Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 04

The Determination of Uncertainties in Critical Crack Tip Opening Displacement (CTOD) Testing

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1. SCOPE

This procedure covers the evaluation of uncertainty in the determination of *Critical Crack Tip Opening Displacement* (CTOD) of metallic materials according to the testing Standards

British Standard, BS 7448. Part 1-1991: Amd 1: August 1999.
"Fracture Mechanics Toughness Tests. Part 1. Method for determination of K_{IC}, critical CTOD and critical J values of metallic materials".

ASTM E1290-93 *"Crack-Tip Opening Displacement (CTOD) Fracture Toughness Measurement"*

These standards give a method for determining critical crack tip opening displacement (***CTOD***) for metallic materials. The method uses fatigue precracked specimens. The tests are carried out in displacement control with monotonic loading, and at a constant rate of increase in stress intensity factor within the range $0.5 - 3\ MPa\sqrt{m}\ s^{-1}$ during the initial elastic deformation. The specimens are loaded to the maximum force associated with plastic collapse. The method is especially appropriate to materials that exhibit a change from ductile to brittle behaviour with decreasing temperature. No other influences of environment are covered.

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

a	nominal crack length
B	specimen thickness
c_i	sensitivity coefficient
CoP	Code of Practice
CT	Compact Tension Test Specimen
d_v	divisor associated with the assumed probability distribution, used to calculate the standard uncertainty
E	Young's modulus of elasticity
F	Applied applied Force
f	mathematical function of (a/W)
g	Poisson's ratio
k	coverage factor used to calculate expanded uncertainty
K	Stress Intensity Factor: the magnitude of the stress field near the crack tip for a particular mode in a homogeneous, ideally linear-elastic body.
n	number of repeat measurements
p	confidence level
q	random variable
$\bar{q}$	arithmetic mean of the values of the random variable q

s	span between outer loading points in three point bend test
S	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
S_y	proof strength
SE (B)	three-point bend specimen
u	standard uncertainty
u_c	combined standard uncertainty
U	expanded uncertainty
V	value of a measurand
V_p	plastic component of notch opening displacement
W	effective width of test specimen
x_i	estimate of input quantity
Y	measurand
y	test (or measurement) mean result
z	distance of the notch opening gauge location above the surface of the specimen
d	crack tip opening displacement

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be monitored more closely. This Code of Practice (CoP) has been prepared within UNCERT, a project to simplify the way in which uncertainties are evaluated funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165. The aim is to produce a series of documents in a common format that is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into six sections as follows with all individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in materials testing, the user may need to refer to *Section 3 in Reference 1*. The individual CoPs are kept as simple as possible by following the same structure:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (AppendixA)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in the determination of the ***CTOD*** parameter.

4. A PROCEDURE FOR ESTIMATING THE UNCERTAINTY IN THE DETERMINATION OF CRITICAL CRACK TIP OPENING DISPLACEMENT (CTOD), USING A THREE POINT BEND TEST SPECIMEN (SE (B))

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameter that is usually reported in the test. This measurand is not measured directly, but is determined from other quantities (or measurements).

Table 1 Measurand and Measurements, their units and symbols

Measurand	Units	Symbol
Crack tip opening displacement	*mm*	**d**
Measurements		
Thickness of the specimen	*mm*	B
Width of the specimen	*mm*	W
Crack length	*mm*	a
Applied force	*N*	F (F_c, F_u, F_m) *(See Fig A1)*
Plastic component of notch opening displacement	*mm*	V_p
Span between outer loading points in three point bend test	*mm*	s
Distance of the notch opening gauge location above the surface notch specimen	*mm*	z

Step 2. Identifying all Sources of Uncertainty in the Test

In the Step 2, the user must identify all possible sources of uncertainty that may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand, as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (e.g. when a plotter is replaced by a computer). To help the user list all sources, five categories have been defined. The following table (*Table 2*) lists the five categories and gives some examples of sources of uncertainty in each category for the test method applied.

It is important to note that *Table 2* is not exhaustive and is for guidance only. Relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 2 Sources of Uncertainty, their Type and their likely contribution to Uncertainties on Measurands and Measurements

*(1 = major contribution, 2 = minor contribution, blank = no influence, * = indirectly affected)*

Sources of uncertainty		Measurand and Measurements							
	Type [1]	d	B	W	a	F	V_P	s	z
1. Apparatus									
Load cell	B	*2				1			
Extensometer	B	*2					1		
Plotter X	B	*2					1		
Plotter Y	B	*2					1		
Caliper	B	*2	1	1	1				
Knife edges thickness [2]	B	*2					1		1
2. Method									
Span	B	*2				2	2	1	
Alignment	B	*2				2	2		
Perpendicularity	B	*2				2	2		
Speed	B	*2				2	2		
3. Environment									
Laboratory ambient temperature and humidity	B	*2				2	2		
4. Operator									
Graph interpretation	A	*1					1		
Distance between knife edges [2]	A	*2					2		
Error in measuring specimen dimensions	A	2							
Crack length measurement	A	*1			1				
5. Test Piece									
Specimen thickness	B	*2	1		2	2	2		
Specimen width	B	*1		1	2	2	2		

1 See Step 3
2 The specimen must be provided with a pair of accurately machined knife-edges that support the gauge arms and serve as the displacement reference points

Step 3. Classifying the Uncertainty According to Type A or B

In this third step, which is in accordance with Reference 2, '*Guide to the Expression of Uncertainties in Measurement*', the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified *Type A*, if it is evaluated by any other means it should be classified *Type B* (see *Table 2*).

The values associated with *Type B* uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For *Type B* uncertainties, it is necessary for the users to estimate for the most appropriate probability distribution for each source (further details are given in *Section 2 of Reference 1*).

It should be noted that, in some cases, an uncertainty could be classified as either *Type A* or *Type B* depending on how it is estimated.

Step 4. Estimating the Standard Uncertainty for each Source of Uncertainty

In this step the standard uncertainty, $u(x_i)$, for each measurement is estimated (see *Appendix A*). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter d_v, which is associated with the assumed probability distribution. The divisors for the distributions most likely to be encountered are given in *Section 2 of Reference 1*.

The significant sources of uncertainty and their influence on the evaluated quantity are summarised on *Table 3.*

This table is structured in the following manner:

- column ①: sources of uncertainty
- column ②: source's value. There are two types:
  - (1) permissible range for the measurement according to the test standard
  - (2) maximum range between measures on the same test made by several trained operators
- column ③: measurements affected by each source
- column ④: measurement values obtained in a real test
- column ⑤: source of uncertainty type
- column ⑥: assumed probability distribution
- column ⑦: correction factor for *Type B* sources (d_v)
- column ⑧: effect on the measurement uncertainty produced by the input quantity uncertainty

  This column is obtained by two different ways:
  - if the influence of the source of uncertainty on the measurement is proportionally direct.

