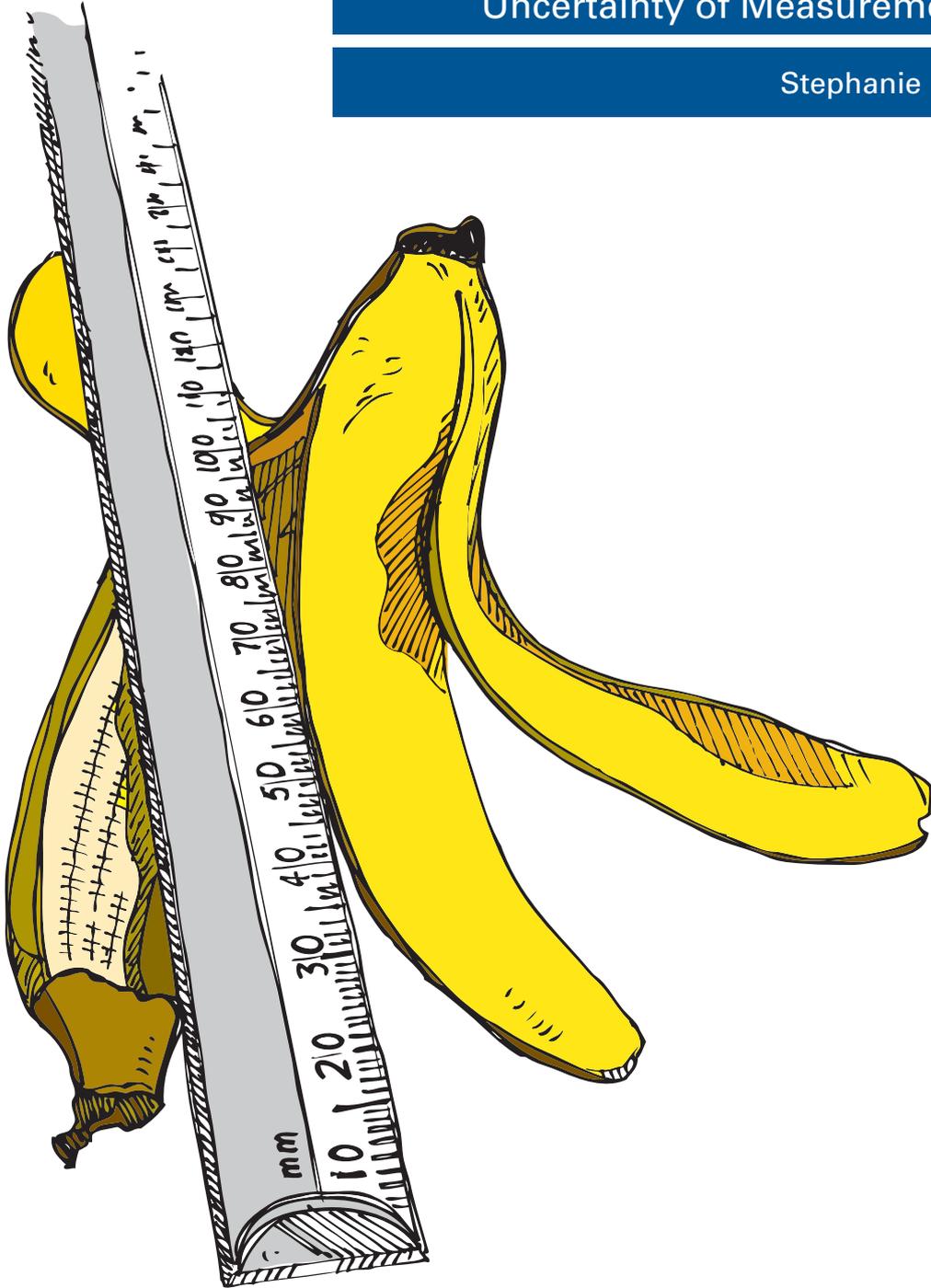


## Good Practice Guide No. 11

### The Beginner's Guide to Uncertainty of Measurement

Stephanie Bell



## **Measurement Good Practice Guide No. 11 (Issue 2)**

### **A Beginner's Guide to Uncertainty of Measurement**

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**Abstract:** The aim of this Beginner's Guide is to introduce the subject of measurement uncertainty. Every measurement is subject to some uncertainty. A measurement result is only complete if it is accompanied by a statement of the uncertainty in the measurement. Measurement uncertainties can come from the measuring instrument, from the item being measured, from the environment, from the operator, and from other sources. Such uncertainties can be estimated using statistical analysis of a set of measurements, and using other kinds of information about the measurement process. There are established rules for how to calculate an overall estimate of uncertainty from these individual pieces of information. The use of good practice – such as traceable calibration, careful calculation, good record keeping, and checking – can reduce measurement uncertainties. When the uncertainty in a measurement is evaluated and stated, the fitness for purpose of the measurement can be properly judged.



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# A Beginner's Guide to Uncertainty of Measurement

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## **Foreword**

This is a beginner's guide for people who know little or nothing about uncertainty of measurement, but need to learn about it. It is for technicians and managers in testing and calibration laboratories, technicians and managers in manufacturing, technical salespeople, research scientists, students, teachers, and everyone who has an interest in measurement.

This Beginner's Guide will not teach you all you will need to know to perform your own uncertainty analysis. But it explains the most important things you need to understand before you can master the subject. It will prepare you to read the more advanced and authoritative texts on uncertainty. In particular, this Guide will be useful preparation for reading the United Kingdom Accreditation Service (UKAS) Publication M 3003, 'The Expression of Uncertainty and Confidence in Measurement', and the Publication EA-4/02 of the European co-operation for Accreditation (EA), 'Expression of the Uncertainty in Measurement and Calibration'.

Many people are daunted by the subject of measurement uncertainty. It is a subject that is widely misunderstood, from the factory floor to the highest academic circles. It is a complicated subject, and still evolving. So there is a great need for a guide that provides clear, down-to-earth explanations, easy enough for non-expert readers. In the development of this Beginner's Guide, care has been taken to make the explanations and examples understandable to anyone who can spare the short time it takes to read it. On most pages, examples are given of uncertainties that we meet in everyday situations.

This Beginner's Guide is not the 'last word' on uncertainty of measurement - far from it. It gives only the basic concepts. Although what you can read here is correct and in line with good practice, it is not complete or rigorous. It does not cover any difficult or special cases. (Section 15, 'Words of warning', briefly lists some cases where the basic procedures given in this Guide would not be sufficient.) For more complete information, the references detailed in the 'Further reading' Section should be consulted.

In the first sections of this Beginner's Guide, the concept and importance of measurement uncertainty are introduced. Following this, details are given of how to estimate uncertainties in real measurement situations. The main steps involved in calculating the uncertainty for a measurement are outlined with easy to follow examples. Finally a glossary, some cautionary remarks and list of publications for further reading are given, to direct you towards the next steps in understanding and calculating measurement uncertainties.

My thanks go to John Hurl of UKAS and Maurice Cox of NPL for their assistance during the writing of this document, and also to the many other people who supplied valuable comments and advice during the review process.

Stephanie Bell



# 1 Measurement

## 1.1 What is a measurement?

A measurement tells us about a property of something. It might tell us how heavy an object is, or how hot, or how long it is. A measurement gives a number to that property.

Measurements are always made using an instrument of some kind. Rulers, stopwatches, weighing scales, and thermometers are all measuring instruments.

The result of a measurement is normally in two parts: a number and a unit of measurement, e.g. 'How long is it? ... 2 metres.'

