Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 07

The Determination of Uncertainties in Tensile Testing

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1. SCOPE

This procedure covers the evaluation of uncertainty in tensile test results obtained from tests at ambient or elevated temperature, carried out according to any of the following Standards:

EN 10002-Part 1-1990: *"Tensile testing - Method of testing at ambient temperature"*

EN 10002-Part 5-1990: *"Tensile testing - Method of testing at elevated temperature"*

ASTM E8-1998: *"Standard Test Methods for Tension Testing of Metallic*

ASTM E111-1997: *"Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus"*

The Code of Practice is restricted to tests performed at ambient and elevated temperatures with a digital acquisition of load and displacement. The tests are assumed to run continuously without interruptions on specimens that have uniform gauge lengths, and the procedure is restricted to tests performed under axial loading conditions.

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

a_0	original thickness of a sheet type specimen, (mm)
a_u	minimum thickness after fracture, (mm)
b_0	width of the parallel length of a sheet type specimen, (mm)
b_u	minimum width after fracture, (mm)
c_i	sensitivity coefficient associated with uncertainty on measurement x_i, [see Appendix A]
d_0	diameter of the parallel length of a circular test specimen, (mm)
d_u	minimum diameter after fracture, (mm)
E	Young's modulus, (GPa)
F	force, (N)
Fe_H	force at Re_H, (N)
Fe_L	force at Re_L, (N)
F_m	maximum force, (N)
L_0	extensometer gauge length = L_e, (mm)
L_u	final gauge length, (mm)
n	evaluated data pairs in the linear regression

Re_H	upper yield strength, (MPa)
Re_L	lower yield strength, (MPa)
R_m	ultimate tensile strength, (MPa)
R_p	stress at a permanent strain, (MPa)
S_0	original cross-sectional area, (mm^2)
S_u	minimum cross-sectional area after fracture, (mm^2)
$u(x_i)$	standard uncertainty
$u_{C(y)}$	combined uncertainty on the mean result of a measurement
Z	percentage reduction in area
ε	strain (extension)
σ	stress
e (ΔL)	displacement increment, (mm)

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be controlled more closely.

This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated. The aim is to avoid ambiguity and provide a common format which is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. The Codes of Practice have been collated in a single Manual [1] that has the following sections.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials.

This CoP can be used as a stand-alone document. For further background information on measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 of the Manual [1]. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure.
- Details of the uncertainty calculations for the particular test type (Appendix A)
- A worked example

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in tensile testing.

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTY IN TENSILE TESTING

Step 1. Identifying the Parameters for which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are usually reported in tensile testing. None of the measurands are measured directly, but are determined for other quantities (measurands).

Table 1 Measurands, measurements, their units and symbols

Measurands	Units	Symbol
Original cross-sectional area	mm^2	S_0
Modulus of Elasticity	GPa	E
Proof strength, non proportional elongation	MPa	$R_{p0.2\%}$
Upper yield strength	MPa	R_{eH}
Lower yield strength	MPa	R_{eL}
Ultimate tensile strength	MPa	R_m
Percentage elongation after fracture	%	A
Percentage reduction of area	%	Z

Measurements	Units	Symbol
Specimen original thickness (rectangular specimen)	mm	a_0
Specimen original width (rectangular specimen)	mm	b_0
Specimen original diameter (circular specimen)	mm	d_0
Original gauge length	mm	L_0
Load applied during test	N	F
Axial displacement during the test	mm	$e(\Delta L)$
Final gauge length	mm	L_u
Mean diameter of a cicular specimen after fracture	mm	d_u

None of the measurands are measured directly, instead they are calculated from the following formulae:

$S_0 = a_0\ b_0$ (rectangular test piece) **(1a)**

$S_0 = d_0^2\ \pi/4$ (round test piece) **(1b)**

$E = (\Delta F\ \ L_0) / (\Delta L\ \ S_0)$ **(2)**

$R_p = F_{Rp} / S_0$ **(3)**

$R_{eH} = F_{eH} / S_0$ **(4)**

$R_{eL} = F_{eL} / S_0$ **(5)**

$R_m = F_m / S_0$ **(6)**

$A = (L_u - L_0)\ 100/L_0$ **(7)**

$Z = (S_0 - S_u)\ 100/S_0$ **(8)**

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2, the user must identify all possible sources of uncertainty which may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand as it is associated uniquely with the individual test procedure and apparatus used.

This means that a new list should be prepared each time a particular test parameter changes (e.g. when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined. The following table (Table 2) lists the four categories and gives some examples of sources of uncertainty in each category.

It is important to note that Table 2 is NOT exhaustive and is for GUIDANCE only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 2 Typical sources of uncertainty and their likely contribution to uncertainties in tensile testing measurands for a cold rolled steel (sheet type specimen) at ambient temperature performed by a screw driven tensile testing machine
[1 = major contribution, 2 = minor contribution, 0 = no contribution (zero effect), ? = unknown]

Source of uncertainty	**Type**	**E**	**$R_{p0.2\%}$**	**R_{eH}**	**R_{eL}**	**R_m**	**A**	**Z**
1. Test specimen								
Dimensional compliance	B	2	2	2	2	2	2	2
Surface finish	B	2	2	2	2	2	2	2
Residual stresses	B	?	?	?	?	?	?	?
Shape and size of specimen	B	1-2	1-2	1-2	1-2	1-2	1-2	1-2
Shape of fracture	B	0	0	0	0	0	1	1
Location of failure	B	0	0	0	0	0	1	1-2
2. Test system								
Cross-sectional area measuring unit	B	1	1	1	1	1	2	2
Original gauge length	B	1	1	0	0	0	1	0
Extensometer angular positioning	B	1	1	0	0	0	2	0
Alignment	B	1	1	1	1	2	2	0
Test machine stiffness	B	1	1	1	1	2	2	2
Uncertainty in force measurement	B	1	1	1	1	1	0	0
Uncertainty in displacement measurement	B	1	1	0	0	0	2	0
3. Environment								
Ambient temperature and humidity	B	2	2	2	2	2	2	2
4. Test Procedure								
Zeroing	B	2	1	1	1	1	2	2
Stress rate	B	1	1	1	1	1	2	2
Strain rate	B	1	1	1	1	1	1	1
Digitizing	B	1	1	1	1	2	2	0

