afford that option.’

We can see from the above story that Sam is an experienced metrologist. She knows that however new the technology, and however wonderful the claims for it are, you still need to stick to some basic principles of dimensional metrology. What applies for micrometers and callipers still applies to Sam’s NAMM. Always question the manufacturer’s specification. Ask how they came to the numbers, run your own tests and make sure all your measurements are traceable. If in doubt think ‘What would John have done?’

We hope you have enjoyed reading this guide. If you would like to know more we suggest you obtain some of the reading material in the next chapter. For the latest developments in dimensional measurement keep an eye on the NPL website.
Reference materials and further reading

IN THIS CHAPTER

- Further reading.
- Books.
- Published standards.
Further reading

Books


Bell S A 2001 A beginner's guide to uncertainty in measurement Measurement good practice guide No. 11 Issue 2 (NPL)


Flack D R 2001 Callipers and micrometers Measurement Good Practice Guide No. 40 (NPL)

Flack D R 2005 Fundamental good practice in the design and interpretation of engineering drawings for measurement processes Measurement Good Practice Guide No. 79 (NPL)

Hebra A 2003 Measure for measure - The story of imperial, metric and other units (The John Hopkins University Press)

Hecht E 2003 Optics (Addison Wesley)

Hewitt P L 1984 Techniques in metrology (World scientific)


Moore W R 1970 Foundations of mechanical accuracy (Bridgeport: Moore Special Tool Company)

Slocum A H 1992 Precision machine design (Society of Manufacturing Engineers)

Smith G T 2002 Industrial metrology - surfaces and roundness (London: Springer)


Williams D C (Ed.) 1993 Optical methods in engineering metrology (Chapman and Hall: London/New York)
Published standards

Standards are published by national or international organisations. In the UK standards are published by the British Standards Institution (BSI). The International Organization for Standardization (ISO) publishes international standards.

Who decides what a published standard should contain?

Before a standard is published the views of all interested parties are taken into account: manufacturers, vendors and users, consumer groups, testing laboratories, governments, engineering professions and research organisations. Standardisation is industry-driven and, therefore, based on voluntary involvement of all interests parties in industry.

The need for a standard is usually expressed by an industry sector, which communicates this need to BSI. BSI proposes the new work item to ISO. Once the need for an international standard has been recognised and formally agreed, the first phase involves definition of the technical scope of the future standard. This phase is usually carried out in working groups that comprise technical experts from countries interested in the subject matter. Technical committee TC 213 Dimensional and geometrical product specifications and verification cover most of the work that this guide covers.

Once technical aspects to be covered in the standard have been decided the various countries negotiate the detailed specifications within the standard. This is the consensus-building phase.

The final phase comprises the formal approval of the resulting draft international standard, following which the agreed text is published as an ISO international standard.

ISO standards are reviewed at intervals of about five years. On occasion, it is necessary to revise a standard earlier.

British Standards Institution

BSI started in 1901 as a committee of engineers determined to standardise the number and type of steel sections in order to make British manufacturers more efficient and competitive. The BSI Group is now the oldest and arguably the most prestigious national standards body in the world and is among the world’s leading commodity and product testing organisations.

International Organization for Standardization

The International Organization for Standardization is a worldwide federation of national standards bodies from some 140 countries.
The mission of ISO is to promote the development of standardisation and related activities in the world with a view to facilitating the international exchange of goods and services, and to developing cooperation in the spheres of intellectual, scientific, technological and economic activity.

ISO's work results in international agreements that are published as International Standards.

**Standards relevant to this guide**

BS ISO 230-1:2012  Test code for machine tools. Geometric accuracy of machines operating under no-load or finishing conditions

BS ISO 230-4:2005  Test code for machine tools. Circular tests for numerically controlled machine tools

BS ISO 230-6:2002  Test code for machine tools. Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests)

BS ISO 1000:1992  SI units and recommendations for the use of their multiples and of certain other units

ISO 3650 Geometrical Product Specifications (GPS) Length Standards Gauge blocks

BS 5317:1976  Specification. Metric length bars and their accessories

BS EN ISO 10360-1:2001 Geometrical Product Specifications (GPS) – Acceptance and reverification tests for co-ordinate measuring machines (CMM)—Part 1: Vocabulary


BS EN ISO 10360-3:2001 Geometrical Product Specifications (GPS) – Acceptance and reverification tests for co-ordinate measuring machines (CMM)—Part 3: CMMs with the axis of a rotary table as the fourth axis

BS EN ISO 10360-4:2001 Geometrical Product Specifications (GPS) – Acceptance and reverification tests for co-ordinate measuring machines (CMM)—Part 4: CMMs used in scanning measuring mode

BS EN ISO 10360-5:2010 Geometrical Product Specifications (GPS) – Acceptance and reverification tests for co-ordinate measuring machines (CMM)—Part 5: CMMs using single and multiple-stylus contact probing systems

BS EN ISO 14253-1:1999 Geometrical Product Specifications (GPS) -- Inspection by measurement of workpieces and measuring equipment -- Part 1: Decision rules for proving conformance or non-conformance with specifications

BS EN ISO 14253-3:2011 Geometrical Product Specifications (GPS) -- Inspection by measurement of workpieces and measuring equipment -- Part 3: Guidelines for achieving agreements on measurement uncertainty statements

BS 5233:1986 Glossary of terms used in metrology

PD 6461-1:1995 Basic and general terms (VIM)

This standard has been withdrawn as it is based on the second edition of the International vocabulary of basic and general terms in metrology (VIM). A new 3rd edition of the VIM was published by ISO as ISO/IEC Guide 99:2007. The relevant BSI committee have decided not to adopt the new edition of the VIM as a BSI Published Document. Printed copies of ISO/IEC Guide 99:2007 are available for purchase from BSI Customer Services. An online copy of the VIM can be found on the BIPM website at www.bipm.org/en/publications/guides/vim.html

PD 6461-2:1980 Vocabulary of legal metrology (VLM)

This standard has been withdrawn as it reproduces verbatim the International Vocabulary of Terms used in Legal Metrology (VIML) published by the International Organization of Legal Metrology, (OIML). The OIML has published later editions of the VIML but PD 6461-2:1980 has not been revised inline with the OIML documents. The latest edition of the VIML was published in 2000 and is available to download free of charge from the OIML web site. Consequently, the responsible BSI Technical Committee has decided to withdraw PD 6461-2:1980 without replacement. An online copy of the 2000 edition of the VIML can be found on the OIML web site at www.oiml.org/publications/V/V001-ef00.pdf

PD 6461-3:1995 Guide to the expression of uncertainty in measurement (GUM)

This standard has been withdrawn as it is based on the 1993 edition of the Guide to the expression of uncertainty in measurement (GUM). ISO has recently republished the 1995 (corrected) edition of the GUM as ISO/IEC Guide 98-3:2008. The relevant BSI committee has decided not to adopt the 2008 edition of the GUM as a BSI Published Document.

PD 6461-4:2004 General metrology. Practical guide to measurement uncertainty

ASME B89.4.22 Methods for Performance Evaluation of Articulated Arm Coordinate Measuring Machines.

ASME B89.4.19 Performance Evaluation Tests and Geometric Misalignments in Laser Trackers
VDI/VDE 2617 Part 10 Accuracy of coordinate measuring machines - Characteristics and their checking - Laser trackers with multiple probing systems

Other relevant NPL Measurement Good Practice Guides

The following Measurement Good Practice guides are available from the NPL website.
Glossary of terms
| **Abbe principle** | The Abbe principle can be paraphrased as ‘maximum accuracy may be obtained only when the standard is in line with the axis of the part being measured.’ |
| **Accuracy of measurement** | The closeness of the agreement between the result of a measurement and the (conventional) true value of the measurand. The use of the term precision for accuracy should be avoided. The statistical definition of ‘Accuracy’ is given in BS 5532. (BS 5233:1986) |
| **Calibration** | The set of operations which establish, under specified conditions, the relationship between values indicated by a measuring instrument or measuring system, or values represented by a material measure, and the corresponding known values of a measurand. (BS 5233:1986) |
| **Coverage factor** | A numerical factor, symbol \( k \), used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty (GUM 2.3.6) |
| **Combined standard uncertainty** | Combined standard uncertainty - standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities. (GUM 2.3.4) |
| **Cosine error** | An error caused by the angular misalignment of the measuring system with the item being measured. |
| **CMM** | Co-ordinate measuring machine. A measuring system with the means to move a probing system and capability to determine spatial co-ordinates on a workpiece surface. (ISO 10360:2001) |
| **Fringe** | A light or dark band observed when using an interferometer. Caused by constructive or destructive interference. |
| **Gauge block** | ...material measure of rectangular section, made of wear-resistant material, with one pair of planar, mutually parallel measuring faces, which can be wrung to the measuring faces of other gauge blocks to make composite assemblies, or to similarly finished
surfaces of auxiliary plates for length measurements.  
(ISO 3650:1999)