    (column ②).(column ④) /(column ⑦)
  - if the influence is not direct it should be obtained by calculating the measurement for both the maximum and minimum values of the range in column ② without variation in the rest of the sources of uncertainty and applying the appropriate correction factor for the probability (column ⑦).

Table 3 Example Worksheet for Uncertainty Calculations in CTOD Tests

Column No. ①	②	③	④	⑤	⑥	⑦	⑧
Sources of Uncertainty		**Measurements**		**Uncertainties**			
Source	**Value** (1) **or** (2)	**Measurement Affected**	**Nominal or Averaged Value (Units)**	**Type**	**Probabl. Distribt.**	**Divisor (d_v)**	**Effect on Uncertainty in Measurement**
Apparatus							
Load cell		F	(kN)	B	Rectang.	$\sqrt{3}$	u(load cell)
Extensometer		V_P	(mm)	B	Rectang.	$\sqrt{3}$	u(extensom)
Plotter Y		V_P	(mm)	B	Rectang.	$\sqrt{3}$	u(plotterY)
Plotter X	The influence on Uncertainty is Negligible						
Knife edges thickness		z V_P	(mm)	B	Rectang.	$\sqrt{3}$	u(knife edges)
Caliper		W B a	(mm) (mm) (mm)	B	Rectang.	$\sqrt{3}$	u(caliper)
Method							
Span		s F V_P	(mm) (N) (mm)	B	Rectang.	$\sqrt{3}$	u(span)
Alignment		F V_P	(N) (mm)	B	Rectang.	$\sqrt{3}$	u(alignm)
Perpendicularity		F V_P	(N) (mm)	B	Rectang.	$\sqrt{3}$	u(perpen)
Distance between knife edges		V_P	(mm)	B	Rectang.	$\sqrt{3}$	u(distance)
Speed		F V_P	(N) (mm)	B	Rectang.	$\sqrt{3}$	u(speed)
Environment							
Room temperature		F V_P	(N) (mm)	B	Rectang.	$\sqrt{3}$	u(room temp)
Operator							
Graph interpretation		V_P	(mm)	A	normal	1	u(graph)
B Measurement		B	(mm)	A	normal	1	u(B msrment.)
W Measurement		W	(mm)	A	normal	1	u(W msrment.)
Crack length measurement		a	(mm)	A	normal	1	u(a msrment.)
Test Piece							
Specimen thickness		B	(mm)	B	Rectang.	$\sqrt{3}$	u(thickness)
Specimen width		W	(mm)	B	Rectang.	$\sqrt{3}$	u(width)

(1) permissible range for the source according to the test standard
(2) maximum range between measures made on the same test by several trained operators

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty of the measurand, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[c_i \cdot u(x_i)\right]^2} \qquad (1)$$

where c_i is the sensitivity coefficient associated with the measurement x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.27%.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, ***U***, is defined in Reference 2 as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand". It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, ***k***, which is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2, which corresponds to a confidence level interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty, ***U***, is, therefore, broader than the combined uncertainty, u_c. Where the customer (such as aerospace and electronics industries) demands a higher confidence level, a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in *Type A* analysis is small), the value of ***k*** should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference 1, Section 4 for more details).

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following format:

$$V = y \pm U$$

where: *V* is the estimated value of the measurand
y is the test (or measurand) mean result
U is the expanded uncertainty associated with ***y***

An explanatory note, such as that given in the following example should be added (change where appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k=2, which for a normal distribution corresponds to a coverage probability, ***p***, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 04:2000.

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials.* Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0 946754 41 1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, *Guide to the expression of Uncertainty in Measurement.* International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. [This Guide is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that drafted it.]

3. British Standard, BS 7448. Part 1-97. *"Fracture Mechanics Toughness Tests. Part 1. Method for determination of K_{IC}, critical CTOD and critical J values of Metallic Materials".*

4. ASTM E1290-93 *"Crack-Tip Opening Displacement (CTOD) Fracture Toughness Measurement"*

5. ASTM E399-90 *"Plane-Strain Fracture Toughness of Metallic Materials"*

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APPENDIX A

Mathematical Formulae for Calculating Uncertainties in Crack Tip Opening Displacement (CTOD) Parameter Determination Testing, using SE (B) specimens

The formula for the calculation of the Crack Tip Opening Displacement (***CTOD***) Fracture Toughness for a SE (B) specimen is:

$$d = \frac{K^2(1-g^2)}{2S_Y E} + \frac{0.4(W-a)V_P}{0.4W + 0.6a + z} \qquad \textbf{(A1)}$$

where ***K*** is a stress intensity factor, calculated from equation

$$K = \frac{Fs}{BW^{1.5}} f \qquad \textbf{(A2)}$$

where ***f*** is a mathematical function of $\frac{a}{W}$, given in next equation

$$f\left(\frac{a}{W}\right) = \frac{3\left(\frac{a}{W}\right)^{1/2}\left[1.99 - \left(\frac{a}{W}\right)\left(1-\frac{a}{W}\right)\right]\left[2.15 - 3.93\left(\frac{a}{W}\right) + 2.7\left(\frac{a}{W}\right)^2\right]}{2\left(1+2\frac{a}{W}\right)\left(1-\frac{a}{W}\right)^{3/2}} \qquad \textbf{(A3)}$$

where:

g	is the Poisson's ratio
S_y	is the 0.2% proof strength at the temperature of the fracture test
E	is the Young's modulus at the test temperature
V_P	is the plastic component of notch opening displacement. It is obtained by drawing a line parallel to the tangent of the initial linear part of the record from point ***F*** (F_c, F_u or F_m); the V_P value is the distance between the origin and the intersection of this parallel with X-axis (see *Figure A1*).