Sampling frequency	B	1	1	1	1	2	2	0
Uncertainty in fracture area measurement	B	0	0	0	0	0	0	1
Software	B	1	1	1	1	2	1	0

To simplify the uncertainty calculations it is advisable to regroup the significant sources affecting the tensile testing results in Table 2 in the following categories:

- Uncertainty due to errors in the measurement of cross-sectional area
- Uncertainty due to errors in the force measurement
- Uncertainty due to errors in the displacement measurement
- Uncertainty due to evaluated quantities (e.g. Young's modulus)

The worked examples in Appendix B use the above categorisation when assessing uncertainties.

Step 3. Classifying the Uncertainty According to Type A or B

In this third step, which is in accordance with the GUM [2], the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified as Type A. If it is evaluated by any other means it should be classified as Type B.

The values associated with Type B uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For Type B uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of Reference 1).

It should be noted that, in some cases, an uncertainty can be classified as either Type A or Type B depending on how it is estimated.

Step 4. Estimating the Standard Uncertainty for each Source of Uncertainty

In this step the standard uncertainty, u, for each major input source identified in Table 2 is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d_v, associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of Reference 1.

The standard uncertainty requires the determination of the associated sensitivity coefficient, c, which is usually estimated from the partial derivatives of the functional relationship between the output quantity (the measurand) and the input quantities. The calculations required to obtain the sensitivity coefficients by partial differentiation can be a lengthy process, particularly when there are many individual contributions and uncertainty estimates are needed for a range of values. If the functional relationship for a particular measurement is not known, the sensitivity coefficients may be obtained experimentally. In many cases the input quantity to the measurement may not be in the same units as the output quantity. For example, one contribution to $R_{p0.2}$ is the test temperature. In this case the input quantity is temperature, but the output quantity is the stress which is MPa. In such a case, a sensitivity coefficient, c_T (corresponding to the partial derivative of the proof strength / test temperature relationship), is used to convert from temperature to MPa (for more information see Appendix A).

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} [c_i u(x_i)]^2} \qquad \textbf{(9)}$$

where c_i is the sensitivity coefficient associated with x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.27%.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, U, is defined in Reference 2 as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand". It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, k, which is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2 which corresponds to a confidence interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty, U, is, therefore, broader than the combined uncertainty, u_c. Where a higher confidence level is demanded by the customer (such as for Aerospace and electronics industries), a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in Type A analysis is small), the value of k should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference 1, Section 4 for more details).

Tables B1 to B4 in Appendix B shows the recommended format of the calculation worksheets for estimating the uncertainty in Young's modulus and proof stress for a rectangular test piece. Appendix A presents the mathematical formulae for calculating uncertainty contributions.

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U \quad (\mathbf{10})$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, U is the expanded uncertainty associated with y. An explanatory note, such as that given in the following example should be added (change when appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k = 2, which for a normal distribution corresponds to a coverage probability, p, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 07:2000.

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0-946754-41-1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, *"Guide to the Expression of Uncertainty in Measurement"*. International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. [This Guide is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that drafted it.]

   BSI (identical), *"Vocabulary of metrology, Part 3. Guide to the expression of uncertainty in measurement"*, PD 6461: Part 3 : 1995, British Standards Institution.

3. ISO 5725 ; Accuracy (trueness and precision) of measurement methods and results
   Part 1: 1994(E) General principles and definitions
   Part 2: 1994(E) Basic method for the determination of repeatability and reproducibility of standard measurement method
   Part 3: 1994(E) Intermediate measures of the precision of a standard measurement method
   Part 4: 1994(E) Basic methods for the determination of the trueness of a standard measurement method
   Part 5: 1998(E) Alternative methods for the determination of the precision of a standard measurement method
   Part 6: 1994(E) Use in practice of accuracy values

4. ISO 3534 Part 3: 1999(E/F) Statistics - Vocabulary and symbols - design of experiments

5. ISO Guide 33: 1989(E) Uses of certified reference materials

6. ISO Guide 35: 1989(E) Certification of reference materials - General and statistical principles

7. Malcolm S. Loveday, "Room Temperature Tensile Testing: A Method for Estimating Uncertainty of Measurement," Measurement Note CMMT (MN) 048, July 1999; Centre for Materials Measurement and Technology, National Physical Laboratory, Teddington, TW11 0LW

8. Thomas G. F. Gray and James Sharp, *"Influence of Machine Type and Strain Rate Interaction in Tension Testing,"* Factors That Affect the Precision of Mechanical Tests, ASTM STP 1025, R. Papirno and H. C. Weiss, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 187-205.

9. Bruce W. Christ, - Fracture and Deformation Division, Center for Materials Science, National Bureau of Standards, *"Effect of Specimen Preparation, Setup, and Test Procedures on Test Results"*.

10. Günter Robiller, *"Problems of the computer-controlled tensile test,"* Materialprüfung 31 (1989) 11-12, Carl Hanser Verlag, München

11. Bodo Hesse, Hans-Martin Sonne, and Günter Robiller, *"Reliable proof stress determination with computerized tensile test,"* Materialprüfung 33 (1991) 7-8, Carl Hanser Verlag, München

12. Hans-Martin Sonne and Alois Wehrstedt, *"Computer-aided Tension Test - Problems of Performance on the Basis of the Standard,"* Materialprüfung 37 (1995) 4, Carl Hanser Verlag, München

13. Thomas H. Courtney, *"Mechanical Behavior of Materials"* McGraw-Hill series in materials science and engineering (ISBN 0-07-013265-8).