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interferometer</strong></td>
<td>A length measuring transducer based on optical interference.</td>
</tr>
<tr>
<td><strong>ISO</strong></td>
<td>International Organization for Standardization</td>
</tr>
<tr>
<td><strong>LASER</strong></td>
<td>Laser is an acronym for light amplification by stimulated emission of radiation. It is, in simple terms, a very bright light source of fixed wavelength.</td>
</tr>
<tr>
<td><strong>Laser tracker</strong></td>
<td>An interferometer based system for measuring the spherical co-ordinates of a moving retroreflector.</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>The mean is one way of obtaining a best estimate of a quantity. It is sometimes referred to as average. Summing all the measurements of a quantity and then dividing by the number of measurements will give you the mean of those measurements.</td>
</tr>
<tr>
<td><strong>Measurand</strong></td>
<td>The particular quantity subject to measurement.</td>
</tr>
<tr>
<td><strong>NIST</strong></td>
<td>National Institute of Standards and Technology</td>
</tr>
<tr>
<td><strong>NPL</strong></td>
<td>National Physical Laboratory</td>
</tr>
<tr>
<td><strong>Optical comparator</strong></td>
<td>An optical device that projects an enlarged shadow of the test item on to a screen.</td>
</tr>
<tr>
<td><strong>Parallax error</strong></td>
<td>… the apparent displacement of an object as seen from two different points that are not on a line with the object.</td>
</tr>
<tr>
<td><strong>Precision</strong></td>
<td>The reproducibility of a measurement.</td>
</tr>
<tr>
<td><strong>PTB</strong></td>
<td>Physikalisch-Technische Bundesanstalt</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td>A quantitative expression of the ability of an indicating device to distinguish meaningfully between closely adjacent values of the quantity indicated. (BS 5233:1986)</td>
</tr>
<tr>
<td>Term</td>
<td>Definition / Description</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Result of measurement</td>
<td>…value attributed to a measurand, obtained by measurement.</td>
</tr>
<tr>
<td>Reversal</td>
<td>A measurement technique that allows separation of the errors in the device being measured from errors in measuring instrument.</td>
</tr>
<tr>
<td>Size</td>
<td>A number expressing, in a particular unit, the numerical value of a linear dimension.</td>
</tr>
<tr>
<td>SMR</td>
<td>Spherically mounted retroreflector</td>
</tr>
<tr>
<td>SI</td>
<td>Système International d'Unités (International System of Units, international abbreviation SI)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>A statistical term that refers to the root mean square deviation of the measurements used to obtain a mean value. It is a useful way to characterise the dispersion of the measurements.</td>
</tr>
<tr>
<td>SEOM</td>
<td>Standard error of the mean. This is calculated by dividing the standard deviation by the square root of the number of readings.</td>
</tr>
<tr>
<td>Standard uncertainty</td>
<td>Uncertainty of a result of measurement expressed as a standard deviation.</td>
</tr>
<tr>
<td>Thermal expansion</td>
<td>How much a material changes length for a given temperature change is known as the coefficient of linear thermal expansion.</td>
</tr>
<tr>
<td>Tolerance</td>
<td>… difference between the upper and lower tolerance limits.</td>
</tr>
<tr>
<td>Traceability</td>
<td>… property of the results of a measurement whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties.</td>
</tr>
<tr>
<td>Triangulation</td>
<td>Triangulation is the process of finding a distance to a point by calculating the length of one side of a triangle, given measurements of angles and sides of the triangle formed by the point and two other reference points. Triangulation is used for many purposes, including surveying and navigation. Surveying problems can involve the solution of large meshes of triangles, with thousands of measurements (for example, the mapping Great Britain).</td>
</tr>
<tr>
<td>Type A contribution</td>
<td>Method of evaluation of uncertainty by the statistical analysis of a series of observations (GUM 2.3.2)</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Type B contribution</td>
<td>Method of evaluation of uncertainty by means other than the statistical analysis of a series of observations (GUM 2.3.3)</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>An estimate characterizing the range of values within which the true value of a measurand lies. (BS 5233:1986)</td>
</tr>
<tr>
<td>Value</td>
<td>…magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number.</td>
</tr>
<tr>
<td>Variance</td>
<td>A number that is the square of the standard deviation.</td>
</tr>
</tbody>
</table>
Appendices

IN THIS SECTION

- Appendix A Links to other useful sources of information.
- Appendix B Basic mathematical concepts.
- Appendix C Metrological nuggets.
- Appendix D Some simple physics.
Appendix A  Links to other useful sources of information

A.1  National and international organisations

A.1.1  National Physical Laboratory

"When you can measure what you are speaking about and express it in numbers you know something about it; but when you can not express it in numbers your knowledge is of a meagre and unsatisfactory kind."

Lord Kelvin, British Scientist (1824 – 1907)

The National Physical Laboratory (NPL) is the UK’s national measurement institute and is a world-leading centre of excellence in developing and applying the most accurate measurement standards, science and technology available. For more than a century NPL has developed and maintained the nation’s primary measurement standards. These standards underpin an infrastructure of traceability throughout the UK and the world that ensures accuracy and consistency of measurement.

NPL ensures that cutting edge measurement science and technology have a positive impact in the real world. NPL delivers world-leading measurement solutions that are critical to commercial research and development, and support business success across the UK and the globe.

Good measurement improves productivity and quality; it underpins consumer confidence and trade and is vital to innovation. NPL undertake research and shares its expertise with government, business and society to help enhance economic performance and the quality of life.
NPL’s measurements help to save lives, protect the environment, enable citizens to feel safe and secure, as well as supporting international trade and companies to innovation. Support in areas such as the development of advanced medical treatments and environmental monitoring helps secure a better quality of life for all.

NPL employs over 500 scientists, based in south west London, in a laboratory, which is amongst the world’s most extensive and sophisticated measurement science buildings.

The National Physical Laboratory is operated on behalf of the National Measurement Office by NPL Management Limited, a wholly owned subsidiary of Serco Group plc. For further information: Switchboard 020 8977 3222 | www.npl.co.uk/contact

A.1.2 National Institute of Standards and Technology (NIST)

NIST is the equivalent of NPL in the United States of America. Founded in 1901, NIST is a non-regulatory federal agency within the U.S. Department of Commerce. NIST’s mission is to promote U.S. innovation and industrial competitiveness by advancing measurement science, standards, and technology in ways that enhance economic security and improve quality of life.

The NIST web site at www.nist.gov often contains documents relevant to this guide in Adobe PDF.

A.1.3 EURAMET

The European Association of National Metrology Institutes (EURAMET) is a Regional Metrology Organisation (RMO) of Europe. It coordinates the cooperation of National Metrology Institutes (NMI) of Europe in fields such as research in metrology, traceability of measurements to the SI units, international recognition of national measurement standards and related Calibration and Measurement Capabilities (CMC) of its members. Through knowledge transfer and cooperation among its members EURAMET facilitates the development of the national metrology infrastructures.

EURAMET serves the promotion of science and research and European co-operation in the field of metrology.

This is realized by the following measures in particular:

- development and support of European-wide research co-operation in the field of metrology and measurement standards;
- development, regular updating and implementation of a European Metrology Research Programme (EMRP);
- support of members and associates when applying for research funds for the purpose of European cooperative projects;
Appendices

- co-ordination of joint use of special facilities;
- improvement of the efficiency of use of available resources to better meet metrological needs and to assure the traceability of national standards;
- technical co-operation with metrology institutes beyond EURAMET and with other regional and international metrology organisations;
- performing the tasks of a Regional Metrology Organisation (RMO) with the objective of worldwide mutual recognition of national measurement standards and of calibration and measurement certificates;
- promotion and co-ordination of scientific knowledge transfer and experience in the field of metrology;
- representing metrology at the European level and promoting best practice to policy and political decision makers with regard to the metrological infrastructure and European co-operation;
- co-operation with European and international organisations responsible for quality infrastructure, in particular by participation in the preparation of harmonized technical documents.

For more information visit the EURAMET web site at: www.euramet.org

A.1.4 Institute for Geometrical Product Specification

More information about GPS can be found at the Institute for Geometrical Product Specification website www.ifgps.com. Click on resources for more information on GPS.

A.2 Networks

A.2.1 Measurement Network

NPL has set up a national network on measurement for UK businesses with the support of National Measurement Office (NMO). The network brings sector and technology based communities closer to the measurement expertise through a number of community engagement mechanisms which include events, workshops, webinars, joint publications etc. In addition to supporting businesses to access measurement resources and capabilities, the network also provide insights on measurement innovations through measurement news, broker relationships with technology experts and provides a forum to engage in discussions and virtual networking with the measurement community through _connect and LinkedIn. A number of free resources including over 100 different good practice guides from ‘beginner’s guides’ to measurement and uncertainty to more specialised guides for measurement professionals and practicing scientists are free to download from the NPL website. To join the growing number of over 3000 businesses in the Measurement Network and
learn what measurement can do for your business visit www.npl.co.uk/measurement-network.