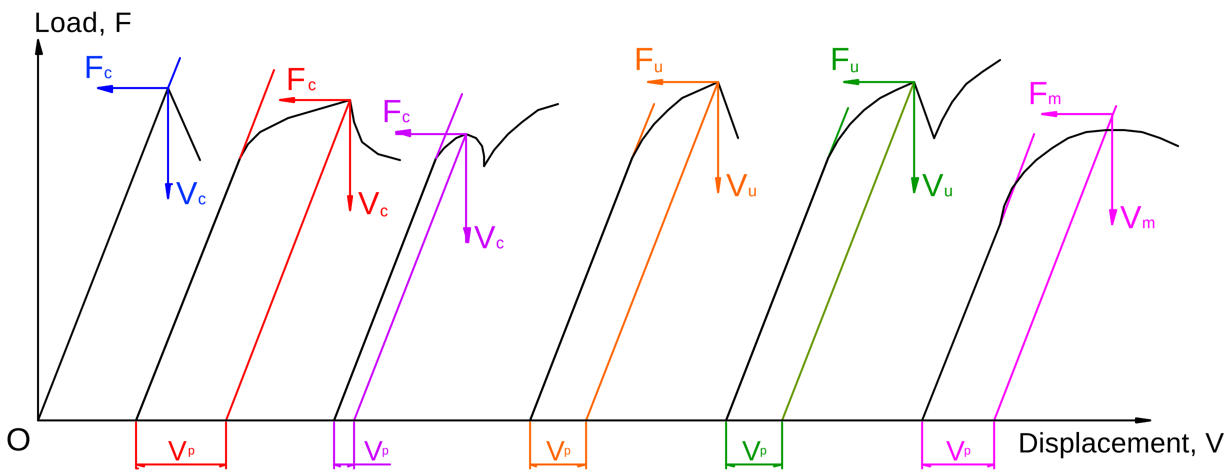

Figure A1. Definition of F (F_c, F_u or F_m) and V_P

The derived measurand **d** (*y*) is a function of eight measurements ***F***, ***f***, V_P, ***z***, ***s***, ***a***, ***B*** and

W (x_i), and each x_i is subject to uncertainty ***u(x_i)***. The general combined standard uncertainty ***u_c(y)*** is expressed by equation (1) in main procedure:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} [c_i u(x_i)]^2} \qquad \textbf{(A4)}$$

where

$$c_i = \frac{\partial y}{\partial x_i} \qquad \textbf{(A5)}$$

Using these formulae, it is possible to write the combined standard uncertainty of **d** parameter:

$$[u_c(\boldsymbol{d})]^2 = c_F{}^2 u(F)^2 + c_f{}^2 u(f)^2 + c_W{}^2 u(W)^2 + c_a{}^2 u(a)^2 + \\ + c_B{}^2 u(B)^2 + c_{V_P}{}^2 u(V_P)^2 + c_s{}^2 u(s)^2 + c_z{}^2 u(z)^2 \qquad \textbf{(A6)}$$

The sensitivity coefficients ($\boldsymbol{c_i}$)for equation **A6** are given by:

$$c_F = \frac{\partial \boldsymbol{d}}{\partial F} = AK\frac{\partial K}{\partial F} \qquad \textbf{(A7)}$$

where

$$\frac{\partial K}{\partial F} = \frac{sf}{BW^{3/2}} \qquad \textbf{(A8)}$$

and

$$A = \frac{1-g^2}{S_y E} \qquad \textbf{(A9)}$$

$$c_f = \frac{\partial \boldsymbol{d}}{\partial f} = AK\frac{\partial K}{\partial f} \qquad \textbf{(A10)}$$

where

$$\frac{\partial K}{\partial f} = \frac{F \times s}{B \times W^{3/2}} \qquad \textbf{(A11)}$$

$$c_W = \frac{\partial \boldsymbol{d}}{\partial W} = AK\frac{\partial K}{\partial W} + \frac{0.4V_P(a+z)}{(0.4W+0.6a+z)^2} \qquad \textbf{(A12)}$$

where

$$\frac{\partial K}{\partial W} = -\frac{1.5Fsf}{BW^{5/2}} \qquad \textbf{(A13)}$$

$$c_a = \frac{\partial \mathbf{d}}{\partial a} = \frac{-0.4V_P(W+z)}{(0.4W+0.6a+z)^2} \qquad \textbf{(A14)}$$

$$c_B = \frac{\partial \boldsymbol{d}}{\partial B} = AK\frac{\partial K}{\partial B} \qquad \textbf{(A15)}$$

where $$\frac{\partial K}{\partial B} = -\frac{Fsf}{B^2 W^{3/2}} \qquad \textbf{(A16)}$$

$$c_{V_P} = \frac{\partial \boldsymbol{d}}{\partial V_P} = \frac{0.4(W-a)}{0.4W + 0.6a + z} \qquad \textbf{(A17)}$$

$$c_s = \frac{\partial \boldsymbol{d}}{\partial s} = AK\frac{\partial K}{\partial s} \qquad \textbf{(A18)}$$

where $$\frac{\partial K}{\partial s} = \frac{Ff}{BW^{3/2}} \qquad \textbf{(A19)}$$

$$c_z = \frac{\partial \boldsymbol{d}}{\partial z} = -\frac{0.4(W-a)V_P}{(0.4W + 0.6a + z)^2} \qquad \textbf{(A20)}$$

A6 equation is composed by eight terms that will be analysed in next paragraphs.

A.1 UNCERTAINTY IN LOAD ($u(F)$)

F is a measurement affected by different sources of uncertainty (see *Table 2* and *Table 3* in the main procedure), but its influence can be considered negligible except for the load cell. So:

$$u(F) = u(load\ cell) \qquad \textbf{(A21)}$$

u(load cell) can be estimated using *Table 3* in this procedure.

A.2 UNCERTAINTY IN *f* ($u(f)$)

f is a mathematical function of ***a/W*** (equation **A3**).

Assuming that this source of uncertainty (***f***) is considered as *Type A*, the contribution to total uncertainty can be calculated from the standard deviation of the arithmetic mean:

$$u(f) = s(\bar{f}) = \frac{s(f_j)}{\sqrt{n}} \qquad \textbf{(A22)}$$

Where:

$$s(f_j) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(f_j - \bar{f})^2} \qquad \textbf{(A23)}$$

Using the maximum and the minimum values to calculate *s(*f_j*)*, (n=2):

$$u(f) = \frac{\sqrt{\frac{1}{2-1}\left[(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2\right]}}{\sqrt{2}} = \sqrt{\frac{(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2}{2}} \quad \textbf{(A24)}$$

Where:

$$f_{max} = f(\frac{a_{max}}{W_{min}}) \quad and \quad f_{min} = f(\frac{a_{min}}{W_{max}}) \quad \textbf{(A25)}$$

and

$$\begin{aligned} a_{max} &= a_0 + 2u(a) \\ a_{min} &= a_0 - 2u(a) \\ W_{max} &= W_0 + 2u(W) \\ W_{min} &= W_0 - 2u(W) \end{aligned} \quad \textbf{(A26)}$$

(assuming that both ***a*** and ***W*** have a normal probability distribution, k=2)

A.3 UNCERTAINTY IN SPECIMEN WIDTH ($u(W)$)

W is a measurement affected by three sources of uncertainty (see *Table 2* and *Table 3*). Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty in ***W*** can be computed using the root sum squares:

$$u(W) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(W\ measurement)^2 + u(width)^2} \quad \textbf{(A27)}$$

A.4 UNCERTAINTY IN CRACK LENGTH ($u(a)$)

a is a measurement affected by two sources of uncertainty (see *Table 2* and *Table 3*). Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty in ***a*** can be computed using the root sum squares:

$$u(a) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(crack\ length\ measurement)^2} \quad \textbf{(A28)}$$

The term ***u(s_i)*** can be calculated using Table 3 in this procedure.