14. Wilfried J. Bartz, Herausgeber, *"Mechanische Werkstoffprüfung - Grundlagen, Prüfmethoden, Anwendungen"* expert verlag (ISBN 3-8169-1035-1)

15. Horst Blumenauer, Herausgeber, *"Werkstoffprüfung"* Dt. Vlg. für Grundstoffindustrie, Leipzig , Stuttgart (ISBN 3-342-00547-5)

16. Friedhelm Richter, *"Physikalische Eigenschaften von Stählen und ihre Temperaturabhängigkeit - Polynome und graphische Darstellungen,"* STAHLEISEN - SONDERBERICHTE HEFT 10, Verlag STAHLEISEN M.B.H.; Düsseldorf 1983 (ISBN 3-514-00294-0)

17. Franz Adunka, *"Meßunsicherheiten - Theorie und Praxis"* Vulkan Vlg Essen (ISBN 3-8027-2186-1)

18. John Mandel, *"The Statistical Analysis of Experimental Data"* Dover Publications (ISBN 0-486-64666-1)

19. Eberhard Scheffler, *"Statistische Versuchsplanung und -auswertung: eine Einführung für den Praktiker"* Dt. Vlg. für Grundstoffindustrie, Leipzig, Stuttgart (ISBN 3-342-00366-9)

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APPENDIX A

MATHEMATICAL FORMULAE FOR CALCULATING UNCERTAINTIES IN TENSILE TESTING

To simplify matters sections A0 to A10 are limited to uncertainty affected by calibration, determination of cross-sectional area, and evaluation procedure. With the exception of A11 and A12 it was not necessary to study the mechanical behavior of metallic materials under different conditions or to consult published analyses. Basic concepts should be used. The methods of DOE (Design of Experiments) should be used for further studies to consider many parameters that affect the results.

A0. Uncertainty of Measurements (see Table 2)

<u>General</u>

The GUM [2] says **"**In other cases it may only be possible to estimate bounds (upper and lower limits) for X_i, in particular, to state that - the probability that the value of X_i lies within the range LL to UL for all practical purposes is equal to 1 and the probability that X_i lies outside this range is essential 0. If there is no specific knowledge about the possible values of X_i within the range, it can only be assumed that it is equally probable for X_i to lie anywhere within it [a uniform or rectangular distribution of possible values]. Then x_i, the expectation or expected value of X_i is the midpoint of the range: $x_i = (LL + UL)/2$, with variance

$$u^2_{(x_i)} = \frac{(UL - LL)^2}{12} \qquad \textbf{(11a)}$$

If the difference between two bounds, UL-LL, is denoted by **2a**, then

$$u^2_{(x_i)} = \frac{a^2}{3} \qquad \textbf{(11b)}$$

In this CoP "a" is replaced by "δ ".

$$u^2_{(x_i)} = \frac{d^2}{3} \qquad \textbf{(11c)}$$

Example: Specimen original thickness (rectangular specimen) a_0 :

$$u^2_{a_0} = \frac{d^2_{a_0}}{3}$$

If the thickness has been measured n times (and at least 5 times,) The recommended procedure for estimating the bounds is as follows:

a) Determine the mean value of a_0 and the standard deviation s
b) Determine the confidence region of the mean value

$$u_{a_0} = \frac{s\,t(P,f)}{\sqrt{n}} \qquad \textbf{(12)}$$

t ... factor of Students' distribution
P... confidence level
f ... (n-1) degrees of freedom
n ... number of measurements

For P = 68.27% and n = 5 the factor t = 1.15

A1. Uncertainty due to Errors in the Measurement of Cross-Sectional Area

- <u>For a Rectangular Test piece</u>:

$S_0 = a_0 b_0$,

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial S_0}{\partial a_0} = b_0 \qquad \textbf{(13)}$$

$$\frac{\partial S_0}{\partial b_0} = a_0 \qquad \textbf{(14)}$$

Uncertainty in S_0 :

$$u_{S_0} = \sqrt{(b_0)^2 u_{a_0}^2 + (a_0)^2 u_{b_0}^2} \qquad \textbf{(15)}$$

- <u>For a Circular Test piece</u>:

$$S_0 = \frac{\pi d_0^2}{4}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial S_0}{\partial d_0} = \frac{\pi d_0}{2} \qquad \textbf{(16)}$$

Uncertainty in S_0 :

$$u_{S_o} = \sqrt{\frac{d_0^2 \pi^2 u_{d_0}^2}{4}} \qquad \textbf{(17)}$$

A2. Uncertainty in Stress

$$\sigma = \frac{F}{S_0} \qquad \textbf{(18)}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial \sigma}{\partial F} = \frac{1}{S_0} \qquad \textbf{(19)}$$

$$\frac{\partial \sigma}{\partial S_0} = -\frac{F}{S_0^2} \qquad \textbf{(20)}$$

Uncertainty in σ :

$$u_s = \sqrt{\left(\frac{1}{S_0}\right)^2 u_F^2 + \left(\frac{F}{S_0^2}\right)^2 u_{S_0}^2} \qquad \textbf{(21)}$$

A3. Uncertainty in Strain

$e = \Delta L$ (displacement)

$$\varepsilon = \frac{e}{L_0} \qquad \textbf{(22)}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial \varepsilon}{\partial e} = \frac{1}{L_0} \qquad \textbf{(23)}$$

$$\frac{\partial \varepsilon}{\partial L_0} = -\frac{e}{L_0^2} \qquad \textbf{(24)}$$

Uncertainty in u_ε :

$$u_e = \sqrt{\left(\frac{1}{L_0}\right)^2 u_e^2 + \left(\frac{e}{L_0^2}\right)^2 u_{L_0}^2} \qquad \textbf{(25)}$$

A4. Uncertainty in Young's Modulus

The determination of Young's modulus is standardized in ASTM E 111-97. This test method only applies to the range of materials, temperatures and stresses where elastic behavior occurs and creep is negligible compared to the strain produced immediately on loading.