## 1.2 What is not a measurement?

There are some processes that might seem to be measurements, but are not. For example, comparing two pieces of string to see which is longer is not really a measurement. Counting is not normally viewed as a measurement. Often, a test is not a measurement: tests normally lead to a 'yes/no' answer or a 'pass/fail' result. (However, measurements may be part of the process leading up to a test result.)

# 2 Uncertainty of measurement

## 2.1 What is uncertainty of measurement?

The uncertainty of a measurement tells us something about its quality.

*Uncertainty of measurement* is the doubt that exists about the result of any measurement. You might think that well-made rulers, clocks and thermometers should be trustworthy, and give the right answers. But for every measurement - even the most careful - there is always a margin of doubt. In everyday speech, this might be expressed as 'give or take' ... e.g. a stick might be two metres long 'give or take a centimetre'.

## 2.2 Expressing uncertainty of measurement

Since there is always a margin of doubt about any measurement, we need to ask 'How big is the

margin?’ and ‘How bad is the doubt?’ Thus, two numbers are really needed in order to quantify an uncertainty. One is the width of the margin, or *interval*. The other is a *confidence level*, and states how sure we are that the ‘true value’ is within that margin.

For example:

We might say that the length of a certain stick measures 20 centimetres plus or minus 1 centimetre, at the 95 percent confidence level. This result could be written:

20 cm  $\pm$  1 cm, at a level of confidence of 95%.

The statement says that we are 95 percent sure that the stick is between 19 centimetres and 21 centimetres long. There are other ways to state confidence levels. More will be said about this later on, in Section 7.

## 2.3 Error versus uncertainty

It is important not to confuse the terms ‘error’ and ‘uncertainty’.

*Error* is the difference between the measured value and the ‘true value’ of the thing being measured.

*Uncertainty* is a quantification of the doubt about the measurement result.

Whenever possible we try to correct for any known *errors*: for example, by applying *corrections* from calibration certificates. But any error whose value we do not know is a source of uncertainty.

## 2.4 Why is uncertainty of measurement important?

You may be interested in uncertainty of measurement simply because you wish to make good quality measurements and to understand the results. However, there are other more particular reasons for thinking about measurement uncertainty.

You may be making the measurements as part of a:

- **calibration** - where the uncertainty of measurement must be reported on the certificate
- **test** - where the uncertainty of measurement is needed to determine a pass or fail

or to meet a

- **tolerance** - where you need to know the uncertainty before you can decide whether the tolerance is met

... or you may need to read and understand a calibration certificate or a written specification for a test or measurement.

### 3 Basic statistics on sets of numbers

#### 3.1 'Measure thrice, cut once' ... *operator error*

There is a saying among craftsmen, 'Measure thrice, cut once'. This means that you can reduce the risk of making a mistake in the work by checking the measurement a second or third time before you proceed.



*'Measure thrice, cut once'. You can reduce the risk of making a mistake by checking the measurement a second or third time.*

In fact it is wise to make any measurement at least three times. Making only one measurement means that a mistake could go completely unnoticed. If you make two measurements and they do not agree, you still may not know which is 'wrong'. But if you make three measurements, and two agree with each other while the third is very different, then you could be suspicious about the third.

So, simply to guard against gross mistakes, or *operator error*, it is wise to make at least three tries at any measurement. But uncertainty of measurement is not really about operator error. There are other good reasons for repeating measurements many times.

### 3.2 Basic statistical calculations

You can increase the amount of information you get from your measurements by taking a number of readings and carrying out some basic statistical calculations. The two most important statistical calculations are to find the average or *arithmetic mean*, and the *standard deviation* for a set of numbers.

### 3.3 Getting the best estimate - taking the average of a number of readings

If repeated measurements give different answers, you may not be doing anything wrong. It may be due to natural variations in what is going on. (For example, if you measure a wind speed outdoors, it will not often have a steady value.) Or it may be because your measuring instrument does not behave in a completely stable way. (For example, tape measures may stretch and give different results.)

If there is variation in readings when they are repeated, it is best to take many readings and take an average. An average gives you an estimate of the ‘true’ value. An *average* or *arithmetic mean* is usually shown by a symbol with a bar above it, e.g.  $\bar{x}$  (‘x-bar’) is the mean value of  $x$ . Figure 1 shows an illustration of a set of values and their mean value. Example 1 shows how to calculate an arithmetic mean.

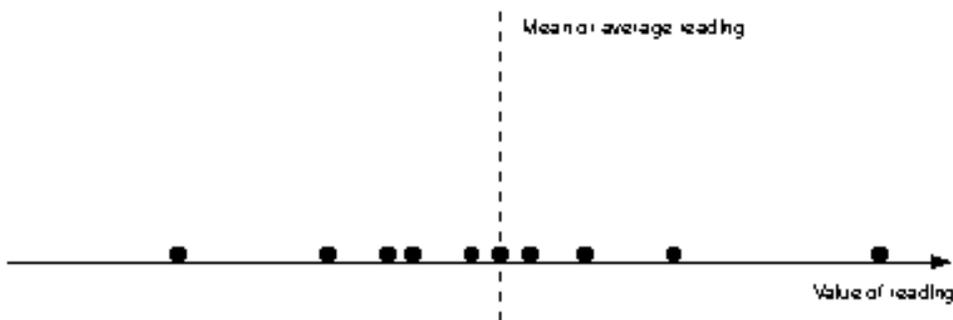


Figure 1. ‘Blob plot’ illustrating an example set of values and showing the mean

### 3.4 How many readings should you average?

Broadly speaking, the more measurements you use, the better the estimate you will have of the ‘true’ value. The ideal would be to find the mean from an infinite set of values. The more results you use, the closer you get to that ideal estimate of the mean. But performing more readings takes extra effort, and yields ‘diminishing returns’. What is a good number? Ten is a popular

choice because it makes the arithmetic easy. Using 20 would only give a slightly better estimate than 10. Using 50 would be only slightly better than 20. As a rule of thumb usually between 4 and 10 readings is sufficient.

**Example 1. Taking the *average* or *arithmetic mean* of a number of values**

Suppose you have a set of 10 readings. To find the average, add them together and divide by the number of values (10 in this case).

The readings are: 16, 19, 18, 16, 17, 19, 20, 15, 17 and 13

The sum of these is: 170

The average of the 10 readings is:  $\frac{170}{10} = 17$

### 3.5 Spread ... *standard deviation*

When repeated measurements give different results, we want to know how widely *spread* the readings are. The spread of values tells us something about the uncertainty of a measurement. By knowing how large this spread is, we can begin to judge the quality of the measurement or the set of measurements.

Sometimes it is enough to know the *range* between the highest and lowest values. But for a small set of values this may not give you useful information about the spread of the readings in between the highest and the lowest. For example, a large spread could arise because a single reading is very different from the others.

The usual way to quantify spread is *standard deviation*. The standard deviation of a set of numbers tells us about how different the individual readings typically are from the average of the set.

As a ‘rule of thumb’, roughly two thirds of all readings will fall between plus and minus ( $\pm$ ) one standard deviation of the average. Roughly 95% of all readings will fall within two standard deviations. This ‘rule’ applies widely although it is by no means universal.

The ‘true’ value for the standard deviation can only be found from a very large (infinite) set of readings. From a moderate number of values, only an estimate of the standard deviation can be found. The symbol  $s$  is usually used for the *estimated standard deviation*.

### 3.6 Calculating an estimated standard deviation

Example 2 shows how to calculate an estimate of standard deviation.

**Example 2. Calculating the estimated standard deviation of a set of values**

It is rarely convenient to calculate standard deviations by hand, with pen and paper alone. But it can be done as follows:

Suppose you have a set of  $n$  readings (let's use the same set of 10 as above). Start by finding the average:

For the set of readings we used before, 16, 19, 18, 16, 17, 19, 20, 15, 17 and 13, the average is 17.

