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Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 09

The Determination of Uncertainties in Bend Tests on Metallic Materials

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CONTENTS

1. SCOPE

This procedure covers the determination of the uncertainties in bend tests for establishing the modulus of elasticity in bending and the bending strength of metallic strips or sheets at room temperature.

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

a	distance from the support to the load applicator
b	specimen width
c_i	sensitivity coefficient
E_b	modulus of elasticity in bending
h	specimen thickness
k	coverage factor used to calculate expanded uncertainty
L	span length between supports
N	number of repeat measurements
P	load increment
p	confidence level
q	random variable
$\bar{q}$	arithmetic mean of the values of the random variable q
S	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
d	deflection increment

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be monitored more closely. This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated. The aim is to produce a series of documents in a common format that is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into six sections as follows, with all the individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty

2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials.

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in Reference 1.

The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in Bend Tests on metallic materials.

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTIES IN BEND TESTING

The objective of this procedure is to evaluate the uncertainty of each calculated quantity in a bend test with a given confidence level. It is assumed throughout the procedure that the test has been carried out and the raw data from the test is available. The final result of this quantity will thus be presented in the following way:

$$V \pm U \text{ with a confidence level of } X\%$$

where V is the displayed value or mean computed value
U is the expanded uncertainty associated with V
X is the confidence level

This document guides the user through six steps to express the above values.

Before starting, the user must be aware of the following:

1. the relevant standard is ASTM E855 - 90 ;
2. the quantities which are to be evaluated and produced as test's results ;
3. the testing procedure followed during the test ;
4. the testing apparatus' specifications and/or calibration certificates ;
5. the raw data gathered during the test ;

6. the required confidence level for each desired quantity (for most applications, a confidence level of 95% will be retained as default value).

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first Step consists in setting the quantities which are to be presented as results of the test:

1. Young's modulus of elasticity in bending

It is useful to fill up a table of all quantities to be evaluated during the test. Below there is the table containing all terms measured, (Part A), all terms calculated, (Part B).

Table 1 Measured, calculated and invariant quantities

	Measurements	Unit of measurement	Symbol
Part A	Specimen width	[m]	B
	Specimen thickness	[m]	H
	Deflection increment	[m]	δ
	Distance from the support	[m]	a
	Span length between supports	[m]	L
	Load increment	[N]	P
Part B	Modulus of elasticity in bending	[MPa]	E_b

Step 2. Identifying all Sources of Uncertainty in the Test

The user must identify and list all possible sources of uncertainty which may have an effect (either directly or indirectly)on the test. This list cannot be defined beforehand since it is tightly linked to the test method and apparatus. Therefore, a new list should be drafted each time a particular test parameter changes (when a plotter is replaced by a computer and printer for example). In the following table (Table 2) the user finds some examples of sources, divided in the five categories.

Table 2 Example of sources of uncertainty

Category	Example of Sources of Uncertainty	Influence on Young's Modulus
Apparatus	Load Cell calibration	Influential
	Load Cell sensitivity	Influential
	Tooling alignment	Influential
	Extensometer	Influential
	Shape and dimensions of the support	Not influential
	Span length measurements	Influential
	Deflection increment	Influential
Method	Crosshead rate	Not influential
	Digitizing rate	Influential
Environment	Temperature	Not influential
Operator	Choice of limits on graph for regression	Influential
	Specimen positioning	Not influential
Test Piece	Thickness	Influential
	Width	Influential

It is important to note that the lists in the above table are NOT exhaustive. Many other sources can be defined depending on specific testing configurations. The user is strongly advised to draft his own list corresponding to his own test facilities.

Step 3. Classifying the Uncertainty According to Type A or B

In accordance with ISO TAG 4 'Guide to the Expression of Uncertainties in Measurement' (Paragraph 0.7), sources of uncertainty can be classified as type A or B, depending on the way their influence is quantified. If a source's influence is evaluated by statistical means (from a number of repeated observations), it is classified type A. If a source's influence is evaluated by any other mean (manufacturer's documents, certification, ...), it is classified type B.

Attention should be drawn to the fact that one same source can be classified as type A or B depending on the way it is estimated. For instance, if the diameter of a cylindrical specimen is measured once, that parameter is considered type B. If the mean value of ten consecutive measurements is taken into account, then the parameter is type A.