A.5 UNCERTAINTY IN *B* ($u(B)$)

B is a measurement affected by three sources of uncertainty (see *Table 2* and *Table 3* in the procedure). Assuming that the individual uncertainty sources are uncorrelated, the combined uncertainty in ***B*** can be computed using the root sum squares:

$$u(B) = \sqrt{u(s_i)^2} = \sqrt{u(caliper)^2 + u(B\ measurement)^2 + u(thickness)^2} \quad \textbf{(A29)}$$

The terms ***u***(s_i) can be calculated using Table 3 in this procedure.

A.6 UNCERTAINTY IN THE PLASTIC COMPONENT OF NOTCH OPENING DISPLACEMENT ($u(V_P)$)

V_P is a measurement affected by five sources of uncertainty (see *Table 2* and *Table 3* in the procedure). Assuming that the individual uncertainty sources are uncorrelated, the combined uncertainty in V_P can be computed using the root sum squares:

$$u(V_P) = \sqrt{u(s_i)^2} = \sqrt{\begin{array}{l} u(extensometer)^2 + u(plotter\ Y)^2 + u(graph\ interpretation)^2 + \\ + u(knife\ edges\ thickness)^2 + u(distance\ between\ knife\ edges)^2 \end{array}} \quad \text{(A30)}$$

(**$u(s_i)$** is the standard uncertainty of each source that contributes to ***F*** combined uncertainty).

The terms **$u(s_i)$** can be calculated using *Table 3* in this procedure.

A.7 UNCERTAINTY IN THE SPAN ($u(s)$)

It is directly obtained from data in *Table 3*.

A.8 UNCERTAINTY IN THE KNIFE EDGES THICKNESS ($u(z)$)

It is directly obtained from data in *Table 3*

The combined uncertainty of each measurement is shown in *Table A1*.

Table A1 Formulae for calculating Combined Uncertainties

Measurement	Sources of uncertainty (s_i)	u (s_i) (Units)	Uncertainty of Measurements u(x_i)
Applied force ***F***	Load cell	(kN)	$u(F) = u(load\ cell)$
Plastic component of notch opening displacement ***V_P***	Extensometer	(mm)	$u(V_P) = \sqrt{\begin{array}{l} u(extensometer)^2 + u(plotter Y)^2 + \\ + u(graph interpretation)^2 + \\ + u(knife edges thickness)^2 + \\ + u(distance between knife edges)^2 \end{array}}$
	Plotter Y	(mm)	
	Graph Interpretation	(mm)	
	Knife Edges Thickness	(mm)	
	Distance between Knife Edges	(mm)	
Knife edges thickness ***z***	Knife Edges Thickness	(mm)	$u(z) = u(knife\ edges\ thickness)$
Specimen width ***W***	Caliper	(mm)	$u(W) = \sqrt{\begin{array}{l} u(caliper)^2 + u(W\ measurement)^2 + \\ + u(width)^2 \end{array}}$
	W measurement	(mm)	
	Specimen Width	(mm)	
Specimen thickness ***B***	Caliper	(mm)	$u(B) = \sqrt{\begin{array}{l} u(caliper)^2 + u(B\ measurement)^2 + \\ + u(thickness)^2 \end{array}}$
	B measurement	(mm)	
	Specimen Thickness	(mm)	
Crack length ***a***	Caliper	(mm)	$u(a) = \sqrt{u(caliper)^2 + u(crack length measurement)^2}$
	Crack Length Measurement	(mm)	
Span ***s***	Span	(mm)	$u(s) = u(span)$

APPENDIX B

A Worked Example for Calculating Uncertainties in Crack Tip Opening Displacement (CTOD) Parameter Determination Testing

B1. Introduction

A customer asked a testing laboratory to carry out a fracture test to determinate the Crack Tip Opening Displacement (***CTOD***) at room temperature, according to British Standard, BS 7448. Part 1-1991, on three point bend specimens. The mechanical properties of the material were:

- proof strength at 0.2% at the test temperature: 602 MPa
- Young's modulus: 210,000 MPa
- Poisson's ratio: 0.3.

The laboratory has considered the sources of uncertainty in its test facility and has found that the sources of uncertainty in the test results are identical to those described in *Table 2* of the Main Procedure.

B2. Estimation of Input Quantities to the Uncertainty Analysis

1 All tests were carried out according to the laboratory's own procedure using an appropriately calibrated tensile test facility. The test facility was located in a temperature-controlled environment, at room temperature.

2 The specimen was a three point bend type, and its dimensions were:

thickness ***B*** = 18 mm ± 0.5%

width ***W*** = 36 mm ± 0.5%

The dimensions of the specimen were measured using a caliper with an uncertainty of *0.05 mm*, typical for calipers used in the laboratory.

3 The crack length (***a***) was measured with the same caliper and the value obtained was 17.57 mm.

4 The test was carried out on a universal test machine using a strain rate of $0.4\ kNs^{-1}$. (A constant loading rate of $0.3 - 1.5\ kNs^{-1}$ for a standard bend specimen corresponds to a rate of increase in stress intensity factor within the range $0.55 - 2.75\ MPa\sqrt{m}\ s^{-1}$, according to *ASTM E 399 Standard, Clause A3.4.2.1*).

5 The machine was calibrated to *Grade 1* of BS 1610.

6 Strain was measured using a clip gauge extensometer with a nominal gauge length of *10 mm*. The extensometer complied with *Class 0.5*, specification according to EN 10002-4:1994.

7 The thickness of the knife-edges attached to the specimen were 1.5 mm (*BS 7448 Clause 5.1.3* permits a thickness of *1.5 - 2 mm*) and were attached at a distance of *10 mm*. This distance was determined by the extensometer, because it is impossible to calibrate the extensometer if the distance between edges is not within the nominal value ± 1%

8 The line of action of the applied force passed midway between the centres of the rollers within ± 1% of the distance between these centres. The crack tip midway squared to the roller axes within ±2º.

9 The span for the test method was *144 mm ±0.5%*, according to the test standard.

10 The accuracy of the plotter used to record the load-displacement curve was within ±0.5% in both axes.

The curve obtained in this test is represented in *Figure B1*. The maximum load value recorded by the machine F_C was *33,800 N*. The value indicated in *Figure B1* as V_P was obtained graphically by the operator (V_P = *0.42 mm*), according to the procedure described in *Appendix A* of this *CoP*.