Next, find the difference between each reading and the average,

i.e. -1 +2 +1 -1 0 +2 +3 -2 0 -4,

and square each of these,

i.e. 1 4 1 1 0 4 9 4 0 16.

Next, find the total and divide by  $n-1$  (in this case  $n$  is 10, so  $n-1$  is 9),

i.e.

$$\frac{1+4+1+1+0+4+9+4+0+16}{9} = \frac{40}{9} = 4.44.$$

The estimated standard deviation,  $s$ , is found by taking the square root of the total, i.e.

$$s = \sqrt{4.44} = 2.1$$

(correct to one decimal place).

The complete process of calculating the estimated standard deviation for a series of  $n$  measurements can be expressed mathematically as:

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}}, \tag{1}$$

where  $x_i$  is the result of the  $i$ th measurement and  $\bar{x}$  is the arithmetic mean of the  $n$  results considered.

**Calculator tip:** It is usually easiest to use the function key on a calculator to find the estimated standard deviation. Enter the readings into the calculator memory according to the instructions for your calculator, then use the ‘estimated standard deviation’ key ( $s$ , or  $\sigma_{n-1}$  ‘sigma n minus one’). See Section 13 for more information on the use of calculators.

### 3.7 How many readings do you need to find an estimated standard deviation?

Again, the more readings you use, the better the estimate will be. In this case it is the estimate of *uncertainty* that improves with the number of readings (not the estimate of the mean or ‘end result’). In ordinary situations 10 readings is enough. For a more thorough estimate, the results should be adjusted to take into account the number of readings. (See Section 16 for further reading which covers this subject.)

## 4 Where do errors and uncertainties come from?

Many things can undermine a measurement. Flaws in the measurement may be visible or invisible. Because real measurements are never made under perfect conditions, errors and uncertainties can come from:

- **The measuring instrument** - instruments can suffer from errors including bias, changes due to ageing, wear, or other kinds of drift, poor readability, noise (for electrical instruments) and many other problems.
- **The item being measured** - which may not be stable. (Imagine trying to measure the size of an ice cube in a warm room.)
- **The measurement process** - the measurement itself may be difficult to make. For example measuring the weight of small but lively animals presents particular difficulties in getting the subjects to co-operate.
- **‘Imported’ uncertainties** - calibration of your instrument has an uncertainty which is then built into the uncertainty of the measurements you make. (But remember that the uncertainty due to not calibrating would be much worse.)



*Visual alignment is an operator skill. A movement of the observer can make an object appear to move. 'Parallax errors' of this kind can occur when reading a scale with a pointer.*

- **Operator skill** - some measurements depend on the skill and judgement of the operator. One person may be better than another at the delicate work of setting up a measurement, or at reading fine detail by eye. The use of an instrument such as a stopwatch depends on the reaction time of the operator. (But gross mistakes are a different matter and are not to be accounted for as uncertainties.)
- **Sampling issues** - the measurements you make must be properly representative of the process you are trying to assess. If you want to know the temperature at the work-bench, don't measure it with a thermometer placed on the wall near an air conditioning outlet. If you are choosing samples from a production line for measurement, don't always take the first ten made on a Monday morning.
- **The environment** - temperature, air pressure, humidity and many other conditions can affect the measuring instrument or the item being measured.

Where the size and effect of an error are known (e.g. from a calibration certificate) a correction can be applied to the measurement result. But, in general, uncertainties from each of these sources, and from other sources, would be individual 'inputs' contributing to the overall uncertainty in the measurement.

## 5 The general kinds of uncertainty in any measurement

### 5.1 Random or systematic

The effects that give rise to uncertainty in measurement can be either:

- **random** - where repeating the measurement gives a randomly different result. If so, the more measurements you make, and then average, the better estimate you generally can expect to get.

or

- **systematic** - where the same influence affects the result for each of the repeated measurements (but you may not be able to tell). In this case, you learn nothing extra just by repeating measurements. Other methods are needed to estimate uncertainties due to systematic effects, e.g. different measurements, or calculations.

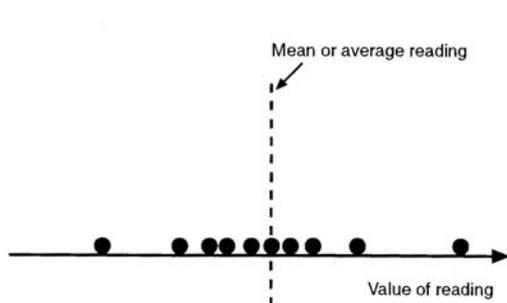
### 5.2 Distribution - the ‘shape’ of the errors

The spread of a set of values can take different forms, or *probability distributions*.

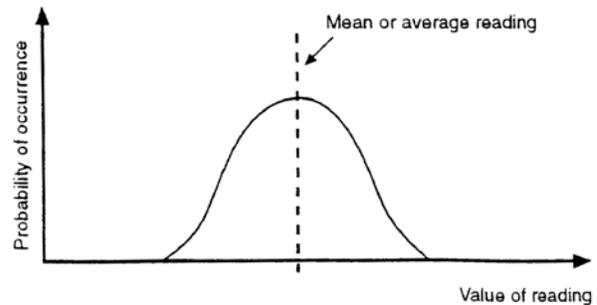
#### 5.2.1 Normal distribution

In a set of readings, sometimes the values are more likely to fall near the average than further away. This is typical of a *normal* or *Gaussian* distribution. You might see this type of distribution if you examined the heights of individuals in a large group of men. Most men are close to average height; few are extremely tall or short.

Figure 2 shows a set of 10 ‘random’ values in an approximately normal distribution. A sketch of a normal distribution is shown in Figure 3.



**Figure 2.** ‘Blob plot’ of a set of values lying in a normal distribution

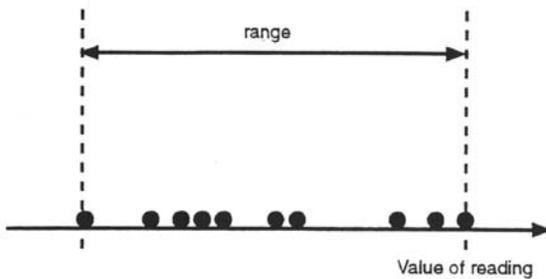


**Figure 3.** Sketch of a ‘normal’ distribution

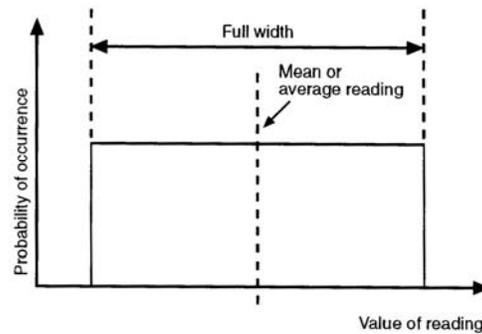
### 5.2.2 Uniform or rectangular distribution

When the measurements are quite evenly spread between the highest and the lowest values, a *rectangular* or *uniform* distribution is produced. This would be seen if you examined how rain drops fall on a thin, straight telephone wire, for example. They would be as likely to fall on any one part as on another.

Figure 4 shows a set of 10 ‘random’ values in an approximately rectangular distribution. A sketch of a rectangular distribution is shown in Figure 5.



**Figure 4.** ‘Blob plot’ of a set of values lying in a rectangular distribution.



**Figure 5.** Sketch of a rectangular distribution.

### 5.2.3 Other distributions

More rarely, distributions can have other shapes, for example, triangular, M-shaped (bimodal or two-peaked), or lop-sided (skew).