It is likely that when a computation table is drafted for the first time, most sources will prove type B (quantified by reference to documentation or estimation). But as experience builds up, more and more sources can be quantified as type A, thus reducing overall uncertainty.

In the following table the sources of uncertainty identified in Step 2 are classified as type A or B:

Table 3 Classification of the sources

Category	Example of Sources of Uncertainty	Classification of the sources
Apparatus	Load Cell calibration	B
	Load Cell sensibility	B
	Tooling alignment	B
	Extensometer	B
	Shape and dimensions of the support	A, B
	Span length measurements	A, B
	Deflection increment	B
Method	Crosshead rate	Not influential
	Digitizing rate	Influential
Environment	Temperature	Not influential
Operator	Choice of limits on graph for regression	Not applicable
	Specimen positioning	Not influential
Test - Piece	Thickness	A, B
	Width	A, B

Step 4. Estimating the Standard Uncertainty for each Source of Uncertainty

In this step the standard uncertainty, $u(x_i)$, for each measurement is estimated. The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter k' associated with the assumed probability distribution. The parameters for the distributions most likely to be encountered are given in Section 2 of reference 1.

Type A: Standard uncertainty S

Type B: Standard uncertainty $u(x) = k' u'$.

The significant sources of uncertainty and their influence on the evaluated quantity are summarized in Table 4

Table 4 Correction factor $\mathbf{d_v}$ according to the estimated distribution

Category	**Measurand**			**Uncertainties**		
Source of Uncertainty	**Measurand Affected**	**Nominal or Averaged Value**	**Type**	**Probabl. Distrib.**	**Divisor d_v**	$u(x_i)$
Apparatus						
Load Cell calibration	P	(N)	B	Rectangular	$1/\sqrt{3}$	$u(P)$
Load Cell sensitivity	P	(N)	B	Rectangular	$1/\sqrt{3}$	$u(P)$
Tooling alignment	P	(N)	B	Rectangular	$1/\sqrt{3}$	$u(P)$
Extensometer	δ	(mm)	B	Rectangular	$1/\sqrt{3}$	$u(\delta)$
Distance from support and the load applicator	a	(mm)	A, B.	Rectangular	$1/\sqrt{3}$	$u(a)$
Span length measurements	L	(mm)	A, B	Rectangular	$1/\sqrt{3}$	$u(L)$
Deflection increment	δ	(mm)	B	Rectangular	$1/\sqrt{3}$	$u(\delta)$
Method						
Crosshead rate			n.i.			
Digitizing rate	P, δ	(N), (mm)	B	Rectangular	$1/\sqrt{3}$	$u(P)$ $u(\delta)$
Environment						
Temperature			n.i.			
Operator						
Choice of limits on graph for regression	P, δ	(N), (mm)	n.a.			
Specimen positioning	n.i.					
Test – Piece						
Thickness	h	(mm)	A, B	Rectangular	$1/\sqrt{3}$	$u(h)$
Width	b	(mm)	A, B	Rectangular	$1/\sqrt{3}$	$u(b)$

1) In Appendix A there are the formulae for the calculation of the relative combined standard uncertainty of the modulus of elasticity in bending for a three point loaded beam.

2) In Appendix B there are the formulae for the calculation of the combined standard uncertainty of the modulus of elasticity in bending for a four point loaded beam.

Step 5. Computing the combined uncertainty u_c

Once each source of uncertainty is estimated, it is possible to calculate the combined standard uncertainty $u_c(x)$ and/or the relative combined standard uncertainty $u_c(x)/x$. These uncertainties correspond to plus or minus one standard deviation on the normal law representing the studied quantity's distribution. This law takes into account all estimated sources as if they were fully independent in the following way :

Combined standard uncertainty:

$$(u_c)^2 = (\sum S^2 + \sum u^2) \qquad \textbf{(1)}$$

It is also possible to calculate the relative standard uncertainty for Type A when the standard uncertainty (S) has been elucidated, by dividing for the average: $u_c(x)/x = S/x_m$. When the standard uncertainty has been obtained for Type B, it can be transformed to a relative standard uncertainty by dividing with the measured value x: i.e.: $u_c(x)/x$.