A.2.2 Mathematics and Modelling for Metrology (MMM)

Mathematics & Modelling for Metrology (MMM) is a programme that underpins the NMS, focusing on the use of mathematics and computing in metrology. It aims to achieve a balance between research and development, whilst also extending the range of techniques and applications available to meet the continually changing needs of metrology. The overall aim of the MMM Programme is to tackle a wide range of generic issues, some of which are problems in metrology that require the application of established software engineering practices, whilst others require advances in mathematics, software engineering or theoretical physics. The programme, thus, includes work in metrology, mathematics, software and theoretical physics, with strong links between the various disciplines.

Further details can be found at website: http://www.npl.co.uk/mathematics-scientific-computing/

A.3 Traceability

Traceability in measurement is the concept of establishing a valid calibration of a measuring instrument or measurement standard, by a step-by-step comparison with better standards up to an accepted or specified standard. In general, the concept of traceability implies eventual reference to an appropriate national or international standard.

The National Physical Laboratory is the United Kingdom's national standards laboratory. It operates at the heart of the National Measurement System (NMS) which is the infrastructure designed to ensure accuracy and consistency in every physical measurement made in the UK. Chains of traceability link UK companies’ measurements directly to national standards held at NPL.

For the majority of industrial applications, companies can establish a link to national measurement standards through the calibration and testing services offered by United Kingdom Accreditation Service (UKAS) accredited laboratories, which are in turn traceable to NPL. However, for challenging or novel measurements to the highest standards of accuracy, which are not catered for by UKAS-accredited laboratories, NPL can often provide a traceable measurement solution directly to industry.

The United Kingdom Accreditation Service is the sole national accreditation body recognised by government to assess, against internationally agreed standards, organisations that provide certification, testing, inspection and calibration services.

Accreditation by UKAS demonstrates the competence, impartiality and performance capability of these evaluators.

UKAS is a non-profit-distributing private company, limited by guarantee. UKAS is independent of Government but is appointed as the national accreditation body by the Accreditation Regulations 2009 (SI No 3155/2009) and operates under a
Memorandum of Understanding with the Government through the Secretary of State for Business, Innovation and Skills.

UKAS accreditation demonstrates the integrity and competence of organisations providing calibration, testing, inspection and certification services.

Further information on UKAS can be found at: [www.ukas.com](http://www.ukas.com).

**A.4 Training courses**

NPL provides training courses that teach the underpinning measurement principles and methods across a range of science and technology subjects from foundation through to expert level. The training is aimed at developing metrology skills and an awareness of why measurement best practice is vital to increase corporate productivity and competitiveness; and to satisfy regulatory requirements. A selection of NPL courses lead to an ‘Award in Metrology’ under the National Qualifications and Credits Framework for work placed learning. Further information is available at: [www.npl.co.uk/training](http://www.npl.co.uk/training).

**Dimensional measurement Training: Level 1 – Measurement User**

A three day training course introducing measurement knowledge focusing upon dimensional techniques.

**Aims & Objectives**

To provide:

- the underpinning knowledge and expertise for anyone who uses measurement tools or requires an appreciation of the importance of measurement,
- the principle knowledge and practical training for people who are required to use dimensional measurement techniques to complete their daily tasks; and
- the tools to instil and encourage questioning culture.
Enabling:

- An understanding of the fundamentals of standards, traceability, calibration, uncertainty, repeatability, drawing symbols and geometrical tolerances, the importance of the relationship between tolerances and measuring equipment and be able to question the measurement.

Level 1 is applicable to all industrial sectors as a stand-alone qualification or as a building block for further NPL Dimensional Measurement Training levels – 2 & 3.

Course Content

**Day 1 - Geometric Product Specification (GPS) A**
Including what is GPS, drawing practice and geometrical tolerances.

**Day 2 - Measurement Principles and Methods A**
Including successful measurements, standards, traceability, calibration, uncertainty, units, relationship between tolerances and measuring equipment using micrometers and callipers, repeatability and reproducibility of measurements.

**Day 3 - Measurement Principles and Methods B**
Including the relationship between tolerances and measuring equipment by the use of height gauges, dial test indicators, dial gauges, plug gauges, gap gauges and temperature effects.

NB: Fundamental Measurement Calculation is incorporated into all 3 days including powers, scientific notation and triangles. This is achieved by understanding the relationship of these calculations when applied to tolerance zones and practical measuring tasks.

A workbook of evidence must be completed successfully during the training course and, where required, post assessment tasks can be set for each individual to be completed in the workplace.

**Dimensional Measurement Training: Level 2 - Measurement Applier**
A four day training course for those who have a good basic understanding of measurement principles gained through the Level 1 training course.

**Aims & Objectives**

To provide:

- the underpinning knowledge and expertise for anyone who uses measurement tools or requires an appreciation of the importance of measurement,
- the principle knowledge and practical training for people who are required to use co-ordinate measurement techniques to complete their daily tasks; and
- the tools to instil and encourage questioning and planning culture
Enabling:

- a visible return on investment for a manufacturing organisation in the form of various production cost savings and an upskilled workforce,
- a reduction in re-work time and waste on the production line - faults and problems will be detected earlier in the production process; and
- An in-depth appreciation of why measurement is carried out and not simply how

Level 2 is applicable to all industrial sectors as a stand-alone qualification or as a building block for further NPL Dimensional Measurement Training levels – 3 & 4.

A workbook of evidence must be completed successfully during the training course and, where required, post assessment tasks can be set for each individual to be completed in the workplace.

**Course Content**

**Geometric Product Specification (GPS) B**
Content covered:
GPS standards; Envelope tolerance; Size Principles; ISO Limits and Fits Projected tolerance; Free state condition; Virtual condition; Maximum Material Condition principles; Geometrical tolerancing measurements using first principle measuring equipment; Surface texture principles.

**Measurement Principles and Methods C**
Content covered:
Calibration; Uncertainties; Traceability; Procedures; First Principle Measurement; Angle plate; Gauge blocks; Surface plate; Height micrometer; Sine bar or sine table.

**Process Control A**
Content covered:
Statistical Process Control theory; Variation – common, special causes; Prevention versus detection; Collecting and calculating data when using measuring tools; Callipers; micrometers; Basic charts – Tally chart/Frequency Table, Histogram, Control Chart; Reacting to variation; Benefits of process control; Standard deviation; Capability indices; Fundamentals of Gauge R&R.

**Measurement Principles and Methods D**
Content covered:
Taper calculations; Angles; Diameters; Searching for triangles; Chords; Radians; Manipulation of formula.

**Co-ordinate Principles A**
Content covered:
*Application of equipment:* First principles; Co-ordinate Measuring
Machine; Optical and vision machines; Articulating arm; Laser tracker; Projector; Microscopes; Height gauge with processor; Contour measurement equipment.

**Machine performance**: Calibration standards; Self-verification/artefacts; Measurement volume.

**Alignment Techniques**: 321/point system alignment; Flat face alignment; Axes alignment; Car line/engine centre line.

**Machine appreciation**: Ownership; Care; Respect; Cost; Contribution to the business.

**Work Holding**: Fixturing; Rotary table; Clamping; How to hold the part; Influence of component weight, size, shape; Free state; Restrained state.

**Co-ordinate geometry**: Points; Plane; Line; Circle; Cylinder; Cone; Sphere; Ellipse.

**Sensor Types**: Probing Strategies; Relevant standards; Environment.

**Measurement Strategies**: Number of points; Partial arc; Contact/non contact.

**Co-ordinate methods A (OEM Training - equipment specific)**

Content covered:

First principles; Co-ordinate Measuring Machine; Optical and vision machines; Articulating arm; Laser tracker; Projector; Microscopes; Height gauge with processor; Contour measurement equipment.
Appendix B  Basic mathematical concepts

To make full use of this guide there are a number of mathematical principles that you will need to be able to understand. This appendix will cover these principles and introduce some more advanced concepts such as matrix algebra. Don’t worry if you cannot grasp the more advanced concepts.