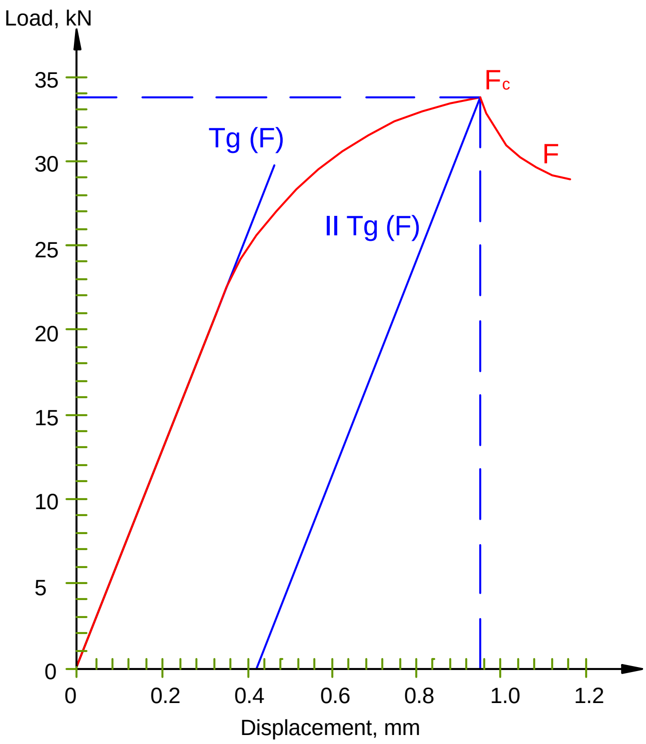

Figure B1. Test Record

11 An indication of the uncertainty associated with interpreting the graph was obtained using another test record (*Figure B2*). Four trained operators analysed the record and obtained the values indicated in *Table B1*.

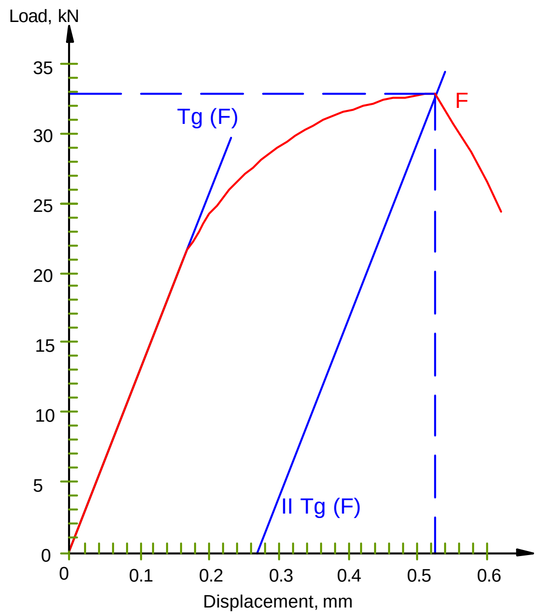

Figure B2. Test Record used to establish the Uncertainty in Interpreting the Graph B1

Table B1 Uncertainty in Interpreting the Graph B2

	V_P (mm)
Operator 1	0.273
Operator 2	0.274
Operator 3	0.265
Operator 4	0.265
Mean	0.269

The uncertainty for this input value is its standard deviation:

$$u = \sqrt{\frac{1}{n-1}\sum_{j=1}^{4}(V_j - \bar{V})^2} =$$

$$= \sqrt{\frac{1}{3}[(0.273-0.269)^2 + (0.274-0.269)^2 + 2\cdot(0.265-0.269)^2]} =$$

$$= 0.005 \text{ mm} \equiv 1.829\%$$

12 The uncertainties associated with the error in measurements (***B***, ***W*** and ***a***) were obtained in the same manner as above (using a compact tension test specimen with the same nominal dimensions than the used in this example ***B*** = 18 *mm* and ***W*** = 36 *mm*)

Table B2 Uncertainty in Dimensions Measurement

	B (mm)	W (mm)	a (mm)
Operator 1	18.02	35.95	19.14
Operator 2	17.98	36.04	19.30
Operator 3	18.01	35.97	19.27
Operator 4	17.98	36.05	19.30
Mean	17.998	36.003	19.253
Standard Deviation	0.021	0.050	0.076
Uncertainty*	0.115%	0.139%	0.396%

(*) $\text{uncertainty}(\%) = \dfrac{s(q_i) \cdot 100}{mean}$

The highest uncertainty associated with all these measurements was in the crack length, so an uncertainty ***of 0.4***% has been selected for both ***B***, ***W*** and ***a***.

13 The influence of the knife-edges thickness was studied using a Computer Assistant Design Program:

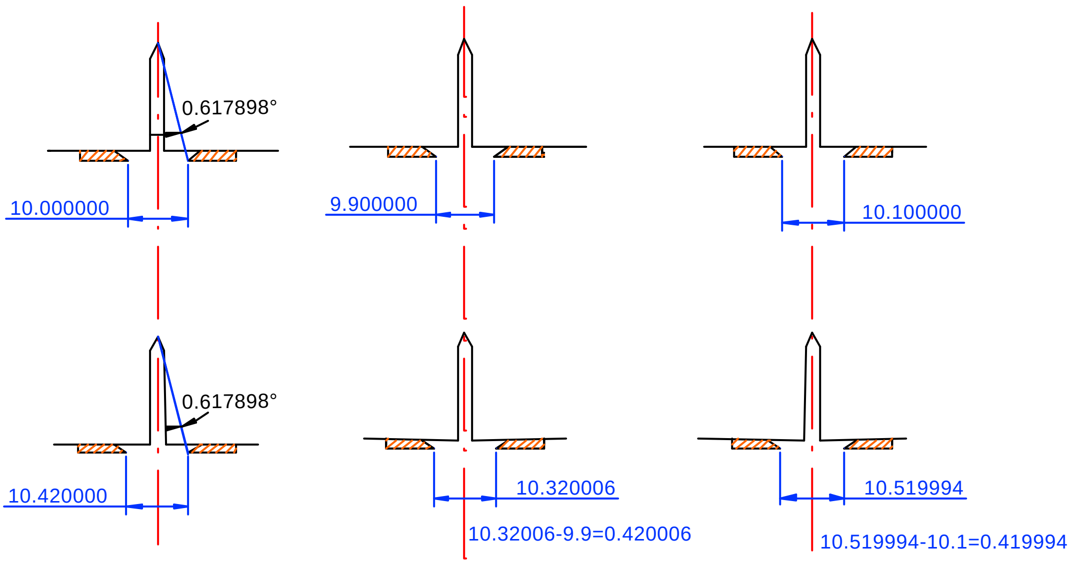

Fig. B3.1 *Fig. B3.2* *Fig. B3.3*

Figure B3. Uncertainty in Distance between Knife Edges

First of all, with a distance of *10 mm* between knife edges, the angle for a V_P of *0.42 mm* is calculated (*Figure B3.1*).