The right formula for direct calculation of relative standard uncertainty is:

$$[u_c(x)/x]^2 = \Sigma^N_{i=1} [c_i\, u(x_i) / x_i]^2 \qquad \textbf{(2)}$$

where c_i = influence coefficient

The influence coefficients c_i of each source of uncertainty necessary for the calculation of the Young's modulus of elasticity in bending for Test A and for Test B, in accordance with document ASTM E855 – 90, are listed in the following tables.

Table 5 Influence coefficients for the calculation of the relative combined standard uncertainty of Young's modulus of elasticity

TEST A 3 point loaded beam

Sources of Uncertainty	Symbol	c_i
Load	P	1
Span length supports	L	3
Specimen width	B	-1
Specimen thickness	H	-3
Deflection increment	δ	-1

Table 6 Influence coefficients for the calculation of the relative combined standard uncertainty of δ, deflection increment

Sources of Uncertainty	Symbol	c_i
Span length supports	L	2
Specimen thickness	H	-1

Table 7 Influence coefficients for the calculation of the relative combined standard uncertainty of Young's modulus of elasticity

TEST B 4 point loaded beam

Sources of Uncertainty	Symbol	c_i
Load	P	1
Span length supports	L	3
Specimen width	B	-1
Specimen thickness	H	-3
Deflection increment	δ	-1
Distace from supports to load applicator	a	3

Table 8 Influence coefficients for the calculation of the relative combined standard uncertainty of δ, deflection increment

Sources of Uncertainty	Symbol	c_i
Span length supports	L	2
Specimen thickness	H	-1
Distace from supports to load applicator	A	2

Step 6. Computing the Expanded Uncertainty U

The final Step is optional and depends on the requirements of the customer. The expanded uncertainty U is broader than the combined standard uncertainty and the confidence level associated with it is greater too. The combined standard uncertainty u_c has a confidence level of 68.27% corresponding to plus or minus one standard deviation. Where a high confidence level is needed (aerospace and electronics industries), the combined standard uncertainty u_c is multiplied by a coverage factor k to obtain the expanded uncertainty U. If u_c is, for example, tripled the corresponding confidence level is 99.73%.

Table 9 presents some coverage factors k leading to an X% confidence level.

Table 9 Coverage factor according to requested Confidence level

Confidence level : X%	Coverage factor k
68.27	1
90	1.645
95	1.960
95.45	2
99	2.576
99.73	3

Step 7. Reporting of Results

Once the expanded standard uncertainty has been computed, the final result can be given as:

$$V \pm U \text{ with a confidence level of } X\%$$

where V is the displayed value or mean computed value
U is the expanded uncertainty associated with V
X is the confidence level

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0-946754-41-1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, *Guide to the expression of Uncertainty in Measurement*. International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. [This Guide is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that drafted it.]

3. ASTM Standard , *Bend Testing of Metallic Flat Materials for Spring Applications Involving Static Loading.* ASTM E 855 - 90

APPENDIX A

Mathematical Formulae for Calculating Uncertainties in Bend Tests on Metallic Materials

This appendix has been devised to explain the formulae need to calculate the combined standard uncertainty and where possible, the relative combined standard uncertainty of the modulus of elasticity E_b, $u_c(E_b)$, in accordance with the ISO TAG4 document *"Guide to the Expression of Uncertainty in Measurement" [2]* .

According to ASTM E855 - 90 [3] there are two methods for determining the modulus of elasticity in bending test. These methods are:

- Test A - for a three point loaded beam;
- Test B - for a four point loaded beam.

TEST A: three point loaded beam

TERMINOLOGY
E_b = modulus of elasticity in bending;
b = specimen width;
h = specimen thickness;
P = load increment relative to pre-load;
δ = deflection increment at mid span relative to pre-load;
L = span length between supports.

The formula for the calculation of the modulus of elasticity is:

$$E_b = (P\,L^3)/(4\,b\,h^3\delta) \tag{A1}$$

In this case it is possible to use the expression of the relative combined standard uncertainty for the calculation of uncertainty. The general formula is:

$$[u_c(Y)/Y]^2 = \Sigma\,[c_i\,u(X_i)/X_i]^2 \tag{A2}$$

where: c_i is the influence coefficient of X_i.