## 5.3 What is not a measurement uncertainty?

Mistakes made by operators are not measurement uncertainties. They should not be counted as contributing to uncertainty. They should be avoided by working carefully and by checking work.

Tolerances are not uncertainties. They are acceptance limits which are chosen for a process or a product. (See Section 10 below, about compliance with specifications.)

Specifications are not uncertainties. A specification tells you what you can expect from a product. It may be very wide-ranging, including ‘non-technical’ qualities of the item, such as its appearance. (See Section 10 below).

Accuracy (or rather inaccuracy) is not the same as uncertainty. Unfortunately, usage of these words is often confused. Correctly speaking, ‘accuracy’ is a qualitative term (e.g. you could say that a measurement was ‘accurate’ or ‘not accurate’). Uncertainty is quantitative. When a ‘plus or minus’ figure is quoted, it may be called an uncertainty, but not an accuracy.

Errors are not the same as uncertainties (even though it has been common in the past to use the words interchangeably in phrases like ‘error analysis’). See the earlier comments in Section 2.3.

Statistical analysis is not the same as uncertainty analysis. Statistics can be used to draw all kinds of conclusions which do not by themselves tell us anything about uncertainty. Uncertainty analysis is only one of the uses of statistics.

## **6 How to calculate uncertainty of measurement**

To calculate the uncertainty of a measurement, firstly you must identify the sources of uncertainty in the measurement. Then you must estimate the size of the uncertainty from each source. Finally the individual uncertainties are combined to give an overall figure.

There are clear rules for assessing the contribution from each uncertainty, and for combining these together.

### **6.1 The two ways to estimate uncertainties**

No matter what are the sources of your uncertainties, there are two approaches to estimating them: ‘Type A’ and ‘Type B’ evaluations. In most measurement situations, uncertainty evaluations of both types are needed.

*Type A evaluations* - uncertainty estimates using statistics (usually from repeated readings)

*Type B evaluations* - uncertainty estimates from any other information. This could be information from past experience of the measurements, from calibration certificates, manufacturer’s specifications, from calculations, from published information, and from common sense.

There is a temptation to think of ‘Type A’ as ‘random’ and ‘Type B’ as ‘systematic’, but this is not necessarily true.

How to use the information from Type A and Type B evaluations is described below.

## 6.2 Eight main steps to evaluating uncertainty

The main steps to evaluating the overall uncertainty of a measurement are as follows.

1. Decide what you need to find out from your measurements. Decide what actual measurements and calculations are needed to produce the final result.
2. Carry out the measurements needed.
3. Estimate the uncertainty of each input quantity that feeds into the final result. Express all uncertainties in similar terms. (See Section 7.1).
4. Decide whether the errors of the input quantities are independent of each other. If you think not, then some extra calculations or information are needed. (See correlation in Section 7.3.)
5. Calculate the result of your measurement (including any known corrections for things such as calibration).
6. Find the combined standard uncertainty from all the individual aspects. (See Section 7.2.)
7. Express the uncertainty in terms of a coverage factor (see Section 7.4), together with a size of the uncertainty interval, and state a level of confidence.
8. Write down the measurement result and the uncertainty, and state how you got both of these. (See Section 8.)

This is a general outline of the process. An example where these steps are carried out is given in Section 9.

## 7 Other things you should know before making an uncertainty calculation

Uncertainty contributions must be expressed in similar terms before they are combined. Thus, all the uncertainties must be given in the same units, and at the same level of confidence.

### 7.1 Standard uncertainty

All contributing uncertainties should be expressed at the same confidence level, by converting them into *standard uncertainties*. A standard uncertainty is a margin whose size can be thought of as ‘plus or minus one standard deviation’. The standard uncertainty tells us about the uncertainty of an average (not just about the spread of values). A standard uncertainty is usually shown by the symbol  $u$  (small u), or  $u(y)$  (the standard uncertainty in  $y$ ).

### 7.1.1 Calculating standard uncertainty for a Type A evaluation

When a set of several repeated readings has been taken (for a Type A estimate of uncertainty), the mean,  $\bar{x}$ , and estimated standard deviation,  $s$ , can be calculated for the set. From these, the estimated standard uncertainty,  $u$ , of the mean is calculated from:

$$u = \frac{s}{\sqrt{n}}, \quad (2)$$

where  $n$  was the number of measurements in the set. (The standard uncertainty of the mean has historically also been called the standard deviation of the mean, or the standard error of the mean.)

### 7.1.2 Calculating standard uncertainty for a Type B evaluation

Where the information is more scarce (in some Type B estimates), you might only be able to estimate the upper and lower limits of uncertainty. You may then have to assume the value is equally likely to fall anywhere in between, i.e. a rectangular or uniform distribution. The standard uncertainty for a rectangular distribution is found from:

$$\frac{a}{\sqrt{3}}, \quad (3)$$

where  $a$  is the semi-range (or half-width) between the upper and lower limits.

Rectangular or uniform distributions occur quite commonly, but if you have good reason to expect some other distribution, then you should base your calculation on that. For example, you can usually assume that uncertainties ‘imported’ from the calibration certificate for a measuring instrument are normally distributed.

### 7.1.3 Converting uncertainties from one unit of measurement to another

Uncertainty contributions must be in the same units before they are combined. As the saying goes, you cannot ‘compare apples with pears’.

For example, in making a measurement of length, the measurement uncertainty will also eventually be stated in terms of length. One source of uncertainty might be the variation in room temperature. Although the *source* of this uncertainty is temperature, the *effect* is in terms of length, and it must be accounted for in units of length. You might know that the material being measured expands in length by 0.1 percent for every degree rise in temperature. In that case, a temperature uncertainty of  $\pm 2$  °C would give a length uncertainty of  $\pm 0.2$  cm in a piece of the material 100 cm long.

Once the standard uncertainties are expressed in consistent units, the combined uncertainty can be found using one of the following techniques.

## 7.2 Combining standard uncertainties

Individual standard uncertainties calculated by Type A or Type B evaluations can be combined validly by ‘summation in quadrature’ (also known as ‘root sum of the squares’). The result of this is called the *combined standard uncertainty*, shown by  $u_c$  or  $u_c(y)$ .

Summation in quadrature is simplest where the result of a measurement is reached by addition or subtraction. The more complicated cases are also covered below for the multiplication and division of measurements, as well as for other functions.

### 7.2.1 Summation in quadrature for addition and subtraction

The simplest case is where the result is the sum of a series of measured values (either added together or subtracted). For example, you might need to find the total length of a fence made up of different width fence panels. If the standard uncertainty (in metres) in the length of each fence panel was given by  $a$ ,  $b$ ,  $c$ , etc., then the combined standard uncertainty (in metres) for the whole fence would be found by squaring the uncertainties, adding them all together, and then taking the square root of the total,

i.e.

$$\text{Combined uncertainty} = \sqrt{a^2 + b^2 + c^2 + \dots \text{etc.}} \quad (4)$$

### 7.2.2 Summation in quadrature for multiplication or division

For more complicated cases, it can be useful to work in terms of relative or fractional uncertainties to simplify the calculations.