B.1  Some useful mathematics

B.1.1  Powers

Powers are a shorthand way of expressing when a number has been multiplied by itself a number of times. Some examples are given below. Note that a negative power means one divided by a number to that power.

\[ a^2 = a \times a \]
\[ a^3 = a \times a \times a \]
\[ a^{10} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a \]

\[ a^{-1} = \frac{1}{a} \]

\[ a^{-2} = \frac{1}{a \times a} \]

B.1.2  Scientific notation

Once we understand the concept of powers we can use it as a short hand way of expressing very large or very small numbers. We do this by raising 10 to either a positive or negative number. For instance:

\[ 1 \times 10^6 = 0.000\,001 \]
\[ 1 \times 10^3 = 0.001 \]
\[ 1 \times 10^3 = 1000 \]
\[ 1 \times 10^6 = 1\,000\,000 \]

B.1.3  Triangles

The majority of problems in dimensional metrology can be solved using right-angled triangles. Many calculations in dimensional metrology involve cylinders or spheres in contact with surfaces and the line through the centre of the cylinder or sphere and the point of contact lies normal to the surface (Figure 78).
From Figure 79 we can determine the lengths $a$, $b$ and $c$ and angle $A$, $B$ and $C$ using

$$a = b \cos C = \frac{c}{\tan C},$$

$$b = \frac{a}{\cos C} = \frac{c}{\sin C},$$

and

$$c = a \tan C = b \sin C.$$

The following trigonometric relationships can also be handy.
\[ \tan \phi = \frac{\sin \phi}{\cos \phi} \]
where \( \phi \) is any angle and
\[ \sin^2 \phi + \cos^2 \phi = 1. \]

Sometimes, however, you may not be able to use a right-angle triangle and the following formulae can be applied to any triangle – acute or obtuse (Figure 80 and Figure 81).
When you know one side and two angles then you can use the Sine Rule. As before $a$ is the side opposite angle $A$, $b$ is the side opposite angle $B$ and $c$ is the side opposite angle $C$,
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,
\]
where $R$ is the radius of the circumscribing circle.

Where two sides and the included angle are known then the remaining side can be found from the Cosine Rule
\[
a^2 = b^2 + c^2 - 2bc(\cos A).
\]

Where two sides and two angles are known, simply use
\[
a = b \cos C + c \cos B.
\]

Since the sum of the interior angles of a triangle equals 180º, knowing any two angles lets you determine the third – but doesn’t let you calculate the size of the triangle.

If only the three sides of the triangle are known and one or two angles have to be determined we can introduce the concept of the semi-perimeter, $s$, where
\[
s = \frac{a + b + c}{2}.
\]

Then we can use
\[
\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}.
\]

### B.1.4 Useful algebra

Factorisation of an expression will sometimes simplify the workings. The sum or difference of two squares can arise in the form $a^2 + b^2$ or $a^2 - b^2$.

Sadly $a^2 + b^2$ can’t be factorised, but $a^2 - b^2 = (a + b)(a - b)$.

Another useful equation is the general solution to a quadratic equation such as
\[
a x^2 + bx + c = 0.
\]

There are two possible solutions to the value of $x$ given by
\[
x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.
\]
It often turns out that one solution is imaginary, *i.e.*, involving the square root of a negative number. In a practical situation the choice of solution is often obvious.

If we consider the example in Figure 82 we may be interested in understanding how sensitive to the uncertainty of measurement a value for $R$ may be, or we may be interested in calculating the variation in apparent height of contact of a ball that has contacted on a slope instead of a horizontal surface.

From the triangles in Figure 82 we can see that

$$R^2 = y^2 + (R - x)^2.$$  

Expanding this equation we get

$$R^2 = y^2 + R^2 - 2Rx + x^2$$

then cancelling terms and rearranging them we get a quadratic in terms of $x$

$$x^2 - 2Rx + y^2 = 0.$$  

This expression can be solved using the method above and gives

$$x_1 = \frac{2R + \sqrt{(2R)^2 - 4y^2}}{2}$$

which simplifies to $x_1 = R + \sqrt{R^2 - y^2}$

and

$$x_2 = \frac{2R - \sqrt{(2R)^2 - 4y^2}}{2}$$

which simplifies to $x_2 = R - \sqrt{R^2 - y^2}$. 

![Figure 82 Chord of a circle](image)
Plugging in some sample numbers for a contact probing situation such as $R = 5 \text{ mm}$ and $y = 2.5 \text{ mm}$ gives the solutions $x_1 = 9.33 \text{ mm}$ and $x_2 = 0.67 \text{ mm}$. The obvious practical solution is 0.67 mm as it is the only solution less than $R$.

Changing the sample values to a large radius but short partial arc such as $R = 5000 \text{ mm}$ and $y = 25 \text{ mm}$ gives $x_1 = 9999.9375 \text{ mm}$ and $x_2 = 0.0625 \text{ mm}$. The obvious practical solution is 0.0625 mm.

**NOTE**

**Bad measurement practice**
As an example of bad measurement practice it is worth looking at the sensitivity of the calculated value of $R$ if there is even a small error in the measurement of $x$. One can rearrange the quadratic equation algebraically to give

$$R = \frac{x^2 + y^2}{2x}.$$  

If we consider a $+ 0.005 \text{ mm}$ error in $x$ we find that $R = 4629.66 \text{ mm}$. In other words a mere $5 \text{ µm}$ error in $x$ has manifested itself as a $370 \text{ mm}$ variation in the calculated radius.

So if you see a drawing with a large radius, a short partial arc and a radius tolerance of $\pm 1 \text{ mm}$ or less then you will know that the designer was having a senior moment!

**B.2 Some more advanced mathematics**

**B.2.1 Triangulation**

Knowing the distances between points is a powerful technique in co-ordinate metrology. Consider a group of three points in space as forming a triangle upon which we will base a co-ordinate system. We can then determine the position and co-ordinates of a fourth point relative to the group if we know the three lengths from the unknown point to each of the points in the group along with the distances between the three points in the base triangle.
There are four points (Figure 83) with three co-ordinates \((x, y, z)\) each making a total of twelve unknowns. We know the six lengths and can make assumptions about six of the point co-ordinates. We are basing the co-ordinate system on the triangle of three points 1, 2, 3 so we can assume that say point 1 is the origin and has the co-ordinates \((0, 0, 0)\) and that point 2 we shall say lies on the \(x\) axis so it has co-ordinates \((L_{12}, 0, 0)\) where \(L_{12}\) is the distance between point 1 and point 2. Finally we can say that point 3 lies in the same plane as point 1 and point 2 so it’s co-ordinates will be \((x_3, y_3, 0)\).

Now we can do a little co-ordinate geometry (basic vector analysis is a good method if you are familiar with the ideas) between points 2 and 3 (Figure 84) and we see that

\[
L_{23} = \sqrt{(x_3-x_2)^2 + (y_3-y_2)^2 + (z_3-z_2)^2}.
\]
As \( x_2 = L_{12}, \ y_2 = 0 \) and both \( z_3 \) and \( z_2 = 0 \) this expression simplifies to

\[ L_{23}^2 = (x_3 - L_{12})^2 + y_3^2. \]

Similarly \( L_{13}^2 = x_3^2 + y_3^2 \) and if we then subtract \( L_{13}^2 \) from \( L_{23}^2 \) and rearrange the terms we get

\[ x_3 = \frac{L_{12}^2 - L_{23}^2 + L_{13}^2}{2L_{12}} \text{ and } y_3 = \sqrt{L_{13}^2 - x_3^2}. \]

We have now fully defined the base triangle and can move on to consider the fourth point, thus

\[ L_{14}^2 = x_4^2 + y_4^2 + z_4^2, \]

\[ L_{24}^2 = (x_4 - L_{12})^2 + y_4^2 + z_4^2, \]

and

\[ L_{34}^2 = (x_4 - x_3)^2 + (y_4 - y_3)^2 + z_4^2. \]

We can difference these equations, expand the terms in brackets and simplify the result to obtain

\[ L_{24}^2 - L_{14}^2 = L_{12}^2 - 2x_4L_{12}. \]

This gives us

\[ x_4 = \frac{L_{12}^2 + L_{14}^2 - L_{24}^2}{2L_{12}}. \]

Similarly, for \( L_{34}^2 - L_{24}^2 \), while the mathematics gets a bit messy to include here, the equation simplifies to

\[ y_4' = \frac{L_{24}^2 - L_{34}^2 + (x_4 - x_3)^2 + y_3^2 - (x_4 - L_{12})^2}{2y_3}. \]

Finally applying trigonometry to the vertical triangle made up of point 1, point 4 and the projection of point 4 into the base triangle, which we have just worked out to be \((x_4, y_4, 0)\), we get

\[ z_4 = \sqrt{L_{14}^2 - x_4^2 - y_4^2}. \]

Do not be put off by these lengthy equations – these are exact solutions and once entered into a spreadsheet or programme can be used as often as necessary to solve
for the co-ordinates of any collection of four points (provided they don’t all lie in a plane or that three of them are in a line).

**B.2.2 Co-ordinate transformation**

Following on from triangulation, we can now consider the transformation of co-ordinate systems so that you can apply the same methods to other nearby points but with different base triangles – thus building up a bigger structure from a series of interconnected nodal points. (You may want to consider looking in a mathematical textbook at this stage to brush up on your matrices and matrix multiplication in particular.)

In practice you don’t need to worry about this sort of mathematics in any detail because the software packages associated with the measuring system will handle the details of co-ordinate and reference frame transformations. However, it is important to have some idea of what is going on, because it helps you to understand the occasional ‘funny result’.