Using this formula the relative combined standard uncertainty $u_c(E_b)/E_b$ is:

$$[u_c(E_b)/E_b]^2 = [1\,u(P)/P\,]^2 + [3\,u(L)/L]^2 + [-1\,u(b)/b]^2 + [-3\,u(h)/h]^2 + [-1\,u(\delta)/\delta\,]^2 \tag{A3}$$

A numerical example of this calculation is included in Appendix B.

TEST B: four point loaded beam

TERMINOLOGY
E_b = modulus of elasticity in bending;
b = specimen width;
h = specimen thickness;
P = load increment relative to pre-load;
δ = deflection increment at mid span relative to pre-load ;
a = distance from the support to the load applicator when the specimen is straight;
L = span length between supports.

The formula for the calculation of the modulus of elasticity for test B is:

$$E_b = P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^3 \delta) \qquad \textbf{(A4)}$$

The general combined standard uncertainty $u_c(Y)$ is expressed by:

$$[u_c(Y)]^2 = \Sigma\, (\partial f / \partial X_i)^2\, u^2(X_i). \qquad \textbf{(A5)}$$

Using this formula it is possible to write the combined standard uncertainty of E_b, $u_c(E_b)$:

$$[u_c(E_b)]^2 = (\partial E_b / \partial P)^2\, \mathbf{u^2_P\, (P)} + (\partial E_b / \partial L)^2\, \mathbf{u^2_L\, (L)} + (\partial E_b / \partial b)^2\, \mathbf{u^2_b\, (b)} + (\partial E_b / \partial h)^2\, \mathbf{u^2_h\, (h)} + (\partial E_b / \partial \delta)^2\, \mathbf{u^2_d (d)} + (\partial E_b / \partial a)^2\, \mathbf{u^2_a\, (a)} \qquad \textbf{(A6)}$$

and:

$$(\partial E_b / \partial P) = a\, (3L^2 - 4\, a^2)/\ 4\, b\, h^3\, \delta$$
$$(\partial E_b / \partial L) = a\, P\, (6L)/\ 4\, b\, h^3\, \delta$$
$$(\partial E_b / \partial b) = -\, P\, a\, (3L^2 - 4\, a^2)/(4\, b^2\, h^3 \delta)$$
$$(\partial E_b / \partial h) = -\, 3\, P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^4 \delta)$$
$$(\partial E_b / \partial \delta) = -\, P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^3 \delta^2)$$
$$(\partial E_b / \partial a) = P\, (3L^2 - 12\, a^2)\ /\ (4\, b\, h^3 \delta)$$

therefore:

$$[u_c(E_b)]^2 = [a(3L^2 - 4\, a^2)/\ 4\, b\, h^3\, \delta]^2\, \mathbf{u^2_P\, (P)} + [a\, P\, (6L)/\ 4\, b\, h^3\, \delta]^2\, \mathbf{u^2_L\, (L)} + [-\, P\, a\, (3L^2 - 4\, a^2)/(4\, b^2\, h^3 \delta)]^2\, \mathbf{u^2_b\, (b)} + [-\, 3\, P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^4\, \delta)]^2\, \mathbf{u^2_h\, (h)} + [-\, P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^3\, \delta^2)]^2 + [P\, (3L^2 - 12\, a^2)\ /\ (4\, b\, h^3 \delta)]^2\, \mathbf{u^2_a (a)}. \qquad \textbf{(A7)}$$

A numerical example of this calculation is included in Appendix B.

APPENDIX B

A Worked Example for Calculating Uncertainties in Bend Tests on Metallic Materials

EXAMPLE 1

This example covers the calculation of the uncertainty in the modulus of elasticity for case A: a three point loaded beam.

Equation (A3) of ***Appendix A*** gives the relative standard uncertainty:

$$[u_c(E_b)/\ E_b]^2 = [1\ u(P)/P\ ]^2 + [3\ u(L)/L\ ]^2 + [-1\ u(b)/b\ ]^2 + [-3\ u(h)/h\ ]^2 + [-1\ u(\delta)/\delta\ ]^2$$

DATA
E_b = modulus of elasticity in bending;
b = 38 [mm];
h = 4.99 [mm];
P = 675 [N];
δ = 1.38 [mm];
L = 200 [mm].