For example, you might need to find the area,  $A$ , of a rectangular carpet, by multiplying the length,  $L$ , by the width,  $W$  (i.e.  $A=L \times W$ ). The relative or fractional uncertainty in the area of the carpet can be found from the fractional uncertainties in the length and width. For length  $L$  with uncertainty  $u(L)$ , the relative uncertainty is  $u(L)/L$ . For width  $W$ , the relative uncertainty is  $u(W)/W$ . Then the relative uncertainty  $u(A)/A$  in the area is given by

$$\frac{u(A)}{A} = \sqrt{\left(\frac{u(L)}{L}\right)^2 + \left(\frac{u(W)}{W}\right)^2}. \quad (5)$$

For a case where a result is found by multiplying three factors together, equation (5) would have three such terms, and so on. This equation can also be used (in exactly the same form) for a case where the result is a quotient of two values (i.e. one number divided by another, for example, length divided by width). In other words, this form of the equation covers all cases where the operations are multiplication or division.

### 7.2.3 Summation in quadrature for more complicated functions

Where a value is squared (e.g.  $Z^2$ ) in the calculation of the final measurement result, then the relative uncertainty due to the squared component is in the form

$$\frac{2u(Z)}{Z}. \quad (6)$$

Where a square root (e.g.  $\sqrt{Z}$ ) is part of the calculation of a result, then the relative uncertainty due to that component is in the form

$$\frac{u(Z)}{2Z}. \quad (7)$$

Of course, some measurements are processed using formulae which use combinations of addition, subtraction, multiplication and division, etc. For example, you might measure electrical resistance  $R$  and voltage  $V$ , and then calculate the resulting power  $P$  using the relationship

$$P = \frac{V^2}{R}. \quad (8)$$

In this case, the relative uncertainty  $u(P)/P$  in the value of power would be given by

$$\frac{u(P)}{P} = \sqrt{\left(\frac{2u(V)}{V}\right)^2 + \left(\frac{u(R)}{R}\right)^2}. \quad (9)$$

Generally speaking, for multi-step calculations, the process of combination of standard uncertainties in quadrature can also be done in multiple steps, using the relevant form for addition, multiplication, etc. at each step. The combination of standard uncertainties for complicated formulae is more fully discussed elsewhere (e.g. UKAS Publication M 3003.).

### 7.3 Correlation

The equations given above in Section 7.2 to calculate the combined standard uncertainty are only correct if the input standard uncertainties are not inter-related or *correlated*. This means we usually need to question whether all the uncertainty contributions are independent. Could a large error in one input cause a large error in another? Could some outside influence, such as temperature, have a similar effect on several aspects of uncertainty at once - visibly or invisibly? Often individual errors are independent. But if they are not, extra calculations are needed. These are not detailed in this Beginner's Guide, but can be found in some of the further reading listed in Section 16.

### 7.4 Coverage factor $k$

Having scaled the components of uncertainty consistently, to find the combined standard uncertainty, we may then want to re-scale the result. The combined standard uncertainty may be thought of as equivalent to 'one standard deviation', but we may wish to have an overall uncertainty stated at another level of confidence, e.g. 95 percent. This re-scaling can be done using a *coverage factor*,  $k$ . Multiplying the *combined standard uncertainty*,  $u_c$ , by a *coverage factor* gives a result which is called the *expanded uncertainty*, usually shown by the symbol  $U$ , *i.e.*

$$U = ku_c . \quad (10)$$

A particular value of coverage factor gives a particular confidence level for the expanded uncertainty.

Most commonly, we scale the overall uncertainty by using the coverage factor  $k = 2$ , to give a level of confidence of approximately 95 percent. ( $k = 2$  is correct if the combined standard uncertainty is normally distributed. This is usually a fair assumption, but the reasoning behind this is explained elsewhere, in the references in Section 16.)

Some other coverage factors (for a normal distribution) are:

$k = 1$  for a confidence level of approximately 68 percent

$k = 2.58$  for a confidence level of 99 percent

$k = 3$  for a confidence level of 99.7 percent

Other, less common, shapes of distribution have different coverage factors.

Conversely, wherever an expanded uncertainty is quoted with a given coverage factor, you can find the standard uncertainty by the reverse process, *i.e.* by dividing by the appropriate coverage

factor. (This is the basis for finding the combined standard uncertainty as shown in Sections 7.1.1 and 7.1.2.) This means that expanded uncertainties given on calibration certificates, if properly expressed, can be ‘decoded’ into standard uncertainties.

## 8 How to express the answer

It is important to express the answer so that a reader can use the information. The main things to mention are:

- The measurement result, together with the uncertainty figure, e.g. ‘The length of the stick was 20 cm  $\pm$ 1 cm.’
- The statement of the coverage factor and the level of confidence. A recommended wording is: ‘The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor  $k = 2$ , providing a level of confidence of approximately 95%.’

and

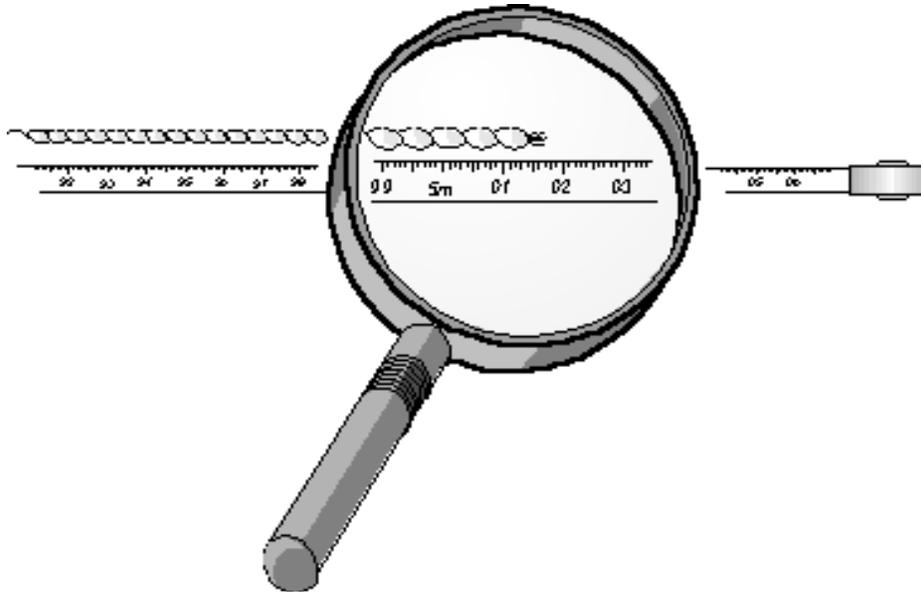
- How the uncertainty was estimated (you could refer to a publication where the method is described, e.g. UKAS Publication M 3003).

## 9 Example - a basic calculation of uncertainty

Below is a worked example of a simple uncertainty analysis. It is not realistic in every detail, but it is meant to be simple and clear enough to illustrate the method. First the measurement and the analysis of uncertainty are described. Secondly, the uncertainty analysis is shown in a table (a ‘spreadsheet model’ or ‘uncertainty budget’).

### 9.1 The measurement - how long is a piece of string?

Suppose you need to make a careful estimate of the length of a piece of string. Following the steps listed in Section 6.2, the process is as follows.



**Figure 6.** How long is a piece of string?

Example 3: Calculating the uncertainty in the length of a piece of string

**Step 1. Decide what you need to find out from your measurements. Decide what actual measurements and calculations are needed to produce the final result.** You need to make a measurement of the length, using a tape measure. Apart from the actual length reading on the tape measure, you may need to consider:

- Possible errors of the tape measure
  - Does it need any correction, or has calibration shown it to read correctly - and what is the uncertainty in the calibration?
  - Is the tape prone to stretching?
  - Could bending have shortened it? How much could it have changed since it was calibrated?
  - What is the resolution, i.e. how small are the divisions on the tape (e.g. millimetres)?
  