Then putting the knife edges at both the maximum and the minimum distances allowed, and with the angle calculated in first step, maximum and minimum V_P values are obtained (*Figures B3.2 and B3.3*).

Then, the influence of the knife edges distance on V_P is:

$$u(distance) = \pm\frac{V_{max} - V_{min}}{2} = \pm\frac{0.420006 - 0.419994}{2} = \pm 6\cdot 10^{-6}\ mm \equiv 0.0014\%$$

14 The influence of the knife edges thickness was obtained using a *CAD* program too:

First of all, with knife edges of 1.5 *mm* thickness, the angle for a V_P of *0.42 mm* is calculated (*Figure B4.1*).

Then with knife edges of 2 *mm* thickness (maximum allowed), and with the angle calculated in first step, V_P value is obtained (*Figure B4.2*).

Then, the influence of the knife edges thickness on V_P is:

$$u(thickness) = \pm\frac{V_{max} - V_{min}}{2} = \pm\frac{0.430784 - 0.42}{2} = \pm 0.005\, mm \equiv 1.284\%$$

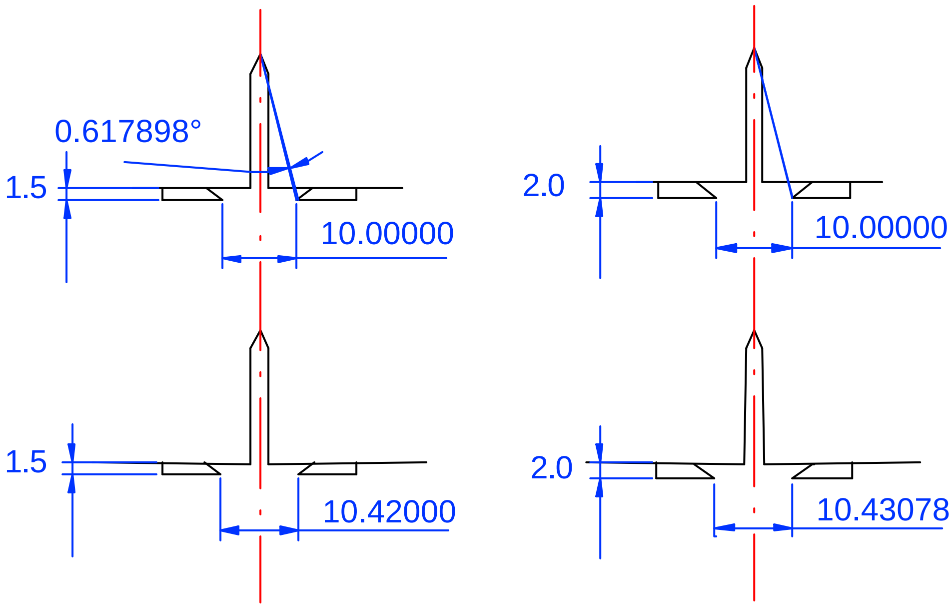

Fig. B4.1 *Fig. B4.2*

Figure B4. Uncertainty in Knife Edges Thickness

B3 Example of Uncertainty Calculations and Reporting of Results

B3.1 Calculations

Firstly, *Table 3* in main procedure should be completed.

Column ①	②	③	④	⑤	⑥	⑦	⑧
Sources of Uncertainty		**Measurements**		**Uncertainties**			
Source	**Value** (1) or (2)	**Measurement Affected**	**Nominal or Averaged Value (Units)**	**Type**	**Probabl. Distribt.**	**Divisor (d_v)**	**Effect on Uncertainty in Measurement**
Apparatus							
Load cell	$\pm 1\%^{(1)}$	F	33,800 N	B	Rectang.	$\sqrt{3}$	195 N$^{(A)}$
Extensometer	$\pm 0.5\%^{(1)}$	V_P	0.42 mm	B	Rectang.	$\sqrt{3}$	0.001 mm$^{(A)}$
Plotter Y	$\pm 0.5\%^{(1)}$	V_P	0.42 mm	B	Rectang.	$\sqrt{3}$	0.001 mm$^{(A)}$
Plotter X	The influence on Uncertainty is Negligible						
Knife edges thickness	1.5-2 mm$^{(1)}$ $\pm 1.284\%^{(B)}$	z V_P	1.5 mm 0.42 mm	B	Rectang.	$\sqrt{3}$	0.144 mm$^{(E)}$ 0.005 mm$^{(B)}$
Caliper	$\pm 0.05mm^{(1)}$	W B a	36 mm 18 mm 17.57 mm	B	Rectang.	$\sqrt{3}$	0.029 mm$^{(C)}$
Method							
Span	$\pm 0.5\%^{(1)}$	s F V_P	144 mm 33,800 N 0.42 mm	B	Rectang.	$\sqrt{3}$	0.416 mm$^{(A)}$ Nglg.$^{(D)}$ Nglg.$^{(D)}$
Alignment	$\pm 1\% \times s$	F V_P	33,800 N 0.42 mm	B	Rectang.	$\sqrt{3}$	Nglg.$^{(D)}$
Perpendicularity	$2°^{(1)}$	F V_P	33,800 N 0.42 mm	B	Rectang.	$\sqrt{3}$	Nglg.$^{(D)}$
Distance between knife edges	$\pm 1\%^{(1)}$ $\pm 0.0014\%^{(B)}$	- V_P	0.42 mm	B	Rectang.	$\sqrt{3}$	- 6×10^{-4} mm$^{(B)}$
Speed	0.3–1.5 kNs^{-1} (1)	F V_P	33,800 N 0.42 mm	B	Rectang.	$\sqrt{3}$	Nglg.$^{(D)}$
Environment							
Room temperature	$\pm 2°C^{(1)}$	F V_P	33,800 N 0.42 mm	B	Rectang.	$\sqrt{3}$	Nglg.
Operator							
Graph Interpretation	$\pm 1.829\%^{(2)}$	V_P	0.42 mm	A	normal	1	0.008 mm$^{(A)}$
B Measurement	$\pm 0.4\%^{(2)}$	B	18 mm	A	normal	1	0.072 mm$^{(A)}$
W Measurement	$\pm 0.4\%^{(2)}$	W	36 mm	A	normal	1	0.144 mm$^{(A)}$
A Measurement	$\pm 0.4\%^{(2)}$	a	17.57 mm	A	normal	1	0.070 mm$^{(A)}$
Test Piece							
Specimen thickness	$\pm 0.5\%^{(1)}$	B	18 mm	B	Rectang.	$\sqrt{3}$	0.052 mm$^{(A)}$
Specimen width	$\pm 0.5\%^{(1)}$	W	36 mm	B	Rectang.	$\sqrt{3}$	0.104 mm$^{(A)}$