**Translation matrix**

To move the point with co-ordinates \((x, y, z)\) by a displacement \(a\) in the \(x\) direction, \(b\) in the \(y\) direction and \(c\) in the \(z\) direction, insert the values \(a\), \(b\) and \(c\) into the \(4 \times 4\) translation matrix and multiply the \(4 \times 1\) matrix of the point co-ordinates matrix. To multiply the two matrices, each row in the first matrix is multiplied by each column in the second matrix (although there is only one column in this case). The method of multiplication is shown below:

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
1 \times x + 0 \times y + 0 \times z + a \times 1 \\
0 \times x + 1 \times y + 0 \times z + b \times 1 \\
0 \times x + 0 \times y + 1 \times z + c \times 1 \\
0 \times x + 0 \times y + 0 \times z + 1 \times 1
\end{bmatrix}
= \begin{bmatrix}
x + a \\
y + b \\
z + c \\
1
\end{bmatrix}.
\]

**Rotation matrices**

To rotate a point we would multiply the matrix containing \(x\), \(y\) and \(z\) by one of the following rotation matrices.

Rotation of \(\phi\) about \(x\) axis

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi & -\sin \phi & 0 \\
0 & \sin \phi & \cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Rotation of $\phi$ about $y$ axis

$$
\begin{bmatrix}
\cos \phi & 0 & \sin \phi & 0 \\
0 & 0 & 0 & 0 \\
-\sin \phi & 0 & \cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Rotation of $\phi$ about $z$ axis

$$
\begin{bmatrix}
\cos \phi & -\sin \phi & 0 & 0 \\
\sin \phi & \cos \phi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

It is worth noting that a combination of the above three rotation matrices will get you to any orientation you may wish to adopt, but you should pay attention to any convention that applies to the order of these rotations as it does make a difference to where the transformed point actually ends up!
Appendix C  Metrological nuggets

C.1 Using a precision tape

In one of the short stories we made use of precision tapes. When using a precision tape for accurate length measurements you have to make allowances for sag, tension, temperature and slope of the tape.

C.1.1 Sag correction

The sag correction $C_s$ is given by the equation

$$C_s = -\frac{n\omega^2 l^3}{24P^2}$$

where $\omega$ is the weight per unit length of the tape, the unsupported length of the tape is $l$, $P$ is the tension applied to the tape and $n$ is the number of unsupported lengths.

A single catenary span using a thin tape with a weight per unit length of 0.15 N m$^{-1}$, a length of 50 m and a tension of 50 N would have a sag correction of

$$\frac{0.15^2 \times 50^3}{24 \times 50^2} = 46.8 \text{ mm}.$$  

The true length will be the measured length plus the correction. In this case the correction is negative and the true length $L$ is given by

$$L = l - \frac{n\omega^2 l^3}{24P^2}.$$  

For a multiple catenary with frictionless rollers we could use twelve spans of 4 m and a single span of 2 m to make up a 50 m length.

The individual sag corrections would be

$$\frac{0.15^2 \times 4^3}{24 \times 50^2} = 24 \mu\text{m} \text{ for the 4 m span}$$

and

$$\frac{0.15^2 \times 2^3}{24 \times 50^2} = 3 \mu\text{m} \text{ for the 2 m span}.$$  

The total sag correction for the twelve span catenary would be $12 \times (24 \mu\text{m}) + (3 \mu\text{m}) = 291 \mu\text{m}$ a little less than 0.3 mm – compare this to the 46.8 mm of the single span catenary.
C.1.2 Tape tension correction

The calibrated length of a tape includes the stretch due to a calibrated tension.

From Hooke’s law we get the equation

\[ l_{\text{stretch}} = \frac{LP}{AE} \]

where \( L \) is the nominal length, \( P \) is the tension, \( A \) is the cross sectional area of the tape and \( E \) is the Young’s modulus of elasticity. For a steel \((E = 200 \times 10^9 \text{ N m}^{-2})\) tape 6 mm wide and 0.3 mm thick with a nominal length of 50 m the stretch would be

\[
\frac{50 \times 50}{0.006 \times 0.0003 \times 200 \times 10^9} = 6.9 \text{ mm.}
\]

Tapes are calibrated at a specific tension – if you apply more tension not only does the tape stretch, but the catenary changes shape – best to stick with the correct tension.

C.1.3 Temperature correction when using a tape

The tape material expands with increased temperature, so the distance on the ground it actually covers is more for the same measured distance – the tape reads short. As the temperature increases the correction needs to get greater. Conversely as the temperature drops below the calibrated temperature the measured distance must be reduced.

The temperature of the tape is influenced not only by the surrounding air temperature, but also by any sunlight falling on it and the temperature of the ground on which it may lie. The tape temperature will normally be measured using a mercury in glass thermometer. Wrapping the thermometer bulb in a similar material as the tape and placing it in similar circumstances can help. Alternatively the tape temperature can be measured by monitoring the electrical resistance along the tape. Usually measuring the air temperature is sufficient.

To calculate the correction, \( C_t \), we use the following equation,

\[ C_t = l\alpha(T - T_s) \]

where \( l \) is the measured length and \( \alpha \) the expansion coefficient of the tape material, \( T \) is the measured temperature and \( T_s \) is the reference temperature.

The correct length, \( L \), is, therefore,

\[ L = l + l\alpha(T - T_s). \]
C.1.4 Slope correction for a tape

It is often necessary to make a slope correction when using a tape. Two methods are available to make this correction. The first is to reduce to the horizontal either by measuring the slope angle, $\alpha$, directly and applying the formula

$$ D = S \cos \alpha, $$

where $D$ is the corrected length and $S$ is the measured length.

An alternative method involves using the known height difference, $\delta h$, and applying Pythagoras’ Theorem, thus

$$ D = \sqrt{(S^2 - \delta h^2)}. $$

C.2 Using reversal to determine the error map of a CMM

We covered simple reversal in an earlier chapter. The following is a more complex extension of the idea of reversal.

A ball plate is an artefact that can be used to calibrate CMMs via a technique developed by PTB. The size of the ball plate and the number of balls in it is chosen to suit the measuring volume of the CMM. Typically a raster or grid pattern of $5 \times 5$ balls is used and the size of the plate varies from $300 \text{ mm} \times 300 \text{ mm}$ to say $600 \text{ mm} \times 600 \text{ mm}$. A multi-tip star stylus is qualified against the reference sphere and then the ball plate is measured in a number of different positions.

Figure 85 A hole plate being measured (an alternative to a ball plate)
Initially the plate is measured in the $xy$ plane while resting on the table of the machine. Next the plate is raised to the top of the measuring volume – still in the $xy$ orientation and measured again. The plate is then positioned in the $xz$ orientation and measured from both sides. Finally the plate is positioned in the $yz$ orientation and again measured from both sides. At the end of the measurement cycle the ball plate has been measured six times. The co-ordinates of the measured balls and the certified ball positions are fed into a computer programme that uses a kinematic model of the CMM to solve for the parametric errors of the CMM being tested. This data can then be used to update the error map of the CMM. Once again, it is essential to undertake a performance verification test to ensure that the data has been gathered, processed and updated correctly.

The initial calibration of a ball plate is usually performed on a high accuracy CMM but exploits the reversal technique to achieve high accuracy independently of the CMM performing the calibration. The measurement procedure involves measuring the balls by starting at ball 1 and then moving in a spiral until eventually the middle ball is measured. Each ball is then measured again while spiralling outwards, ending at ball 1. This spiral in and spiral out motion tends to eliminate any backlash in the machine.

The reversal process involves rotating the ball plate about each of the machine axes in turn and repeating the spiral in and spiral out measurements. Thus a rotation about the $z$ axis forms one of the reversals, a flip of the ball plate about the $x$ axis forms another, and the final flip about the $y$ axis (relative to the initial position in each case) forms the final reversal. The beauty of the two dimensional ball-plate as an artefact is that it can be easily probed from both sides, as too can a hole-plate.

At the end of the process the spiral in and spiral out measurements in four positions yields eight sets of co-ordinates for each of the balls. The calibrated co-ordinate of each ball is simply the average of all eight results. One is cautioned to the fact that strict control of the temperature of the plate is required during the measurement process and that some care must be taken to provide a traceable scale factor in all axes during the measurements.
Appendix D  Some useful physics

D.1  Mechanics of beams

The stiffness or ability to resist bending of a beam can be calculated using knowledge of the beam dimensions and the properties of the material from which it is made. The properties of the cross section of the beam are termed the second moment of area and given the symbol \( I \).

It is important to have an appreciation of the stiffness of things because we often make measurements relative to something and if that reference is moving we will be in error.