ASTM E 111 says: "For most loading systems and test specimens, effects of backlash, specimen curvature, initial grip alignment, etc., introduce significant errors in the extensometer output when applying a small load to the test specimen. Measurements should therefore be made from a preload, known to be high enough to minimize these effects, to some higher load, still within the proportional limit of the material."

The procedure includes two steps:

1. Determination of the upper limit (end of proportional region) by linear regression. The upper limit is reached if the variance of the slope is a minimum (see Eqn. 35). The starting point of the calculation by linear regression depends on the preload, and is adjustable by the operator of the tensile test machine.

2. After step 1 the linear regression starts again at the upper limit but in the opposite direction to determine the lower limit of the proportional region. If the variance of the slope is a minimum we get the lower limit and the associated slope for further calculation of Young's modulus.

Formulae for Linear Regression:

$$y = mx + b \qquad \textbf{(26)}$$

Slope:

$$m = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \qquad \textbf{(27)}$$

Intercept equation:

$$b = \frac{\sum_{i=1}^{n} y_i - m \sum_{i=1}^{n} x_i}{n} \qquad \textbf{(28)}$$

Empirical covariance (S_{xy}):

$$S_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}\right] \qquad \textbf{(29)}$$

Standard deviation of x-values:

$$S_x = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]} \qquad \textbf{(30)}$$

Standard deviation of y-values:

$$S_y = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]} \qquad \textbf{(31)}$$

Correlation coefficient (r):

$$r = \frac{S_{xy}}{S_x S_y} \qquad \textbf{(32)}$$

Standard deviation of the slope (S_m):

$$S_m = \sqrt{\frac{(1-r^2)S_y^2}{(n-2)S_x^2}} \quad \textbf{(33)}$$

Standard deviation of the intercept (S_b):

$$S_b = \sqrt{S_m^2 \frac{(n-1)S_x^2 + \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n}} \quad \textbf{(34)}$$

Bound regarding the upper and the lower proportional limit for the determination of Young's modulus:

$$S_{m(rel)} = \frac{S_m}{m} \rightarrow \text{ minimum} \quad \textbf{(35)}$$

The data pair for the minimum of $S_{m(rel)}$ represents the upper and the lower proportional limit.

Combined Uncertainty of E :

The linear regression is used to determine the linear relationship between force and displacement.

$$E = \frac{FL_0}{eS_0} = m_E \frac{L_0}{S_0} \quad \textbf{(36)}$$

$$F = m_E e + b_E \quad \textbf{(37)}$$

Therefore:

$F = y$; *see Eqn.*26 $\qquad e = x$; *see Eqn.*26

$m_E = m$; *see Eqn.*27 $\qquad b_E = b$; *see Eqn.*28

$S_{e,F} = S_{xy}$; *see Eqn.*29 $\qquad S_e = S_x$; *see Eqn.*30

$S_F = S_y$; *see Eqn.*31 $\qquad r_E = r$; *see Eqn.*32

$S_{m_E} = S_m ; \; see \;\; Eqn.33$ $\qquad$ $S_{b_E} = S_b ; \; see \;\; Eqn.34$

$S_{m_{E(rel)}} = S_{m(rel)}; \; see \;\; Eqn.35$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial E}{\partial m_E} = \frac{L_0}{S_0} \qquad \textbf{(38)}$$

$$\frac{\partial E}{\partial L_0} = \frac{m_E}{S_0} \qquad \textbf{(39)}$$

$$\frac{\partial E}{\partial S_0} = -\frac{m_E L_0}{S_0^2} \qquad \textbf{(40)}$$

$$u_E = \sqrt{\left(\frac{L_0}{S_0}\right)^2 S_{m_E}^2 + \left(\frac{m_E}{S_0}\right)^2 u_{L_0}^2 + \left(\frac{m_E L_0}{S_0^2}\right)^2 u_{S_0}^2} \qquad \textbf{(41)}$$

A5. Uncertainty in the Determination of Proof Stress

$$e_p = e_{IP} + e_z - e_{el} \quad \text{(permanent displacement)} \qquad \textbf{(42)}$$

e_{IP} ... Input data n displacement; (e.g. recorded in ASCII-file)

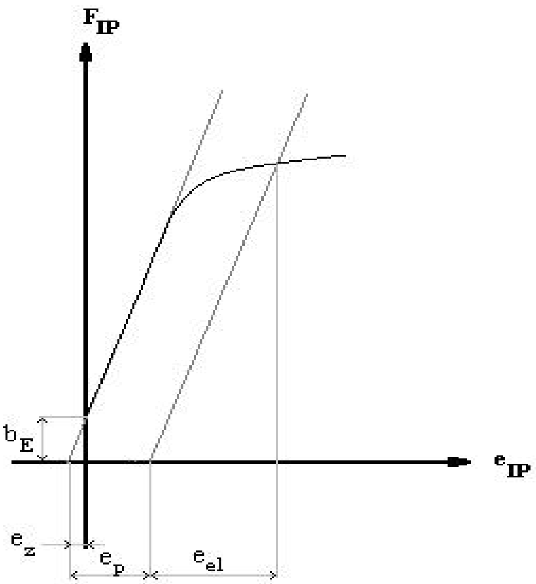
$$e_z ...calculated \;\; Zero-point; \;\; F = 0 \Rightarrow e_z = -\frac{b_E}{m_E} \quad \text{see Eqn. (37)} \qquad \textbf{(43)}$$

$$e_{el} = \frac{F_{IP}}{m_E} \quad \text{(elastic displacement)} \qquad \textbf{(44)}$$

F_{IP} ... Input data of force ; (e.g. recorded in ASCII-file)

$$b_E \geq 0 \Rightarrow e_z \leq 0 \;\; \Rightarrow \;\; e_p = e_{IP} + \frac{b_E - F_{IP}}{m_E} \qquad \textbf{(45)}$$