From equation (A1) $E_b = (P\ L^3)/(4\ b\ h^3\delta)$:

E_b = 207.2 [GPa]

1. Table I - A shows the combined standard uncertainty for the *span length between supports L* , **u(L)**.
2. Table I - B shows the combined standard uncertainty for of the *specimen width b*, **u_c(b)**.
3. Table I - C shows the combined standard uncertainty for the *specimen thickness h*, **u(h)**.
4. Table I - D shows the combined standard uncertainty for the deflection increment at mid span as measured from pre load δ, **u(d)**.
5. Table I - E is shows the combined standard uncertainty for the *load increment P*, **u_c(load)**.

From the calculation:
u(L) = 0.0543 [mm];
u(b) = 0.00879 [mm];
u(h) = 0.007955 [mm];
u(d) = 0.00118 [mm];
u_c(P) = 2.136 [N]

The relative combined standard uncertainty can now be calculated:

$$[u_c(E_b)/\ E_b]^2 = [2.136/675]^2 + [3*0.0543/200]^2 + [-1*0.00879/38]^2 + [-3*0.007955/4.99\ ]^2 + [-1*0.00118/1.38\ ]^2$$

$[u_c(E_b)/ E_b]^2 = 1.00137E-5 + 6.634E-7 + 0.5351E-7 + 2.28729E-5 + 7.31E-7 = 3.4335E-5$

so:

$[u_c(E_b)/ E_b] = 0.00586$

therefore:

$u_c(E_b) = 1.21$ [GPa]

This uncertainty corresponds to a 68.5% probability of survival, with a coverage factor k = 1.

For a 95% probability , a coverage factor k = 2 should be used , so it is necessary to obtain the expanded standard uncertainty:

$U (E_b)= k\ u_c(E_b) = 2 \times 1.21 = 2.42$ [GPa]

This result can be presented as % of the modulus of elasticity:

2.42/207.2 x 100 = 1.17 %

The final result for the calculation is:

207.2 GPa ± 2.42 GPa with a coverage factor k = 2

EXAMPLE 2

This example deals with the calculation of the uncertainty in the modulus of elasticity for case B: a four point loaded beam.

Equation (A6) of ***Appendix A*** gives the uncertainty requested:

$[u_c(E_b)]^2 = [\ a\ (3L^2 - 4\ a^2)/\ 4\ b\ h^3\ \delta]^2\ \mathbf{u^2_P\ (P)} + [a\ P\ (6L)/\ 4\ b\ h^3\ \delta]^2\ \mathbf{u^2_L\ (L)} + [-\ P\ a\ (3L^2 - 4\ a^2)/(4\ b^2\ h^3\delta)]^2\ \mathbf{u^2_b\ (b)} + [-3\ P\ a\ (3L^2 - 4\ a^2)/(4\ b\ h^4\ \delta)]^2\ \mathbf{u^2_h\ (h)} + [-\ P\ a\ (3L^2 - 4\ a^2)/(4\ b\ h^3\ \delta^2)]^2\ \mathbf{u^2_d\ (d)} + [\ P\ (3L^2 - 12\ a^2)\ /\ (4\ b\ h^3\delta)]^2\ \mathbf{u^2_a(a)}$.

DATA
E_b = modulus of elasticity in bending;
b = 38 [mm];
h = 4.74 [mm];
P = 563 [N];
δ = 1.25 [mm];
L = 200 [mm].
a = 75 [mm]

From equation (A4) $E_b = P\ a\ (3L^2 - 4\ a^2)/(4\ b\ h^3\delta)$ E_b = 203.5 [GPa]

Table I - A shows the combined standard uncertainty for the *span length between supports L* , **u(L)**.
Table I - B shows the combined standard uncertainty for the *specimen width b*, $\mathbf{u_c(b)}$.
Table I - F shows the combined standard uncertainty for the *specimen thickness h*, **u(h)**.
Table I - I shows the combined standard uncertainty for the *load increment P*, **u(P)**.
Table I - G shows the combined standard uncertainty for the deflection increment at mid span as measured from pre load δ, **u(d)**.
Table I - H shows the combined standard uncertainty for the *distance from the support a*, **u(a)**.