![Figure 86 Simple beam cross-sections](image)

For a simple case like a beam of rectangular cross-section (Figure 86)

\[
I = \frac{bd^3}{12}
\]

and for a beam of circular cross-section (Figure 86)

\[
I = \frac{\pi d^4}{64}.
\]

The key point to notice here is that, for a rectangular beam, if you double the breadth of the beam, \( b \), you simply double \( I \), but if you double the depth of the beam, \( d \), you increase \( I \) by a factor of eight because of the cubic term in the equation. Note – if you go to extremes and use a beam that has a very high depth to breadth ratio then other effects come into play and the beam buckles sideways and can fail under load.
The strength of a material is described by the Young’s modulus (represented by the symbol \( E \)) and is the ratio of the stress to the strain in the beam. In other words the ratio of the load per unit cross sectional area divided by the extension per unit length.

Typical values of \( E \) for steel may be \( 207\,000\,000\,000\,000\,000 \text{ N m}^{-2} \) and this can be expressed as \( 207\,\text{GN m}^{-2} \).

Provided you are consistent with your units and don’t mix metric and imperial values, you should be able to work out the deflections, \( \delta \), associated with simple loading cases such as a cantilevered beam, a simply supported beam and a constrained beam (Figure 87).

\[
d = \frac{WL^3}{3EI}
\]

\[
d = \frac{WL^3}{48EI}
\]
The load applied, $W$, is calculated using the mass of the object multiplied by the local 
acceleration due to gravity (typically 9.81 m s$^{-2}$ and usually it’s good enough to round 
it up to 10 m s$^{-2}$).

Thus a 150 mm steel ruler, 19 mm wide and 0.5 mm thick would have a second 
moment of area

$$I = \frac{0.019 \times 0.0005^3}{12} = 1.979 \times 10^{-13} \text{ m}^4.$$

Simply supported at its ends and loaded with a big apple weighting 100 g (0.1 kg) we 
can use the following equation to calculate the expected maximum deflection.

$$\delta = \frac{WI^3}{48EI} = \frac{1 \times 0.15^3}{48 \times 207 \times 10^9 \times 1.979 \times 10^{-13}} = 1.716 \times 10^{-3} \text{ m.}$$

This is about 2 mm and intuitively we know that this is about right, because we can 
play with a ruler and try it out with our lunch!

We move on to consider two important cases from the metrologist’s point of view that 
we covered earlier.

- Bending of a beam simply supported at the Bessel points this results in the 
deflection in the mid-point equalling the deflection at the end points and is very useful for supporting straightedges. The Bessel points are 0.554$L$ apart 
and equally spaced from the end of the beam.

- The bending of a beam supported at the Airy points results in the end faces 
being parallel to one another – this is very useful for the calibration of length
bars by interferometry for instance. The Airy points are $0.5773L$ apart and equally spaced from the end of the beam.

The final point to note regarding the bending of beams and the measurement of length is that along the neutral plane of a beam – in other words where there is neither compressive nor tensile strains – the length of the beam does not change.

![Figure 88 An H section beam](image)

Normally neutral plane of the beam would be inaccessible but a useful design by H E Tresca in around 1875 uses an H section (Figure 88) to expose the neutral plane, increase the stiffness (second moment of area, $I$) while reducing the weight per unit length of a scale bar.

**D.2 Some useful optics**

We don’t need to get too involved with the physics of optics, but as a number of useful metrological instruments (see Chapter 8) rely on optical elements it is useful to consider a few of the basic principles.

Light is part of the electromagnetic spectrum - that includes radio waves, microwaves, infrared, visible light, ultraviolet, x-rays and gamma rays. Light has an interesting characteristic – its speed in a vacuum is constant – the exact value is defined by the CGPM as 299 792 458 m s$^{-1}$. The apparent speed of light in a transparent media is, however, slower and this causes some interesting effects that can be useful.

<table>
<thead>
<tr>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>But the speed of light is constant</strong></td>
</tr>
<tr>
<td>We have previously stated that the speed of light is constant. In contradiction to this we have also stated that the speed of light changes from one medium to another. As the light passes through a medium it is continuously being absorbed and re-emitted by the atoms in the medium. We can then say that between being absorbed and reemitted light travels at 299 792 458 m s$^{-1}$ but that there is a delay during absorption and emission. Because of the delay the speed of light in the medium appears slower.</td>
</tr>
</tbody>
</table>
D.2.1 Reflection

Figure 89 shows the reflection of a ray of light by a plane mirror.

![Reflection from a plane mirror](image)

Figure 89 Reflection from a plane mirror

We note that the angle between the normal to the mirror surface and the incident ray (the incident angle) is the same size as the angle between the normal and the reflected ray (angle of reflection).

![Reflection on rotation](image)

Figure 90 Reflection on rotation

If we now rotate the mirror by a small amount (Figure 90) then the angle of incidence will increase by the same amount relative to the normal of the mirror surface. The reflected ray still has the same angle of reflection as the angle of incidence, but the mirror normal has moved and so the angle of reflection has changed by twice as much as the tilt of the mirror relative to the incident ray – a very useful magnification effect.

In fact this magnification effect can be exploited by introducing a second fixed mirror (Figure 91) and for a rather limited range of rotation we can increase the sensitivity by a factor of four. There are, however, practical limits imposed by the physical size of the mirrors and the requirement for them to be very flat.
Some elaborate schemes exist whereby multiple mirrors and polygons can be used to multiply the sensitivity of an angular measuring system, but the limited angular range and the need for very flat mirrors hamper such schemes (Figure 92).

D.2.2 Anti-reflection coatings

A very important consideration in the design of optical systems is the use of anti-reflection (AR) coatings. These are used to cut down on the number of false images and stray reflections that can sometimes occur within an optical system.

D.2.3 Refraction

The apparent bending of a stick dipped into a pool of water is a simple example of the refraction of light as it passes from one transparent medium (in this case water) to another of a different refractive index (in this case air). The propagating wave front speeds up as it enters the air and the net result of the wave front being inclined to the
surface interface is that the direction of the whole wave front changes – this is refraction (Figure 93).

![Figure 93 Refraction at the air water boundary](image)

The refractive index, \(n\), of a medium is given by

\[
n = \frac{\sin i}{\sin r}
\]

where \(i\) is the incident angle and \(r\) is the refracted angle. Typical values for refractive index are given in Table 8.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
</tr>
<tr>
<td>Air</td>
<td>1.00027</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Glass (BK7)</td>
<td>1.51</td>
</tr>
</tbody>
</table>

The refractive index for any transparent material varies with the wavelength of the light. Thus white light (made up of many colours) passing through a dispersing prism is separated into its different component colours.

The shorter wavelengths (blue) are refracted through a greater angle than the longer wavelengths (red). The complete spread of all wavelengths is called a spectrum, for example, a rainbow.

A lens, while usually made of simple curved surfaces, can be considered as a stack of different angled prisms and by choosing the appropriate angle for each prism, light at different radial distances can be brought to a common focal point.

A convex lens is thicker in the middle than at the outer edges and causes a beam of parallel rays passing through it to converge at a single point known as the principle focus. A concave lens is thinner in the middle than at the outer edges and causes a beam of parallel rays to diverge in such a way that the beam appears to come from behind the lens form a single point called the virtual focal point.
D.2.4 Double refraction

If we take at face value that light is made up of two sine waves at right angles to each other. We also need to accept that one of these two sine waves may travel faster through a transparent media than the other due to the interactions of the electromagnetic waves with the lattice structure of the material. The slow wave is referred to as the ordinary or $O$ ray and this obeys Snell’s law of refraction for instance, the other is the extraordinary or $E$ ray and it doesn’t! Figure 94 illustrates this. The material effectively has two refractive indexes $n_E$ and $n_O$, one for the velocity of the $O$ wave and one for the velocity of the $E$ wave.

An illustration of this is provided by a phenomenon called double refraction that is observed when a beam of ordinary unpolarised light is incident on a calcite crystal. In addition to the reflected beam there will be two refracted beams instead of the single refracted beam that would be observed in a glass medium. Calcite and quartz are examples of anisotropic crystals – their physical properties vary with direction and calcite is an example of a uniaxial crystal in that it has only a single optical axis (an axis of symmetry relative to both the crystal form and arrangement of atoms). If any property of the material is measured for different directions it will be found to be the same along any line perpendicular to the optic axis. At other angles it changes reaching a maximum or minimum along the axis. There is no separation of $O$ and $E$ rays along the axis.

![Figure 94 Double refraction or birefringence](image)

Huygens discovered the polarisation of light by double refraction in calcite in 1678. This principle is used in quarter and half wave plates and in the Wollaston prism.

D.2.5 Polarisation

Natural or un-polarised light can be thought of as a whole bunch of out of step sine waves travelling along a light ray all at different angles to each other. If we then polarise the light by allowing it to pass through a polarising filter we effectively block all the sine waves except the ones that are oriented in such a way that they can pass through the polariser.
D.2.6 Diffraction

When waves pass through a small aperture (dimensions less than the wavelength of the light) or pass the edge of an obstacle they always spread out beyond the region that is directly exposed (Figure 95) – this is diffraction. A quick check on Huygens’ principle in any decent optics book will expand on this short statement.