- Possible errors due to the item being measured
  - Does the string lie straight? Is it under- or over-stretched?
  - Does the prevailing temperature or humidity (or anything else) affect its actual length?
  - Are the ends of the string well-defined, or are they frayed?

- Possible errors due to the measuring process, and the person making the measurement
  - How well can you line up the beginning of the string with the beginning of the tape measure?
  - Can the tape be laid properly parallel with the string?
  - How repeatable is the measurement?

Can you think of any others?

**Step 2. Carry out the measurements needed.** You make and record your measurements of length. To be extra thorough, you repeat the measurement a total of 10 times, aligning the tape measure freshly each time (probably not very likely in reality!). Let us suppose you calculate the mean to be 5.017 metres (m), and the estimated standard deviation to be 0.0021 m (i.e. 2.1 millimetres).

For a careful measurement you might also record:

- when you did it
- how you did it, e.g. along the ground or vertically, reversing the tape measure or not, and other details of how you aligned the tape with the string
- which tape measure you used
- environmental conditions (if you think these could affect your results)
- anything else that could be relevant.

**Step 3. Estimate the uncertainty of each input quantity that feeds into the final result. Express all uncertainties in similar terms (standard uncertainty,  $u$ ).** You would look at all the possible sources of uncertainty and estimate the magnitude of each. Let us say that in this case:

- The tape measure has been calibrated. It needs no correction, but the calibration uncertainty is 0.1 percent of reading, at a coverage factor  $k = 2$  (for a normal distribution). In this case, 0.1 percent of 5.017 m is close to 5 mm. Dividing by 2 gives the standard uncertainty (for  $k = 1$ ) to be  $u = 2.5$  mm.
- The divisions on the tape are millimetres. Reading to the nearest division gives an error of no more than  $\pm 0.5$  mm. We can take this to be a uniformly distributed uncertainty (the true readings could lie variously anywhere in the 1 mm interval - i.e.  $\pm 0.5$  mm). To find the standard uncertainty,  $u$ , we divide the half-width (0.5 mm) by  $\sqrt{3}$ , giving  $u = 0.3$  mm, approximately.
- The tape lies straight, but let us suppose the string unavoidably has a few slight bends in it. Therefore the measurement is likely to underestimate the actual length of the

string. Let us guess that the underestimate is about 0.2 percent, and that the uncertainty in this is also 0.2 percent at most. That means we should correct the result by adding 0.2 percent (i.e. 10 mm). The uncertainty is assumed to be uniformly distributed, in the absence of better information. Dividing the half-width of the uncertainty (10 mm) by  $\sqrt{3}$  gives the standard uncertainty  $u = 5.8$  mm (to the nearest 0.1 mm).

The above are all Type B estimates. Below is a Type A estimate.

- The standard deviation tells us about how repeatable the placement of the tape measure is, and how much this contributes to the uncertainty of the mean value. The estimated standard deviation of the mean of the 10 readings is found using the formula in Section 3.6:

$$\frac{s}{\sqrt{n}} = \frac{2.1}{\sqrt{10}} = 0.7 \text{ mm (to one decimal place).}$$

Let us suppose that no other uncertainties need to be counted in this example. (In reality, other things would probably need to be included.)

**Step 4. Decide whether the errors of the input quantities are independent of each other. (If you think not, then some extra calculations or information are needed.)** In this case, let us say that they are all independent.

**Step 5. Calculate the result of your measurement (including any known corrections for things such as calibration).** The result comes from the mean reading, together with the correction needed for the string lying slightly crookedly,

i.e.  $5.017 \text{ m} + 0.010 \text{ m} = 5.027 \text{ m}.$

**Step 6. Find the combined standard uncertainty from all the individual aspects.** The only calculation used in finding the result was the addition of a correction, so summation in quadrature can be used in its simplest form (using the equation in Section 7.2.1). The standard uncertainties are combined as

$$\begin{aligned} \text{Combined standard uncertainty} &= \sqrt{2.5^2 + 0.3^2 + 5.8^2 + 0.7^2} \\ &= 6.4 \text{ mm (to one decimal place)} \end{aligned}$$

**Step 7. Express the uncertainty in terms of a coverage factor (see Section 7.4 above), together with a size of the uncertainty interval, and state a level of confidence.** For a coverage factor  $k = 2$ , multiply the combined standard uncertainty by 2, to give an expanded uncertainty of 12.8 mm (i.e. 0.0128 m). This gives a level of confidence of about 95 percent.

**Step 8. Write down the measurement result and the uncertainty, and state how you got both of these.** You might record:

‘The length of the string was 5.027 m  $\pm$ 0.013 m. The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor  $k = 2$ , providing a level of confidence of approximately 95%.

‘The reported length is the mean of 10 repeated measurements of the string laid horizontally. The result is corrected for the estimated effect of the string not lying completely straight when measured. The uncertainty was estimated according to the method in *A Beginner’s Guide to Uncertainty of Measurement*.’

## 9.2 Analysis of uncertainty - spreadsheet model

To help in the process of calculation, it can be useful to summarise the uncertainty analysis or ‘uncertainty budget’ in a spreadsheet as in Table 1 below.

**Table 1. Spreadsheet model showing the ‘uncertainty budget’**

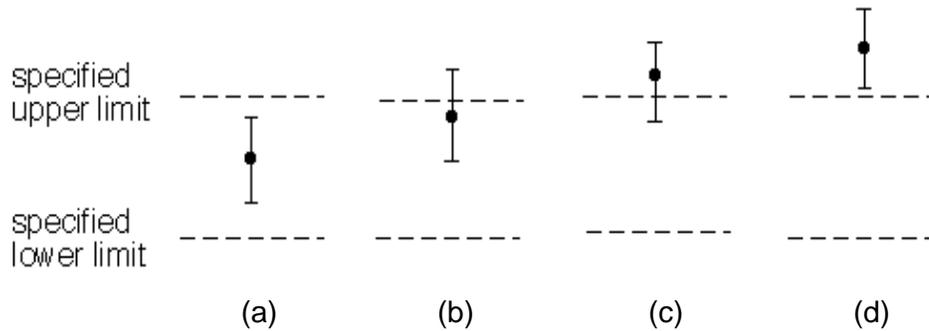
Source of uncertainty	Value $\pm$	Probability distribution	Divisor	Standard uncertainty
Calibration uncertainty	5.0 mm	Normal	2	2.5 mm
Resolution (size of divisions)	0.5 mm*	Rectangular	$\sqrt{3}$	0.3 mm
String not lying perfectly straight	10.0 mm*	Rectangular	$\sqrt{3}$	5.8 mm
Standard uncertainty of mean of 10 repeated readings	0.7 mm	Normal	1	0.7 mm
Combined standard uncertainty		Assumed normal		6.4 mm
Expanded uncertainty		Assumed normal ( $k = 2$ )		12.8 mm

\*Here the ( $\pm$ ) half-width divided by  $\sqrt{3}$  is used.

## 10 Other statements (e.g. compliance with specification)

When conclusions are drawn from measurement results, the uncertainty of the measurements must not be forgotten. This is particularly important when measurements are used to test whether or not a specification has been met.

Sometimes a result may fall clearly inside or outside the limit of a specification, but the uncertainty may overlap the limit. Four kinds of outcome are shown in the illustration in Figure 7.



**Figure 7.** Four cases of how a measurement result and its uncertainty may lie relative to the limits of a stated specification. (Similarly, an uncertainty may overlap the lower limit of a specification.)

In Case (a), both the result and the uncertainty fall inside the specified limits. This is classed as a ‘compliance’.

In Case (d), neither the result nor any part of the uncertainty band falls within the specified limits. This is classed as a ‘non-compliance’.