From the calculations:

u(L) = 0.0543 [mm];
u(b) = 0.00879 [mm];
u(h) = 0.0082 [mm];
$\mathbf{u_c(P)}$ = 1.781 [N];
u(d) = 0.00118 [mm];
u(a) = 0.00879 [mm];

and

$(\partial E_b / \partial P) = a\ (3L^2 - 4\ a^2)/\ 4\ b\ h^3\ \delta = 361.39$

$(\partial E_b / \partial L) = 6\ a\ PL/\ 4\ b\ h^3\ \delta = 2504.16$

$(\partial E_b / \partial b) = - P\, a\, (3L^2 - 4\, a^2)/(4\, b^2\, h^3 \delta) = - 5354.29$

$(\partial E_b / \partial h) = - 3\, P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^4 \delta) = - 128774.1$

$(\partial E_b / \partial \delta) = - P\, a\, (3L^2 - 4\, a^2)/(4\, b\, h^3 \delta^2) = - 162770.5$

$(\partial E_b / \partial a) = P\, (3L^2 - 12 a^2) / (4\, b\, h^3 \delta) = 1460.76$

It is now possible to calculate the combined standard uncertainty:

$[u_c(E_b)]^2 = 361.39^2 \times 1.781^2 + 2504.16^2 \times 0.0543^2 + (- 5354.29)^2 \times 0.00879^2 + (- 128774.1)^2 \times 0.0082^2 + (- 162770.5)^2 \times 0.00118^2 + 1460.76^2 \times 0.00879^2$

$[u_c(E_b)]^2 = 414268.9 + 18489.46 + 2215.04 + 1115025.6 + 36890.6 + 164.86 = 1587054.4$

$[u_c(E_b)] = 1.26$ [GPa]

It is possible to calculate the relative combined standard uncertainty:

$[u_c(E_b)]/ (E_b) = 0.006192$

This uncertainty corresponds to 68.5% probability, with a coverage factor k = 1.

For a 95% probability, a coverage factor k = 2 should be , so it is necessary to obtain the expanded standard uncertainty:

$U (E_b) = k\ u_c (E_b) = 2 \times 1.26 = 2.52$ [GPa]

This result can be presented as % of the modulus of elasticity:

2.52/203.5 x 100 = 1.24 %

Finally the global result for the calculation is:

203.5 GPa ± 2.52 GPa , with a coverage factor k = 2

Table I – A

Calculation
Uncertainty in Span length between supports
L = 200 [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution/ factor k'**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard Deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST PIECE							
Sensitivity of the		B	Rectangular	± 0.01			0.00577
Instrument			1/√3				
Correct dimension of L		A		200.1	200.1	0.12	0.054
				200.2			
				200.1			
				199.9			
				200.2			
COMBINED STANDARD UNCERTAINTY $u_c(L) = (S u^2_{s,i})^{1/2}$							**0.0543** [mm]

n.i. = not influential; n.a. = not applicable
(*) The standard deviation s = $(\Sigma^n_{i=1} (x_i - x_m)^2/(n-1))^{1/2}$
(**) The standard uncertainty is:
for an uncertainty Type A : $u_{s,i} = s / (n^{1/2})$
for an uncertainty Type B: $u_{s,i} = x * k'$

Table I - B

Calculation
Uncertainty in Specimen width
b = 38 [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i/ coverage Factor k**	**Average X_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST PIECE							
Original section	n.a.						
Sensitivity of the instrument		B	Rectangular	± 0.01			0.00577
			1/√3				
Correct dimension of b		A		38.01	38.01	0.014832	0.006633
				38.01			
				37.99			
				38.00			
				38.03			
COMBINED STANDARD UNCERTAINTY $u_c(b) = (S u^2_{s,i})^{1/2}$							**0.00879 [mm]**

n.i. = not influential; n.a. = not applicable (*), (**) see notes table I - A

Table I - C

Calculation
Uncertainty in Specimen Thickness
h = 4.99 [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST PIECE							
Sensibility of the Instrument		B	Rectangular	± 0.01			0.00577
			1/√3				
Correct dimension of h		A		4.98	4.99	0.01225	0.005477
				5.01			
				4.99			
				4.98 4.99			
COMBINED STANDARD UNCERTAINTY **$u_c(h) = (S\,u^2_{s,i})^{1/2}$**							**0.007955** [mm]