Figure 1



$$\varepsilon_p = \frac{e_{IP}}{L_0} + \frac{b_E - F_{IP}}{m_E L_0} \qquad \text{(permanent strain)} \tag{46}$$

$\varepsilon_p = 0.002(e.g.R_{p0.2}) \Rightarrow e_{e_p}, F_{e_p}$ is the associated data pair for the proof stress

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i

$$\frac{\partial \varepsilon_p}{\partial e_{e_p}} = \frac{1}{L_0} = c_1 \tag{47}$$

$$\frac{\partial \varepsilon_p}{\partial L_0} = -\frac{e_{e_p}}{L_0^2} - \frac{(b_E - F_{e_p})}{m_E L_0^2} = c_2 \tag{48}$$

$$\frac{\partial \varepsilon_p}{\partial b_E} = \frac{1}{m_E L_0} = c_3 \tag{49}$$

$$\frac{\partial \varepsilon_p}{\partial F_{e_p}} = -\frac{1}{m_E L_0} = c_4 = \tag{50}$$

$$\frac{\partial \varepsilon_p}{\partial m_E} = -\frac{(b_E - F_{e_p})}{m_E^2 L_0} = c_5 \tag{51}$$

Uncertainty in permanent strain ε_p :

$$u_{\varepsilon_p} = \sqrt{c_1^2 u_{e_{\varepsilon_p}}^2 + c_2^2 u_{L_0}^2 + c_3^2 u_{b_E}^2 + c_4^2 u_{F_{\varepsilon_p}}^2 + c_5^2 u_{m_E}^2} \quad \textbf{(52)}$$

Eqn. 52 leads to the uncertainty in the force at ε_p. From the recorded force-displacement diagram we obtain a polynomial to determine $u_{F_{\varepsilon_p}}$.

$$F_{\varepsilon_p} = \alpha_2 \varepsilon_p^2 + \alpha_1 \varepsilon_p + \alpha_0 \qquad \text{(example)} \quad \textbf{(53)}$$

$$\frac{\partial F_{\varepsilon_p}}{\partial \varepsilon_p} = 2\alpha_2 \varepsilon_p + \alpha_1 \quad \textbf{(54)}$$

$$u_{F_{\varepsilon_p}} = \sqrt{(2\alpha_2 \varepsilon_p + \alpha_1)^2 u_{\varepsilon_p}^2} \quad \textbf{(55)}$$

Combined Uncertainty in force at ε_p :

$$u_{F_{C(\varepsilon_p)}} = \sqrt{u_{F_{\varepsilon_p}}^2 + u_F^2} \quad \textbf{(56)}$$

Combined Uncertainty in proof stress:

$$R_{p0.2} = \frac{F_{\varepsilon_p}}{S_0} \quad \textbf{(57)}$$

$$u_{Rp_{0.2}} = \sqrt{\left(\frac{1}{S_0}\right)^2 u_{F_{C(\varepsilon_p)}}^2 + \left(\frac{F_{\varepsilon_p}}{S_0^2}\right)^2 u_{S_0}^2} \quad \textbf{(58)}$$

A6. Uncertainty in Determination of Ultimate Tensile Strength

$$R_m = \frac{F_m}{S_0}$$

and the sensitivity coefficients c_i associated with the uncertainty on the measurement of ultimate tensile strength are :

$$\frac{\partial R_m}{\partial F_m} = \frac{1}{S_0} \quad \textbf{(59)}$$

$$\frac{\partial R_m}{\partial S_0} = -\frac{F_m}{S_0^2} \quad \textbf{(60)}$$

Uncertainty of R_m :

$$u_{R_m} = \sqrt{\left(\frac{1}{S_0}\right)^2 u_{F_m}^2 + \left(\frac{F_m}{S_0^2}\right)^2 u_{S_0}^2} \quad \textbf{(61)}$$

A7. Uncertainty in Determination of Upper Yield Strength

The calculation of the uncertainty of R_{eH} follows the same procedure as R_m .

$$R_{eH} = \frac{F_{eH}}{S_0}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial R_{eH}}{\partial F_{eH}} = \frac{1}{S_0} \quad \textbf{(62)}$$

$$\frac{\partial R_{eH}}{\partial S_0} = -\frac{F_{eH}}{S_0^2} \quad \textbf{(63)}$$

Uncertainty of R_{eH} :

$$u_{R_{eH}} = \sqrt{\left(\frac{1}{S_0}\right)^2 u_{F_{eH}}^2 + \left(\frac{F_{eH}}{S_0^2}\right)^2 u_{S_0}^2} \quad \textbf{(64)}$$

A8. Uncertainty in Determination of Lower Yield Strength

Similarly for the lower yield strength R_m .

$$R_{eL} = \frac{F_{eL}}{S_0}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement are :

$$\frac{\partial R_{eL}}{\partial F_{eL}} = \frac{1}{S_0} \quad \textbf{(65)}$$

$$\frac{\partial R_{eL}}{\partial S_0} = -\frac{F_{eL}}{S_0^2} \quad \textbf{(66)}$$

Uncertainty of R_{eL} :

$$u_{R_{eL}} = \sqrt{\left(\frac{1}{S_0}\right)^2 u_{F_{eL}}^2 + \left(\frac{F_{eL}}{S_0^2}\right)^2 u_{S_0}^2} \quad \textbf{(67)}$$

A9. Uncertainty in the Determination of Percentage Elongation After Fracture

- **Automatic extensometer**

$$A(a) = \left[e_{(RUPT)} - \frac{s_{RUPT}}{E} + C_{A(m)} \right] 100 \quad \textbf{(68)}$$

The value of A(a) depends on the location of the fracture within the parallel length of the specimen. $C_{A(m)}$ is the correction in comparison with the percentage elongation value measured by hand.