![Diffraction at a single slit](image)

Figure 95 Diffraction at a single slit (slit width exaggerated)

Light does not always travel in straight lines and in fact when the aperture or slit is narrow (compared to the wavelength of the light), the waves can spread out from the slit in the form of semi-circles.

D.2.7 Interference

Young’s double slit experiment (Figure 96) provides a graphic illustration of interference between two coherent light sources (coherent simply means that the sine waves from each source are in step with each other). In a modern reproduction of Young’s experiment a narrow single slit (of the order of 0.001 mm wide) is illuminated by a bright source such as a filament lamp and the diffracted light that passes through the slit falls onto a nearby double slit arrangement. The light is diffracted through these two slits and is observed on a screen. Interference fringes are observed on the screen – they are alternating bands of dark and bright – representing the destructive and constructive interference of the waves coming from the two slits.
The interference described here should be compared with the description of interference described in chapter 9.

### D.2.8 Optical flats

An optical flat is typically a quartz disk 50 mm to 75 mm in diameter and between 10 mm and 15 mm thick. One surface of the flat has been polished and lapped to form an extremely flat surface – usually expressed as a fraction of a wavelength of light, for example, $\lambda/10$. The wavelength is usually stated, but the default value is the middle of the visible – green light – with a wavelength of say 0.5 µm. Thus an optical flat with a flatness of $\lambda/10$ is actually flat to 0.05 µm.

The optical flat is used to measure the flatness of a polished surface, say the anvil of a micrometer. The observer looks for interference fringes between the face of the anvil and the face of the optical flat when they are placed in intimate contact with each other and illuminated by a monochromatic light source such as a sodium lamp.

The flatness of the micrometer anvil can be estimated from the curvature of the fringes. Note it is not the number of fringes that is important – that is related to the angle between the flat and the anvil – rather it is the distortion of an individual fringe as a proportion of the fringe spacing that needs to be estimated. (A detailed account of using optical flats is given in Dotson C, Harlow R, Thompson R L 2003 *Fundamentals of dimensional metrology, Fourth Edition*, (New York: Thomson Delmar Learning)).

### D.2.9 Retarder plates

Various optically transparent materials, such as quartz and mica, have a crystalline structure that enables them to be orientated in such a way as to selectively retard the $O$ ray relative to the $E$ ray and this can be exploited to produce some useful optical devices.

A mica quarter wave plate has a thickness of about 25 µm and the retardance is such that the $E$ and $O$ rays change phase relative to each other by 90° converting the linear
polarised light into circularly polarised light. This is a very handy feature as it can be combined with a linear polariser to form an isolator that blocks reflections from surfaces beyond the isolator. An isolator works because circularly polarised light changes its direction of rotation upon reflection and when the reflected light passes back through the quarter wave plate it is retarded again and converted into linearly polarised light that has been rotated by 90° relative to the input ray. Having passed back through the quarter wave plate the reflected ray is then absorbed by the linear polariser - effectively isolating the source of the input ray from reflections. This is a very important device for preventing feedback into the laser cavity of stabilised lasers that are used for metrology purposes.

A half wave plate changes the direction of linear polarisation so for instance a vertically polarised ray would become horizontally polarised. This can be handy in complicated optical arrangements when you are combining several beams from different directions and want to ensure that interference can occur.

D.2.10 Total internal reflection

Total internal reflection occurs when a ray of light passing through a dense medium such as glass comes to the glass/air interface at an incident angle that exceeds the critical angle and instead of being refracted and emerging from the glass, as we would expect from Snell’s law, it is reflected back into the glass.

Frustrated total internal reflection occurs when a ray of light travelling through glass strikes an interface at an angle exceeding the critical angle. It should be totally internally reflected at the glass/air interface, but if another piece of glass is placed close to (but not quite touching) the interface, some light will couple through the thin gap and propagate. Both the reflected and transmitted beams will be affected, depending on the thickness of the gap. In the limit of the gap having zero thickness, the light will continue as if there were no boundary. In the limit of a large gap, more than a fraction of a wavelength, then virtually all the light is perfectly reflected.

D.2.11 Beam-splitters

Frustrated total internal reflection is exploited in the construction of beam-splitters and by varying the nature of the optical coatings that make up the gap between the right angle prisms, the opticians can construct cube beam-splitters that reflect and transmit different proportions of the incident light and can also arrange to separate light with different polarisations.

D.2.12 Fibre optic cables

Fibre optic cables can be thought of as special light pipes allowing a user to mount a light source such as a laser in some remote location and then couple the light into one end of the fibre using a short focal length lens. Some fancy alignment aids are used to ensure that the maximum amount of light enters the fibre.

At the other end of the fibre it is usual to plug the fibre into a collimator – essentially a lens mounted in a housing with the polished face of the fibre at the focal point of the lens. You’ll need to match the lens to the numerical aperture (NA) or cone angle of
the fibre and check that the light that emerges from the collimator is in fact parallel – this can be done simply enough by shining the beam across the room and checking that the diameter of the beam doesn’t change from close to the collimator all the way across to the other side of the room. Be particularly careful to check that the beam is not focused to a tiny dot in the middle of the room by looking at the beam on a piece of paper or card as you walk across the room.

Optical fibre comes in three basic forms:

- **Multimode fibre** has a relatively large central core and can support reflections of light from a range of different angles. This means that the light will travel different distances along the fibre, so even though it all started at the same time (in phase) the output will have a mishmash of jumbled phases. Multimode fibre is typically used for intensity-based applications where phase information is not important.

- **Single mode fibre** has a much smaller central core and essentially can support reflections of light at only one incident angle. Thus all the light that enters the fibre exits it at the same time having travelled the same distance – this preserves the phase information in the waveform and is very useful in high-speed telecommunications systems and signal processing.

- **Polarisation maintaining fibre** can be thought of as a modified single mode fibre that has a graded refractive index structure running through it. So we can imagine that the fibre appears oval to the incoming waveform. Thus only light with its polarisation aligned with the major axis of the oval can enter the fibre at the appropriate angle to be reflected by the single mode fibre and thus propagate. Any other orientation of polarised light will be quickly attenuated in the fibre, as it is not reflected at the appropriate angle. Polarisation maintaining fibre can, therefore, be used for interferometry systems as both the phase and the polarisation is maintained. A word of caution, however – polarisation maintaining fibre is very sensitive to physical disturbances, so don’t move it about or shake it or subject it to nasty thermal shocks *etc.*, it does not like it at all!

### D.2.13 Retro-reflectors

The retro-reflector is an extremely useful optical element and has the unusual ability to reflect an incoming ray so that the return beam is parallel to the original input beam. One way this can be accomplished is by reflecting the beam from three mutually orthogonal surfaces. This type of retro-reflector is also known as a corner-cube reflector as it can be imaged to be the corner of a glass cube that has been sliced off and then rounded by a cutting plane that is orthogonal to the line running through the centre of the cube and the corner. The solid glass corner cube does suffer slightly from apparent wander of the optical centre as it is tilted due to refraction at the air/glass interface; in addition it suffers from absorption and chromatic aberration.

In contrast, the hollow corner cube is constructed from three mirrors arranged into a corner cube with mutually orthogonal surfaces. As the optical path is entirely in air it
is insensitive to position and movement of the retro-reflector and is the preferred option in spherically mounted retro-reflectors (SMRs) for laser trackers.

An alternative retro-reflector is the cat’s eye. This generally consists of a lens with a mirror at the focus.

D.2.14 The laser

This section explains some of the physics behind the operation of a laser. The description is quite detailed, hence it’s positioning in the appendix of this guide. Don’t worry too much if you don’t understand this section.

The word laser is an acronym for Light Amplification by the Stimulated Emission of Radiation. All materials exhibit bright line spectra if they are excited in some way say by an electric current or radio frequency field or even a bright flash of light. The spectral lines are the result of spontaneous transitions of electrons within the material from higher to lower energy levels. Because light is quantised, only a discrete set of energy levels and transitions are permitted.

The He-Ne laser depends on energy level transitions in the neon gas and there are many possible wavelengths of light in its spectrum including a weak one at 632.8 nm. The helium in the laser is used to couple energy from the discharge to the neon via collisions with the neon atoms. This has the effect of pumping up the neon to a higher energy state and results in a population inversion – that is there are more atoms in the excited state than there are atoms in the ground state.

The excited state of neon that produces the weak spectral line at 632.8 nm has an energy level that almost matches the energy level of helium’s lowest excited state and the vibrational coupling between the two states is very efficient.

To turn the spontaneous emission of all the possible transitions into the stimulated emission of the laser we have to selectively amplify one of the wavelengths and provide feedback to sustain the oscillation. Placing a mirror at each end of the gas discharge tube to form a resonant cavity usually does this. The mirrors are usually specially coated for high reflectivity at the wavelength of interest and one mirror is fully reflective, while the other is partially reflective to allow the laser beam to exit the cavity.