Cases (b) and (c) are neither completely inside nor outside the limits. No firm conclusion about compliance can be made.

Before stating compliance with a specification, always check the specification. Sometimes a specification covers various properties such as appearance, electrical connections, interchangeability, etc., which have nothing to do with what has been measured.

## 11 How to reduce uncertainty in measurement

Always remember that it is usually as important to minimise uncertainties as it is to quantify them. There are some good practices which can help to reduce uncertainties in making measurements generally. A few recommendations are:

- Calibrate measuring instruments (or have them calibrated for you) and use the calibration corrections which are given on the certificate.
- Make corrections to compensate for any (other) errors you know about.
- Make your measurements traceable to national standards - by using calibrations which can be traced to national standards via an unbroken chain of measurements. You can place particular confidence in measurement traceability if the measurements are quality-assured through a measurement accreditation (UKAS in the UK).

- Choose the best measuring instruments, and use calibration facilities with the smallest uncertainties.
- Check measurements by repeating them, or by getting someone else to repeat them from time to time, or use other kinds of checks. Checking by a different method may be best of all.
- Check calculations, and where numbers are copied from one place to another, check this too.
- Use an uncertainty budget to identify the worst uncertainties, and address these.
- Be aware that in a successive chain of calibrations, the uncertainty increases at every step of the chain.

## **12 Some other good measurement practices**

Overall, use recognised good practices in measurements, for example:

- Follow the maker's instructions for using and maintaining instruments.
- Use experienced staff, and provide training for measurement.
- Check or validate software, to make sure it works correctly.
- Use rounding correctly in your calculations. (See Section 13.4.)
- Keep good records of your measurements and calculations. Write down readings at the time they are made. Keep a note of any extra information that may be relevant. If past measurements are ever called into doubt, such records can be very useful.

Many more good measurement practices are detailed elsewhere, for example in the international standard ISO/IEC 17025 'General requirements for the competence of testing and calibration laboratories' (See 'Further Reading', Section 16).

## 13 Use of calculators

When using calculators or computers to work out uncertainties, you need to know how to avoid mistakes when using them.

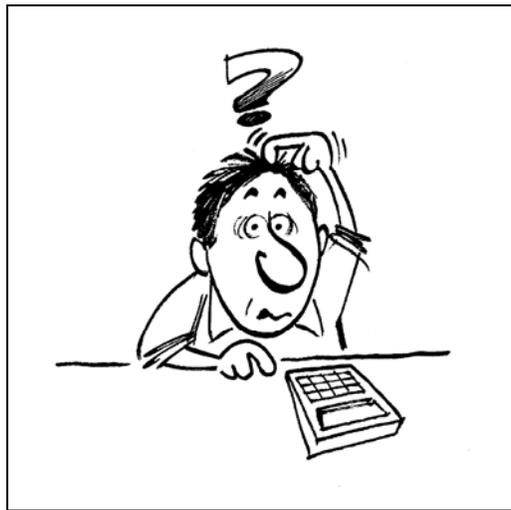
### 13.1 Calculator keys

The  $\bar{x}$  ('x bar') key gives you the average (arithmetic mean) of the numbers you have entered into the calculator memory.

The  $\sigma_{n-1}$  ('sigma n minus one') key (sometimes marked 's') gives you the estimated standard deviation of the 'population' based on your limited sample. (In practice, any set of readings is a small sample from an 'infinite population' of possible readings).  $\sigma_{n-1}$ , or  $s$ , is the estimate of standard deviation you should use when calculating standard uncertainty for a 'Type A evaluation' as in Section 7.1.1 of this Beginner's Guide.

Your calculator may also have a key marked  $\sigma_n$ . You should not normally use  $\sigma_n$  for your estimate of uncertainty:  $\sigma_n$  gives the standard deviation of the sample itself, and does not give the 'estimate' for the larger 'population' you are trying to characterise. For a very large number of readings,  $\sigma_n$  is very close to  $\sigma_{n-1}$ . But in real measurement situations, with moderate numbers of readings, you would not use  $\sigma_n$ .

### 13.2 Calculator and software errors



*A calculator is useful for complicated arithmetic, but it can also be a source of error.*

Calculators can make mistakes! In particular, they can sometimes give unexpected results when dealing with very long numbers. For example, some calculators would wrongly give:

$$0.000\ 000\ 2 \times 0.000\ 000\ 2 = 0 \text{ (exactly),}$$

when the correct answer is 0.000 000 000 000 04. (Of course, this would be better expressed as  $2 \times 10^{-7} \times 2 \times 10^{-7} = 4 \times 10^{-14}$ .) Even computers can suffer from this form of rounding error. To identify this problem, spreadsheet software should be checked by running through a typical calculation ‘by hand’ to make sure both methods agree. To avoid these problems with rounding, it is good practice to work with ‘transformed’ numbers in your calculations (this is sometimes called ‘scaling’ or ‘coding the data’).

### 13.3 Scaling

Example 4 shows how to perform ‘scaling’ to avoid software and calculator errors, and to make the arithmetic easier if you are working without a calculator.

**Example 4: Find the average and estimated standard deviation of 1.000 000 03, 1.000 000 06 and 1.000 000 12**

Working in whole numbers, you can find the average of 3, 6 and 12 (which is 7), and then deduce that the average of the original numbers is 1.000 000 07.

Step by step: you subtract the whole number (1) from 1.000 000 03, 1.000 000 06 and 1.000 000 12, giving

$$0.000\ 000\ 03 \quad 0.000\ 000\ 06 \quad \text{and} \quad 0.000\ 000\ 12$$

then multiply by 100 000 000 ( $10^8$ ) to bring the entire calculation into whole numbers, i.e.

$$3 \quad 6 \quad \text{and} \quad 12$$

After taking the average,

$$\frac{3+6+12}{3} = 7,$$

you reverse the steps, by dividing the average by  $10^8$ , i.e.

$$7 \div 10\ 000\ 000 = 0.000\ 000\ 07$$

and adding 1 again, i.e.

$$1.000\ 000\ 07$$

‘Scaling’ to calculate estimated standard deviation is done in a similar way. The transformed data are as before:

3                      6                      and                      12,

and the transformed average is 7.

The estimated standard deviation is found either using a calculator or as before (in Section 3.6) using

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}},$$

by finding the difference between each reading and the average, i.e.

-4                      -1                      and                      5,

squaring each of these, i.e.

16                      1                      and                      25,

finding the total and dividing by  $n - 1$ , i.e.

$$\frac{16 + 1 + 25}{2} = 21$$

and taking the square root. i.e.

$$\sqrt{21} = 4.6 \text{ (to one decimal place).}$$

The result (4.6) is then transformed back to the original scale, to give an estimated standard deviation of 0.000 000 046. (Note that it is *not* 1.000 000 046, since the standard deviation of a 'shifted' set of numbers is unchanged.)

## 13.4 Rounding

Calculators and spreadsheets can give an answer to many decimal places. There are some recommended practices for rounding the results:

- Use a meaningful degree of rounding in calculations. The uncertainty in a measurement result may define how many decimal places you should report. For example, if the uncertainty in your result is in the first decimal place, then the measurement result should probably also be stated to one decimal place,

e.g. 20.1 cm  $\pm$  0.2 cm.

- Make your calculations to at least one more significant figure than you eventually require.

Be aware of how many significant figures you need to use when multiplying or dividing or carrying out more complex calculations.

- Rounding of values should be carried out only at the end of the calculation, to avoid rounding errors. For example, if 2.346 is rounded up to 2.35 at an early stage in a calculation, it could later be rounded up to 2.4. But if 2.346 is used throughout a calculation it would be correctly rounded to 2.3 at the final stage.
- Although *results* are finally rounded either up or down, depending on which is the nearest figure, the rule for rounding *uncertainties* is different. The final uncertainty is rounded up to the next largest figure, not down.