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial A(a)}{\partial e_{(RUPT)}} = 1 \quad \textbf{(69)}$$

$$\frac{\partial A(a)}{\partial s_{(RUPT)}} = -\frac{1}{E} \quad \textbf{(70)}$$

$$\frac{\partial A(a)}{\partial E} = \frac{s_{(RUPT)}}{E^2} \quad \textbf{(71)}$$

$$\frac{\partial A(a)}{\partial C_{A(m)}} = 1 \quad \textbf{(72)}$$

Uncertainty of A(a):

$$u_{A(a)} = \sqrt{u_{e_{(RUPT)}}^2 + \frac{1}{E^2} u_{s_{(RUPT)}}^2 + \frac{s_{(RUPT)}^2}{E^4} u_{c(E)}^2 + S_{C_{A(m)}}^2} \quad \textbf{(73)}$$

- **Determination by hand (e.g. Vernier calliper)**

$$A(m) = \left(\frac{L_u - L_0}{L_0} \right) 100$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement are:

$$\frac{\partial A(m)}{\partial L_u} = \frac{1}{L_0} \qquad \textbf{(74)}$$

$$\frac{\partial A(m)}{\partial L_u} = \frac{1}{L_0} - \frac{L_u - L_0}{L_0^2} \qquad \textbf{(75)}$$

Uncertainty of A(m):

$$u_{A(m)} = \sqrt{\frac{1}{L_0^2} u_{L_u}^2 + \left(\frac{1}{L_0} - \frac{L_u - L_0}{L_0^2} \right)^2 u_{L_0}^2} \qquad \textbf{(76)}$$

A10. Uncertainty in the Determination of the Percentage Reduction of the Area

- **Determination of the reduced area - rectangular**

$$S_u = a_u \; b_u$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial S_u}{\partial a_u} = b_u \qquad \textbf{(77)}$$

$$\frac{\partial S_u}{\partial b_u} = a_u \qquad \textbf{(78)}$$

Uncertainty in S_u :

$$u_{S_u} = \sqrt{(b_u)^2 u_{a_u}^2 + (a_u)^2 u_{b_u}^2} \qquad \textbf{(79)}$$

- **Determination of the reduced area - circular**

$$S_u = \frac{\pi d_u^2}{4}$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial S_u}{\partial d_u} = \frac{\pi d_u}{2} \qquad \textbf{(80)}$$

Uncertainty of S_u :

$$u_{S_u} = \sqrt{\frac{d_u^2 \pi^2 u_{d_u}^2}{4}} \qquad \textbf{(81)}$$

- **Determination of the percentage reduction area**

$$Z = \left(\frac{S_0 - S_u}{S_0}\right) 100$$

Sensitivity coefficients c_i associated with the uncertainty on the measurement x_i :

$$\frac{\partial Z}{\partial S_0} = \frac{S_u}{S_0^2} \qquad \textbf{(82)}$$

$$\frac{\partial Z}{\partial S_u} = -\frac{1}{S_0} \qquad \textbf{(83)}$$

Uncertainty of Z:

$$u_Z = \sqrt{\frac{S_u^2}{S_0^4} u_{S_0}^2 + \frac{1}{S_0^2} u_{S_u}^2} \qquad \textbf{(84)}$$

A11. Strain-rate Sensitivity (Short introduction - it is a typical scope of Design of Experiments)

Reference [13] says: ... An increase in strain rate generally increases the flow stress of a material, although the degree to which it does so is a strong function of the temperature and is specific to the material. There are a number of reasons for the strain-rate sensitivity of flow stress, and they are all related to the atomistic and/or microscopic mechanisms of permanent deformation.The strain-rate sensitivity of the flow stress is often adequately represented by the empirical equation:

$$s_T = K^{'}(\dot{e}_T)^m \quad \textbf{(85)}$$

Where $\dot{\varepsilon}_T$ is the true strain rate, *m* is the strain rate sensitivity, and *K'* a constant that signifies that it is the material flow stress at a true strain rate of unity...

$$\varepsilon_T = \ln(1+\varepsilon) \quad \textbf{(86)}$$

$$\sigma_T = \sigma(1+\varepsilon) \quad \textbf{(87)}$$

A12. Temperature Uncertainty Consideration

<u>**ANNEX A12 has been prepared by: V. Bicego,**</u>
Generation Area,
ENEL Research

A12.1. Background

Explicit formulae are given here for yield stress (0.2 strain), indicated as R_e, but identical relationships are intended to be applicable to other measurands, namely Young's modulus E and ultimate tensile strength R_m.

It is assumed that for any type of metal and alloy, the following universal relationship to account for temperature (T) dependence is valid:

$$R_e = s_{yo}\left(1-\exp\left(-C\left(\frac{T_1-T}{T+273}\right)\right)\right) \quad \textbf{(88)}$$

$$C = C_o\left(\frac{\frac{d\varepsilon}{dt}}{\frac{d\varepsilon}{dt}_{max}}\right)^n \quad \textbf{(89)}$$

$\frac{d\varepsilon}{dt}_{max}$ is the max. strain rate allowed by the test standard code; e.g. $10^{-3}s^{-1}$ (e.g. ASTM)

T is the test temperature, in °C

s_{yo} is a coefficient that needs not to be determined (as relative uncertainties are dicussed below, not absolute uncertainties)

n, T_1 and C_o are numerical coefficients, whose values are reported below.

A12.3. Uncertainty Evaluation Procedure

This method provides values of uncertainties in tensile data due to temperature uncertainties, and is applicable to tests at room temperature and above for 4 classes of metals and alloys. The explicit coefficients contained in the uncertainty equations given here below have not been derived yet for other materials.

No evaluation of uncertainties due to temperature uncertainties are necessary provided that the temperature and the strain rate limits indicated in the test standard procedure (e.g. ASTM) are followed, and provided the test temperature is lower than

300°C for iron and ferritic steels,
300°C for austenitic steels,
600°C for Ni and Ni base superalloys,
100°C for Aluminium and its alloys.