(1) permissible range for the source according to the test standard (see next paragraphs)
(2) maximum range between measures made on the same test by several trained operators
Nglg. = negligible if compliant to standard (see next paragraphs)
(A toE) see next paragraphs (Column ⑧)

Where:

Column ②:

- Load Cell: according to the BS 7448, the force sensing device shall comply with grade 1 of BS 1610 (accuracy within $\pm 1\%$)
- Extensometer: our extensometers comply with grade 0.5 (accuracy within $\pm 0.5\%$)
- Plotter: our plotter complies with grade 0.5 (accuracy within $\pm 0.5\%$)
- Knife edges: according to the test standard the knife edges thickness must be between 1.5 and 2 mm. Their influence has been calculated geometrically using a *CAD* program (see *B2. Estimation of input quantities to the uncertainty analysis*)
- Caliper: generally uncertainty of 0.05 mm is typical for calipers used to measure both test piece dimensions and crack length
- Span: according to the test standard, span must be adjusted to $\pm 0.5\%$
- Alignment: according to the test standard, the line of action of the applied force must pass midway between the centres of the rollers within $\pm 1\%$ of the distance between these centres.
- Perpendicularlity: according to the test standard, the crack tip midpoint must be perpendicular to the roller within $2°$
- Distance between edges: this distance is determined by the extensometer, because it is impossible to calibrate the extensometer if the distance between edges is not within the nominal value $\pm 1\%$ Their influence has been calculated geometrically using a *CAD* program (see *B2. Estimation of input quantities to the uncertainty analysis*)
- Room temperature: this influence is considered negligible for metallic materials, because only small variations $\pm 2°C$ are allowed by the test standard
- Graph Interpretation: this is an important contribution to total uncertainty because of the manual method used to obtain V_P value from the test record
- Specimen Thickness: according to the test standard, the tolerance for

the specimen thickness is ± 0.5%

- Specimen Width: according to the test standard, the tolerance for the specimen width is ± 0.5%

Column ④: are the values obtained in the real test

Column ⑧:
- (A) (column⑧) = (column②) (column④) / (column⑦)
- (B) it has been calculated geometrically using a *CAD* program (see *B2. Estimation of input quantities to the uncertainty analysis*)
- (C) (column ⑧) = (column②) / (column⑦)
- (D) These sources have a minor influence on both ***F*** and $\boldsymbol{V_P}$, and these influences are opposed mutually (if ***F*** tends to be greater, $\boldsymbol{V_P}$ tends to be smaller), so the influence on **d** is compensated and negligible
- (E) (column⑧) = (column②) /[2 (column⑦)]

Filling in *Table 4* in main procedure:

Measurement	Sources of uncertainty (x_i)	u (x_i)	Uncertainty of Measurands u(X_j)
F	Load cell	195 N	$u(F) = 195\ N$
V_P	Extensometer	0.001 mm	$u(V_P) = \sqrt{(0.001)^2 + (0.001)^2 + (0.008)^2 + (0.005)^2 + (6 \cdot 10^{-4})^2} = 0.021\ mm$
	Plotter Y	0.001 mm	
	Graph Interpretation	0.008 mm	
	Knife Edges Thickness	0.005 mm	
	Distance between Knife Edges	$6 \cdot 10^{-4}$ mm	
z	Knife Edges Thickness	0.144 mm	$u(z) = 0.144\ mm$
W	Caliper	0.029 mm	$u(W) = \sqrt{(0.029)^2 + (0.144)^2 + (0.104)^2} = 0.180\ mm$
	W Measurement	0.144 mm	
	Specimen Width	0.104 mm	
B	Caliper	0.029 mm	$u(B) = \sqrt{(0.029)^2 + (0.072)^2 + (0.052)^2} = 0.093\ mm$
	B Measurement	0.072 mm	
	Specimen Thickness	0.052 mm	
a	Caliper	0.029 mm	$u(a) = \sqrt{(0.029)^2 + (0.07)^2} = 0.076\ mm$
	Crack Length Measurement	0.070 mm	
s	Span	0.416 mm	$u(s) = 0.416\ mm$

To calculate the uncertainty of the ***f*** factor, equations **A24** to **A26** are applied:

$$a_{max} = a_0 + 2 \times u(a) = 17.57 + 2 \times 0.076 = 17.722$$
$$a_{min} = a_0 - 2 \times u(a) = 17.57 - 2 \times 0.076 = 17.418$$
$$W_{min} = W_0 - 2 \times u(W) = 36 + 2 \times 0.180 = 35.64$$
$$W_{min} = W_0 - 2 \times u(W) = 36 - 2 \times 0.180 = 36.36$$

(A26)

$$\bar{f} = f(\frac{a}{W}) = f(\frac{17.57}{36}) = 2.564$$

$$f_{max} = f(\frac{a_{max}}{W_{min}}) = f(\frac{17.722}{35.64}) = 2.639 \quad \textbf{(A25)}$$

$$f_{min} = f(\frac{a_{min}}{W_{max}}) = f(\frac{17.418}{36.36}) = 2.494$$

$$u(f) = \sqrt{\frac{(f_{max} - \bar{f})^2 + (f_{min} - \bar{f})^2}{2}} = \sqrt{\frac{(2.639 - 2.564)^2 + (2.494 - 2.564)^2}{2}} = 0.073 \quad \textbf{(A24)}$$