## 14 Learning more and putting it into practice

You now know the basics of uncertainty estimation. However, you will need further guidance before you can put this knowledge into practice.

More information can be found in the texts listed in Section 16, 'Further reading'. Detailed guidelines on how to make a correct and thorough analysis of measurement uncertainty are given in the document M 3003 'The Expression of Uncertainty and Confidence in Measurement' published by UKAS (United Kingdom Accreditation Service). Similar guidelines are given in EA-4/02 'Expression of the Uncertainty of Measurement in Calibration'. These documents are aimed mainly at laboratories seeking accreditation for calibration or testing. They give full directions for estimating measurement uncertainties, complete with worked examples for various different types of measurements. They give technical definitions of terms relating to uncertainty, and they list the symbols commonly used for these. They also deal with special cases, and some extra points which sometimes need to be considered to make fully correct calculations of uncertainty.

## 15 Words of warning

Uncertainty analysis is an evolving subject area. There have been subtle changes in approach over the years. What is more, the rules given in this Beginner's Guide are not 'absolute'. There are plenty of special cases where slightly different rules apply. There is even room for debate on the finer points of how to account for particular uncertainties. But still the advice given in this publication represents normal good practice.

What is given here is not the full story. Special cases have not been dealt with in this Guide. Extra rules apply:

- if you use statistics on very small sets of data (less than about 10)

- if one component of uncertainty is much bigger than all the others involved
- if some inputs to the calculation are correlated
- if the spread or distribution is unusual in shape
- if the uncertainty is not for a single result, but for fitting a curve or line to a number of points

These cases are covered by some of the texts listed below in ‘Further reading’.

## 16 Further reading

- [1] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. *Guide to the Expression of Uncertainty in Measurement*. International Organization for Standardization, Geneva. ISBN 92-67-10188-9, First Edition 1993, corrected and reprinted 1995. (BSI Equivalent: BSI PD 6461: 1995, *Vocabulary of Metrology, Part 3. Guide to the Expression of Uncertainty in Measurement*. British Standards Institution, London.)
- [2] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. *International Vocabulary of Basic and General Terms in Metrology* Second Edition 1993. International Organization for Standardization, Geneva.
- [3] Chatfield, C. (1983) *Statistics for Technology*. Third Edition. (New York: Chapman and Hall.)
- [4] Dietrich, C.F. (1991), *Uncertainty, calibration and probability*. Second Edition. (Bristol: Adam Hilger.)
- [5] EA-4/02 *Expression of the Uncertainty of Measurement in Calibration*, 1999, European co-operation for Accreditation.
- [6] International Standard ISO 3534-1 *Statistics - vocabulary and symbols - Part I: Probability and General Statistical Terms*, First Edition 1993, International Organization for Standardization, Geneva.
- [7] PD 6461 : Part 1 : 1995 *Vocabulary of Metrology, Part 1. Basic and general terms (international)*, British Standards Institution, London.
- [8] EURACHEM/CITAC Guide: *Quantifying Uncertainty in Analytical Measurement*, Second Edition 2000.
- [9] UKAS publication M 3003 *The Expression of Uncertainty and Confidence in Measurement* Edition 1, December 1997.

- [10] International Standard ISO/IEC 17025 *General Requirements for the competence of testing and calibration laboratories*, First Edition 1999, International Organization for Standardization, Geneva.

At the time of publication, the following websites contained useful information on estimating uncertainty of measurement:

<http://physics.nist.gov/cuu/Uncertainty/index.html>

[http://www.ukas.com/new\\_docs/technical-uncertain.htm](http://www.ukas.com/new_docs/technical-uncertain.htm)

## Annex A - Understanding the terminology

In the 'glossary' below, a few important words are explained. Precise or rigorous definitions are not given here. They can be found elsewhere, for example in the *International Vocabulary of Basic and General Terms in Metrology*. A useful and correct set of definitions can also be found in UKAS publication M 3003 *The Expression of Uncertainty and Confidence in Measurement* (See Further Reading in Section 16).

### **accuracy**

closeness of the agreement between measurement result and true value. (Accuracy is a qualitative term only.)

### **bias (of a measuring instrument)**

systematic error of the indication of a measuring instrument

### **calibration**

comparison of an instrument against a reference or standard, to find any errors in the values indicated by the instrument. In some cases, calibration assigns a relationship between the input and output of an instrument; for example, calibration of a resistance thermometer could relate its output (in ohms) to an input temperature (in degrees Celsius, or in kelvins).

### **confidence level**

number (e.g. 95 %) expressing the degree of confidence in a result

### **correction (calibration correction)**

number added to an instrument reading to correct for an error, offset, or bias. (Similarly, a reading may be multiplied or divided by a *correction factor* to correct the value.)

### **correlation**

interdependence, or relationship, between data or measured quantities

### **coverage factor**

number which is multiplied by the combined standard uncertainty to give an expanded uncertainty for a particular level of confidence

### **error**

offset or deviation (either positive or negative) from the correct value

### **estimated standard deviation**

estimate of the standard deviation of the 'population' based on a limited sample

**expanded uncertainty**

standard uncertainty (or combined standard uncertainty) multiplied by a coverage factor  $k$ , to give a particular level of confidence

**Gaussian distribution**

(See *normal distribution*)

**interval (confidence interval)**

margin within which the ‘true value’ being measured can be said to lie, with a given level of confidence

**level of confidence**

number (e.g. 95 %) expressing the degree of confidence in the result

**mean (arithmetic mean)**

average of a set of numbers

**measurand**

particular quantity subject to measurement

**normal distribution**

distribution of values in a characteristic pattern of spread (Gaussian curve) with values more likely to fall near the mean than away from it

**operator error**

mistake

**precision**

a term meaning ‘fineness of discrimination’ but often misused to mean ‘accuracy’ or ‘uncertainty’. Its use should be avoided if possible.

**random error**

error whose effects are observed to vary randomly

**range**

difference between the highest and the lowest of a set of values

**reading**

value observed and recorded at the time of measurement

**rectangular distribution**

distribution of values with equal likelihood of falling anywhere within a range

**repeatability (of an instrument or of measurement results)**

closeness of the agreement between repeated measurements of the same property under the same conditions

**reproducibility (of an instrument or of measurement results)**

closeness of the agreement between measurements of the same property carried out under changed conditions of measurement (e.g. by a different operator or a different method, or at a different time)

**resolution**

smallest difference that can be meaningfully distinguished (e.g. a change of one (1) in the last place of a digital display)

**result (of a measurement)**

value obtained from a measurement, either before or after correction or averaging

**sensitivity**

change in response (of an instrument) divided by the corresponding change in the stimulus

**standard deviation**

a measure of the spread of a set of results, describing how values typically differ from the average of the set. Where it is not possible to obtain an infinite set of results (in practice it never is) we instead use the estimated standard deviation.

**standard uncertainty**

uncertainty of a measurement expressed as a margin equivalent to plus and minus ( $\pm$ ) one standard deviation.

**systematic error**

bias or offset (either positive or negative) from the correct value

**true value**

the value that would be obtained by a perfect measurement

**Type A evaluation of uncertainty**

evaluation of uncertainty by statistical methods

**Type B evaluation of uncertainty**

evaluation of uncertainty by non-statistical methods

**uncertainty budget**

summary of the uncertainty calculations

**uncertainty of measurement**

quantified doubt about the result of a measurement

**uniform distribution**

distribution of values with equal likelihood of falling anywhere within a range