At higher temperatures, or when slower strain rates or larger temperature errors than those in standards are involved in a test, the uncertainty in tensile results due to temperature uncertainties shall be evaluated as follows:

The following uncertainty formula apply:
(capital U = absolute uncertainty, small u = relative uncertainty, e.g. x 100%)

$$\frac{U_{s_y}}{s_y} = u_{s_y} = -\frac{C\frac{T_1 + 273}{(T + 273)^2}}{\exp\left(\frac{T_1 - T}{T + 273}\right) - 1} U_T \qquad \textbf{(90)}$$

The sign **-** (minus) is from the partial derivative, it can be dropped.

The above eq. provides the relative uncertainty (e.g. x100, in %) due to the uncertainty of T, e.g. U_T = 3 or 5 ...°C.

A12.3. Explicit Values for the Coefficients

Explicit values of the material dependent coefficients T_1 and C_o are given in the table below. They were evaluated from an analysis of actual tensile test results at several temperatures reported in the book *Harmonisation of Testing ..., Elsevier, eds. Loveday and Gibbons, Proceedings of NPL Conference 1992,* already referred to in other UNCERT reports, with some additional ENEL tensile data on 1CrMoV steels, particularly for strain rate dependence. In essence, eqn 1 was forced to fit the actual temperature trends. The upper temperature limits of validity of this method are also indicated in the table.

Table of coefficients values

	Values of the coefficients n, T_1 and C_o that have to be taken for evaluating the uncertainties of measurands σ_y, σ_{UTS} and E					
material	for σ_y			for σ_{UTS} and E		
	n	T_1	C_o	n	T_1	C_o
Ferritic steels, 25 - 600°C	0.1	870	3.2	0.1	930	2.5
Stainless steels, 25 - 600°C	0.1	870	3.2	0.1	930	2.5
Ni and its alloys, 25 - 900°C	0.1	950	18.0	0.1	1000	8.0
Al and its alloys, 25 - 400°C	0.1	not available	not available	0.1	not available	not available

It is judged that the uncertainties evaluated according to such coefficients in the uncertainty formula above have an uncertainty (uncertainty of the estimated uncertainty, i.e. maximum expected errors in uncertainty predictions) not larger than 15% (15% of the uncertainty value which is calculated).

APPENDIX B

A WORKED EXAMPLE FOR CALCULATING UNCERTAINTIES IN TENSILE TESTING AT AMBIENT TEMPERATURE FOR YOUNG'S MODULUS AND PROOF STRESS

B1. Introduction

The subject of this worked example is a sheet type specimen of a cold rolled steel. It is an example of an uncertainty study of a **single test** in comparison to the uncertainty of the mean value of a **test series** consisting of **7 specimens** at a confidence level of 95%. The specimen for the study lies near the mean value of the test series.

B2. Testing conditions

Testing Means	
Load Cell (F)	Class 1 Cell; 100kN nom. capacity
Extensometer (e)	Class 0.5 Extensometer; System 1: 5 mm nom. capacity System 2: 60 mm nom. capacity
Cross-sectional area	Robot measuring unit in the testing system. The thickness and the width are measured with an accuracy of ±5µm
Original gauge length L_o	80 mm
Tooling alignment (angular)	VASL-Equipment guarantees compliance to standard
Tooling alignment (coaxiality)	VASL-Equipment guarantees compliance to standard
Test machine stiffness	It is also depend on clamping system. Parallel (hydraulic) clamping device

Test Method	
Zero Check Frequency	automatic zeroing
Calibration	it is calibrated at the same time once a year (according to EN 10002)
Formula (decimals)	could be of interest for intensive calculations
Digitizing	15 Bit
Sampling Frequency	50 Hz according the draft of annex to EN 10002-1
Stress Rate	10 MPa/sec.
Strain Rate	5% /min above R_{p1}, 25%/min otherwise
Software	Roell & Korthaus

Test Environment	
Temperature	air conditioned lab. (23°C ±2°)
Operator	
Choice of limits on graph, Elasticity modulus	normally automatic calculation is used
Extensometer angular positioning	precision positioning is given by automatic alignment
Specimen	
Section (S_o; mm^2)	S_o = 23.81 mm^2
Tolerance of shape	±0.05mm; compliant to standard
Parallelism	±0.1mm; compliant to standard
Cylindricity	not relevant
Surface aspect	R_z is less 6.3μm; compliant to standard

B3. Example of Uncertainty Calculations and Reporting of Results

All calculations are based on the formulae in Appendix A. Every table is produced for a certain measurand or evaluated quantity. The worked example shows the procedure for **Young's Modulus** and **Proof Stress**.

The test series have been prepared in two steps and the material is a Bake-Hardening Steel (ZSTE 220 BH).

1. Punching of the shape according EN 10002 ANNEX B - Type 2
2. Finishing by milling under cooling medium

Test results:

No.	**a_0 [mm]**	**b_0 [mm]**	**S_0 [mm^2]**	**E [GPa]**	**Rp0.2 [MPa]**
1	1.185	20.057	23.768	206.4	241.2
2	1.185	20.073	23.787	207.9	241.6
3	1.183	20.085	23.761	208.2	241.8
4	**1.185**	**20.093**	**23.810**	**207.5**	**241.4**
5	1.185	20.092	23.809	207.5	240.7
6	1.179	20.081	23.675	207.7	241.6
7	1.177	20.067	23.619	208.9	241.8
Mean Value $\bar{x}$				**207.7**	**241.4**
Standard deviation s_x				0.76	0.39
Uncertainty of $\bar{x}$, see Eqn. 12 t = 2.45; P = 95%				**0.70**	**0.36**
Calculated Uncertainty - of ANNEX B - based on specimen No. 4					
Expanded Uncertainty				**1.71**	**3.06**

Uncertainty study on specimen No. 4

Result of the linear regression:

At the minimum $S_{m(rel)} = 1.6x10^{-3}$ (0.16%) the software detected the upper and 0.15% for the lower proportional limit. The preload for this material is defined at the VASL-laboratory with 20MPa.