n.i. = not influential; n.a. = not applicable (*), (**) see notes table I - A

Table I - D

Calculation
Uncertainty of d

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage (mm)**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
APPARATUS							
Load Cell - calibration	n.i.						
Load Cell – sensitivity	n.i.						
Dynamic control of load	n.i.						
Drift of static control	n.i.						
Tooling alignment	n.i.						
Shape and dimension of the support	n.i.						
Extensometer	n.i.	B	Rectangular 1/√3	± 0.002			0.0011547
TEST METHOD							
Number of decimals	n.i.						
Digitising rate		B	Rectangular 1/√3	± 0.0004			0.00023
COMBINED STANDARD UNCERTAINTY **$u_c(d) = (S\,u^2_{s,i})^{1/2}$**							**0.00118** [mm]

n.i. = not influential; n.a. = not applicable (*), (**) see notes table I - A

Table I – E

Calculation
Uncertainty in Load Measurement
P = 675 [N]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [N]
APPARATUS							
Load Cell - calibration		B	Rectangular 1/√3	0.5% (of the value F) k=1			1.95
Load Cell - sensitivity		B	Rectangular 1/√3	± 0.1 [N]			0.0577
Drift of static control		B	Rectangular 1/√3	0.1% (of the value F) k=1			0.39
Tooling alignment		B	Rectangular 1/√3	0.2% (of the value F) k=1			0.80
TEST METHOD							
Specimen failure criteria	n.i.						
Digitising rate		B	Rectangular 1/√3	± 0.04			0.023
COMBINED STANDARD UNCERTAINTY **$u_c(load) = (S\, u^2_{s,i})^{1/2}$**							**2.136** [N]

n.i. = not influential; n.a. = not applicable; (*), (**) see notes table I - A

Table I – F

Calculation
Uncertainty in Specimen Thickness
h = 4.74 [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST PIECE							
Sensitivity of the		B	Rectangular	± 0.01			0.00577
Instrument			1/√3				
Correct dimension of h		A		4.74	4.74	0.01304	0.00583
				4.76			
				4.75			
				4.73 4.73			
COMBINED STANDARD UNCERTAINTY **$u_c(h) = (S\, u^2_{s,i})^{1/2}$**							**0.0082** [mm]

n.i. = not influential; n.a. = not applicable (*), (**) see notes table I - A

Table I – G

Calculation
Uncertainty in d
1.25 mm

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage (mm)**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
APPARATUS							
Load Cell - calibration	n.i.						
Load Cell - sensitivity	n.i.						
Dynamic control of load	n.i.						
Drift of static control	n.i.						
Tooling alignment	n.i.						
Shape and dimension of the support	n.i.						
Extensometer	n.i.	B	Rectangular 1/√3	± 0.002			0.0011547
TEST METHOD							
Number of decimals	n.i.						
Digitising rate		B	Rectangular 1/√3	± 0.0004			0.00023
COMBINED STANDARD UNCERTAINTY **$u_c(d) = (S\,u^2_{s,i})^{1/2}$**							**0.00118** [mm]

n.i. = not influential; n.a. = not applicable (*), (**) see notes Table I - A

Table I – H

Calculation
Uncertainty in measuring the distance from the support to the load applicator
L = 75 [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST PIECE							
Original section	n.a.						
Sensitivity of the		B	Rectangular	± 0.01			0.00577
Instrument			1/√3				
Correct dimension of a		A		75.01	75.008	0.014832	0.006633
				75.03			
				74.99			
				75.00 75.01			
COMBINED STANDARD UNCERTAINTY **$u_c(a) = (S\,u^2_{s,i})^{1/2}$**							**0.00879** [mm]

n.i. = not influential; n.a. = not applicable (*), (**) see notes table I - A

Table I – I

Calculation
Uncertainty in Load Measurement
P = 563 [N]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [N]
APPARATUS							
Load Cell - calibration		B	Rectangular 1/√3	0.5% (of the value F) k=1			1.625
Load Cell - sensitivity		B	Rectangular 1/√3	± 0.1 [N]			0.0577
Drift of static control		B	Rectangular 1/√3	0.1% (of the value F) k=1			0.325
Tooling alignment		B	Rectangular 1/√3	0.2% (of the value F) k=1			0.65
TEST METHOD							
Specimen failure criteria	n.i.						
Digitising rate		B	Rectangular 1/√3	± 0.04			0.023
COMBINED STANDARD UNCERTAINTY **u_c(load) = (S $u^2_{s,i}$)$^{1/2}$**							**1.781** [N]

n.i. = not influential; n.a. = not applicable; (*), (**) see notes table I - A