Now partial derivatives (equations **A7** to **A20**) are obtained:

$$c_F = \frac{\partial \boldsymbol{d}}{\partial F} = AK\frac{\partial K}{\partial F} \quad \textbf{(A7)}$$

$$A = \frac{1 - g^2}{S_y E} = \frac{1 - 0.3^2}{602 \times 210{,}000} = 7.198 \times 10^{-9} \ MPa^{-2} \quad \textbf{(A9)}$$

$$K = \frac{Fs}{BW^{3/2}} f = \frac{33{,}800 \times 144}{18 \times 36^{3/2}} \times 2.564 = 3{,}210.25 \ N\,mm^{-3/2} \quad \textbf{(A2)}$$

$$\frac{\partial K}{\partial F} = \frac{sf}{BW^{3/2}} = \frac{144 \times 2.564}{18 \times 36^{3/2}} = 0.095 mm^{-3/2} \quad \textbf{(A8)}$$

$$c_F = \frac{\partial \boldsymbol{d}}{\partial F} = AK\frac{\partial K}{\partial F} = 7.198 \times 10^{-9} \times 3{,}210.25 \times 0.095 = \quad \textbf{(A7)}$$
$$= 2.19 \times 10^{-6} \ N^{-1} mm$$

$$c_f = \frac{\partial \boldsymbol{d}}{\partial f} = AK\frac{\partial K}{\partial f} \quad \textbf{(A10)}$$

$$\frac{\partial K}{\partial f} = \frac{Fs}{BW^{3/2}} = \frac{33{,}800 \times 144}{18 \times 36^{3/2}} = 1{,}251.8 \ N\,mm^{-3/2} \quad \textbf{(A11)}$$

$$c_f = \frac{\partial \boldsymbol{d}}{\partial f} = AK\frac{\partial K}{\partial f} = 7.198 \times 10^{-9} \times 3{,}210.25 \times 1{,}251.8 = 2.89 \times 10^{-2} \ mm \quad \textbf{(A10)}$$

$$c_W = \frac{\partial \boldsymbol{d}}{\partial W} = AK\frac{\partial K}{\partial W} + \frac{0.4 V_P (a + z)}{(0.4W + 0.6a + z)^2} \quad \textbf{(A12)}$$

$$\frac{\partial K}{\partial W}=-\frac{1.5Fsf}{BW^{5/2}}=-\frac{1.5\times 33{,}800\times 144\times 2.564}{18\times 36^{5/2}}=-133.76\ N\,mm^{-5/2} \quad \textbf{(A13)}$$

$$c_W=\frac{\partial \boldsymbol{d}}{\partial W}=AK\frac{\partial K}{\partial W}+\frac{0.4V_P(a+z)}{(0.4W+0.6a+z)^2}=$$
$$=7.198\times 10^{-9}\times 3{,}210.25\times(-133.76)+ \quad \textbf{(A12)}$$
$$+\frac{0.4\times 0.42(17.57+1.5)}{(0.4\times 36+0.6\times 17.57+1.5)^2}=1.49\times 10^{-3}$$

$$c_a=\frac{\partial \mathbf{d}}{\partial a}=\frac{-0.4V_P(W+z)}{(0.4W+0.6a+z)^2}=\frac{-0.4\times 0.42(36+1.5)}{(0.4\times 36+0.6\times 17.57+1.5)^2}=-0.009 \quad \textbf{(A14)}$$

$$c_B=\frac{\partial \boldsymbol{d}}{\partial B}=AK\frac{\partial K}{\partial B} \quad \textbf{(A15)}$$

$$\frac{\partial K}{\partial B}=-\frac{Fsf}{B^2W^{3/2}}=-\frac{33{,}800\times 144\times 2.564}{18^2\times 36^{3/2}}=-178.347N\,mm^{-5/2} \quad \textbf{(A16)}$$

$$c_B=\frac{\partial \boldsymbol{d}}{\partial B}=AK\frac{\partial K}{\partial B}=7.198\times 10^{-9}\times 3{,}210.25\times(-178.347)=-4.12\times 10^{-3} \quad \textbf{(A15)}$$

$$c_{V_P}=\frac{\partial \boldsymbol{d}}{\partial V_P}=\frac{0.4(W-a)}{0.4W+0.6a+z}=\frac{0.4(36-17.57)}{0.4\times 36+0.6\times 17.57+1.5}=0.279 \quad \textbf{(A17)}$$

$$c_s=\frac{\partial \boldsymbol{d}}{\partial s}=AK\frac{\partial K}{\partial s} \quad \textbf{(A18)}$$

$$\frac{\partial K}{\partial s}=\frac{Ff}{BW^{3/2}}=\frac{33{,}800\times 2.564}{18\times 36^{3/2}}=22{,}293N\,mm^{-5/2} \quad \textbf{(A19)}$$

$$c_s=\frac{\partial \boldsymbol{d}}{\partial s}=AK\frac{\partial K}{\partial s}=7.198\times 10^{-9}\times 3{,}210.25\times 22.293=5.15\times 10^{-4} \quad \textbf{(A18)}$$

$$c_z=\frac{\partial \boldsymbol{d}}{\partial z}=-\frac{0.4(W-a)V_P}{(0.4W+0.6a+z)^2}=-\frac{0.4\times(36-17.57)\times 0.42}{(0.4\times 36+0.6\times 17.57+1.5)^2}=-0.004 \quad \textbf{(A20)}$$

The combined standard uncertainty for **d** will be (equation **A6**):

$$
\begin{aligned}
[u_c(\delta)]^2 &= c_F{}^2 u(F)^2 + c_f{}^2 u(f)^2 + c_W{}^2 u(W)^2 + c_a{}^2 u(a)^2 + \\
&\quad + c_B{}^2 u(B)^2 + c_{V_P}{}^2 u(V_P)^2 + c_s{}^2 u(s)^2 + c_z{}^2 u(z)^2 = \\
&= (2.195\times10^{-6})^2 \times 195^2 + (2.89\times10^{-2})^2 \times 0.073^2 + \\
&\quad + (1.49\times10^{-3})^2 \times 0.180^2 + (-0.009)^2 \times 0.076^2 + \\
&\quad + (-4.12\times10^{-3})^2 \times 0.093^2 + 0.279^2 \times 0.021^2 + \\
&\quad + (5.15\times10^{-4})^2 \times 0.416^2 + (-0.004)^2 \times 0.144^2 \\
&= 4.24\times10^{-5}\ \text{mm}^2
\end{aligned}
\quad \textbf{(A6)}
$$

$u_c(d) = 0.006\, mm$

The expanded uncertainty with a confidence interval of 95.4% is obtained multiplying the combined standard uncertainty by a coverage factor of 2:

$U(d) = 0.006 \times 2 = 0.012\, mm$

The value of **d** is calculated with equation **A1**:

$$
d = \frac{K^2(1-g^2)}{2S_Y E} + \frac{0.4\times(W-a)\times V_P}{0.4W + 0.6a + z} =
$$

$$
= \frac{3{,}210.25^2 \times (1-0.3^2)}{2\times603\times210{,}000} + \frac{0.4\times(36-17.57)\times0.42}{0.4\times36 + 0.6\times17.57 + 1.5} = 0.154\, mm
$$

B3.2 Reported Results

The Critical Crack Tip Opening Displacement (***CTOD***) is 0.154 ± 0.012 mm

The above reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k=2, which for a normal distribution corresponds to a coverage probability, ***p****, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 04:2000.*