When a spontaneously emitted photon of the selected wavelength happens to be emitted in a direction nearly parallel to the long axis of the tube, it stimulates additional transitions in excited atoms. These excited atoms then emit photons at the same wavelength, phase and direction and these photons bounce back and forth in the resonant cavity between the mirrors. Each pass through the discharge results in amplification of the light and provided the gain due to stimulated emission exceeds the losses due to imperfect reflectors and other losses, then the intensity builds up and a coherent beam of laser light emerges via the partially reflecting mirror at one end.

18 A good introductory text is Hecht 1987, while Sam Goldwasser maintains an interesting online resource at http://www.eio.com/repairfaq/sam/lasersam.htm and the Zygo Primer on displacement measuring interferometers can be found at http://www.zygo.com/.
Spontaneously emitted photons that travel in other directions may miss the mirrors completely or result in stimulated photons that are themselves lost after only a few reflections. The mirrors will poorly reflect photons with different wavelengths and any light at these wavelengths dies out.

The cavity will only resonate strongly if there is a standing wave pattern present – this occurs when an integral number of half wavelengths fit between the mirrors. The laser output tends not to be a single peak, but rather a series of peaks spaced \( \frac{c}{2L} \) Hz apart, where \( L \) is the cavity length and \( c \) is the velocity of light. As an example a cavity length of 150 mm results in a longitudinal mode spacing of about 1 GHz. The strongest spectral lines will be nearest the combined peak of the lasing medium and the mirror reflectivity but many lines will be present hence the term multimode operation.

Since the standing wave pattern is determined by the separation of the mirrors it is common practice with high precision laboratory lasers to use either passive stabilisation – that is to use low expansion materials – or active stabilisation – that is to use optical feedback and a piezoelectric actuator to move one of the mirrors, to compensate for thermal effects and hold the mirrors fixed relative to one another.

To refine the spectral line of the multimode laser and produce a single monochromatic mode, one can introduce another resonator into the cavity – this can be an iodine absorption cell as used in the NPL designed stabilised lasers, or a short etalon with parallel plates that introduces a set of modes of it’s own but with much wider mode spacing due to the short length. Only those modes that are supported by both the etalon and the main laser cavity will produce enough gain to sustain laser output. By adjusting the angle of the inter-cavity etalon, a spectral line can be selected to coincide with a peak in the main gain function – this will result in single mode operation.

Normal He-Ne lasers have a coherence length of 100 mm to 300 mm or so, but single mode He-Ne lasers can have coherence lengths running into the 100’s of metres and these are the lasers we use for measurement.

For now we can simply consider a laser to be a nice bright monochromatic light source with a coherence length determined by its construction and how much we are prepared to spend buying one!
NOTE

Important safety information
A rough guide to laser safety stickers would say that any laser system with a visible output of less than 0.2 mW is considered a Class 1 laser and is not dangerous. While any visible laser of between 0.2 mW and 1.0 mW output power is considered a Class 2 and relies on sensible people blinking before any damage is done to their vision. Class 3B refers to power levels above 5.0 mW and can cause damage to your retina and should on the whole be treated with a great deal of respect because once damaged your eyes are irreparable and irreplaceable! Class 4 involves powers above 0.5 W and will blind you, burn holes through your hand and generally ensure that you have a really bad day. (For a more detailed description of the classes have a look at BS EN 60825-1 1994.)

The He-Ne gas laser has been around since the early 1960’s and has typical output powers in the range 0.5 mW to 5.0 mW. The He-Ne laser has been used as the light source for a number of commercial displacement measuring systems.

D.2.15 Commercial laser interferometer systems

In 1970 one commercial manufacturer brought out a laser interferometer system that used a He-Ne gas laser with a cylindrical magnet around the cavity that causes the laser to oscillate at two slightly different frequencies $f_1$ and $f_2$. The interaction of the magnetic field with the electromagnetic waves within the cavity causes a lasing action with two opposed circularly polarised states separated in frequency by about 2 MHz – this is Zeeman splitting. The two circularly polarised waves emerging from the laser tube pass through a quarter wave plate and are converted into linearly polarised waves at right angles to each other. A portion of the emerging beam is picked off by a beamsplitter and used to establish the difference in frequency between the two optical components.

In the measurement process the laser beam passes through the optics in the measurement path and returns to the receiver. The receiver detects the frequency difference $f_1 - f_2$ plus any Doppler shifted frequency component $\Delta f$ that would be present if there had been any relative motion of the optics. The two polarisations do not actually form fringes until they reach the demodulating polariser mounted at 45˚ in front of the detector.

Homodyne and heterodyne

A homodyne interferometer uses a single frequency or wavelength of light and relies on the counting of fringes – seen as intensity changes - and the ability to interpolate partial fringes to get the desired resolution. Multiple detectors and suitable optics allow one to generate quadrature signals – in other words the sine and the cosine of the incident wave – and this allows the direction of motion to be determined and also
minimises errors caused by fluctuations in the laser intensity. This is made much easier these days with high-speed electronics and digital signal processing and speeds of 6 ms\(^{-1}\) and resolutions of 1 nm are possible.

A heterodyne interferometer measures changes in optical phase between a measurement signal of unknown frequency and a reference signal of known frequency at discrete time intervals. The phase change represents the Doppler shifted frequency that results with movement of the optical target. This shift is measured using a photodetector and has a frequency \( f = f_2 - (f_1 \pm \Delta f_1) \). The phase difference is measured every cycle and any phase changes are counted using digital accumulators. High resolution is made possible using phase interpolation schemes such as digital look-up tables.

### D.3 Kinematic design

James Clark Maxwell (1890) was one of the first scientists to rigorously consider kinematic design. He stated that:

> 'The pieces of our instruments are solid, but not rigid. If a solid piece is constrained in more than six ways it will be subject to internal stress, and will become strained or distorted, and this in a manner which, without the most micromechanical measurements, it would be impossible to specify.'

These sentences capture, essentially, the main concepts of kinematic design. Kinematics is a branch of mechanics that deals with relationships between the position, velocity and acceleration on a body. Kinematic design aims to impart the required movements on a body by means of constraints.

A rigid body possesses six degrees of freedom in motion - three linear and three rotational. In Cartesian co-ordinates the degrees of freedom are in the \( x, y \) and \( z \) directions plus rotations about each of the axes. A constraint is that which prevents minimally motion in just one of the degrees of freedom. There are two lemmas of kinematic design:

(i) any unconstrained rigid body has six degrees of freedom;
(ii) the number of contact points between any two perfectly rigid bodies is equal to the number of constraints.

This means that

\[
\text{Number of constraints} + \text{remaining number of degrees of freedom} = 6.
\]

There are often many assumptions applied when carrying out kinematic design. Real bodies are not perfectly rigid and will experience both elastic and possibly plastic deformations under a load. Such deformations will exclude perfect point contacts and cause unwanted motions. For this reason it is often important to choose with care the materials, shapes and surface texture of a given part. Despite this, kinematic design is an extremely important concept that the designer must master.
The principle of kinematic design states that point contact should be established at the minimum number of points required to constrain a body in a desired position and orientation.

We may wish to completely constrain the body in which case we could rest it on three widely spaced points that lie in a plane. This would stop the body from moving down through the plane and from pitching up or rolling. The body would still be free to yaw – rotate on the plane and it could of course slide along the plane in two directions.

We can now use two points of contact to form a line that contacts the side of the free body and prevents the yaw and movement through the line. However, the body can still slide along the line.

To completely constrain the free body we introduce a sixth point of contact that constrains the movement along the line. The result is a fully constrained body and a support configuration often referred to as 3-2-1. There are practical limitations due to the high contact stresses and compromises may need to be made over the area of contact and the number of contact points leading to the concept of elastic averaging.

A kinematic trihedral socket is the correct location for a sphere. It is as an inverted three faced pyramidal indentation which can be successfully implemented by gluing three small steel balls in a cluster.

The Type I and Type II Kelvin clamps are examples of fully constrained systems, *i.e.* those with six constraints. When designed properly these clamps are very effective where accurate re-positioning is required and are stable to within nanometres.

Both clamps have a top-plate (on which, for example the object to be measured is placed) that has three rigid spheres spaced on a diameter. The three spheres then contact on a flat and in a vee and a trihedral hole, as in the left-hand image of Figure 97, or in three vee-grooves, as in the right-hand image in Figure 97. In the Type II
clamp it is easy to see where the six points of contact, \textit{i.e.} constraints are - two in each vee-groove. In the Type I clamp one contact point is on the flat, two more are in the vee-groove and the final three are in the trihedral hole. The Type I clamp has the advantage of a well-defined translational location based on the position of the trihedral hole, but it is much harder to make. A trihedral hole is produced by pressing three spheres together in a flat bottomed hole (the contacting sphere will then touch at a common tangent) or by complex angled machining techniques. The Type II is more symmetrical and less influenced by thermal variations. Note that the symmetrical groove pattern confers its own advantages but is not a kinematic requirement: any set of grooves will do provided that they are not all parallel.