n = 56 (number of data pairs)

m_E = 61744 N/mm

S_{m_E} = 99.1 N/mm

b_E = 187.5 N

S_{b_E} = 0.337 N

TABLE B1: Uncertainty Budget Calculations for Cross-Sectional Area - Rectangular
(sensitivity coefficient is not dimensionless - see Appendix A)

Symbol	Measurands or evaluated quantities	Symbol of uncertainty	Value	Type	Probability Distribution	Divisor d_v	Sensitivity coefficient c_i	u(Xi)	v_i of v_{eff}
a_0	Thickness	u_{a_0}	±0.005mm	B	rectan.	√3	20.093	±5.81E-2mm^2	∞
b_0	Width	u_{b_0}	±0.005mm	B	rectan.	√3	1.185	±3.42E-3 mm^2	∞
S_0	Cross-sectional area	u_{S_0}	**Combined uncertainty**	B	triangular	±0.24%		**±5.82E-2mm^2**	∞

Steps:

δ_{a_0} = 0.005mm ⇒ Eqn. 11c leads to $u_{a_0} = 2.89\times10^{-3}$mm

δ_{b_0} = 0.005mm ⇒ Eqn. 11c leads to $u_{b_0} = 2.89\times10^{-3}$mm

Sensitivity coefficient ⇒ Eqn. 1a and 13 leads to 20.093mm

Sensitivity coefficient ⇒ Eqn. 1a and 14 leads to 1.185mm

1st term (not squared) of Eqn. 15 ⇒ $2.89\times10^{-3}\times20.093 = 5.81\times10^{-2}$mm^2

2nd term (not squared) of Eqn. 15 ⇒ $2.89\times10^{-3}\times 1.185 = 3.42\times10^{-3}$mm^2

Eqn. 15 ⇒ square root of 1st term^2 + 2nd term^2 = 5.82×10^{-2}mm^2

TABLE B2: Uncertainty Budget Calculations for Young's Modulus
(sensitivity coefficient is not dimensionless - see appendix A)

Symbol	Measurands or evaluated quantities	Symbol of uncertainty	Value	Type	Probability Distribution	Divisor dv	Sensitivity coefficient c_i	u(Xi)	v_i of v_{eff}
L_0	Original gauge length	u_{L_0}	±0.4mm	B	rectan.	√3	2.593E+3	±599MPa	∞
S_0	Original cross sectional area	u_{S_0}	±5.82E-2mm^2	B	rectan.	1	8.713E+3	±507MPa	∞
m_E	Slope	S_{m_E}	±99.1N/mm	A	normal	1	3.36	±334MPa	∞
E	Young's Modulus	$u_{c(E)}$	**Combined uncertainty**	A+B	normal	±0.41%		**±0.85GPa**	∞
		$u_{e(E)}$	**Expanded uncertainty**	A+B	normal	**k = 2**	±0.82%	**±1.71GPa**	∞

Steps:

δ_{L_0} = 0.4mm (Class 0.5) ⇒ Eqn. 11c leads to $u_{L_0} = 2.31\times 10^{-1}$mm

Sensitivity coefficient ⇒ Eqn. 36 and 39 leads to 2593

Sensitivity coefficient ⇒ Eqn. 36 and 40 leads to 8713

Sensitivity coefficient ⇒ Eqn. 36 and 38 leads to 3.36

2nd term (not squared) of Eqn. 41 ⇒ $2593 \times 2.31\times 10^{-1} = 599$ MPa
3rd term (not squared) of Eqn. 41 ⇒ $8713 \times 5.82\times 10^{-2} = 507$ MPa
1st term (not squared) of Eqn. 41 ⇒ $3.36 \times 99.1 = 334$ MPa

Eqn. 41 ⇒ square root of 1st term^2 + 2nd term^2 + 3rd term^2 = 853 MPa

TABLE B3: Uncertainty Budget Calculations for the 0.2% Permanent Strain
(sensitivity coefficient is not dimensionless - see appendix A)

Source of uncertainty									
Symbol	**Measurands or evaluated quantities**	**Symbol of uncertainty**	**Value**	**Type**	**Probability Distribution**	**Divisor dv**	**Sensitivity coefficient c_i**	**u(Xi)**	**v_i of v_{eff}**
e_{ε_p}	Associated displacement at ε_p = 2.00E-3	$u_{e_{\varepsilon_p}}$	±1.5E-3mm	B	rectan.	√3	1.25E-2	±1.083E-5	∞
L_0	Original gauge length	u_{L_0}	±0.4mm	B	rectan.	√3	-2.51E-5	±5.798E-6	∞
b_E	Intercept value	S_{b_E}	±0.337N	A	normal	1	2.02E-7	±6.81E-8	∞
F_{ε_p}	Associated force at ε_p = 2.00E-3	$u_{F_{\varepsilon_p}}$	±57.5N	B	rectan.	√3	2.02E-7	±6.71E-6	∞
m_E	Slope	S_{m_E}	±99.1N/mm	A	normal	1	-1.82E-8	±1.8E-6	∞
ε_P	Permanent strain	u_{ε_P}	**Combined uncertainty**	A+B	normal	±0.71%		**±1.41E-5**	∞

Steps:

$\delta_{e_{\varepsilon_p}} = 1.5\times10^{-3}$ mm (Class 0.5) ⇒ Eqn. 11c leads to $u_{e_{Rp}} = 8.66\times10^{-4}$mm

e_{ε_p} and F_{ε_p} obtained from the recorded ASCII-file

Sensitivity coefficient ⇒ Eqn. 46 and 47 leads to 1.25×10^{-2}
Sensitivity coefficient ⇒ Eqn. 46 and 48 leads to -2.51×10^{-5}
Sensitivity coefficient ⇒ Eqn. 46 and 49 leads to 2.02×10^{-7}
Sensitivity coefficient ⇒ Eqn. 46 and 50 leads to 2.02×10^{-7}
Sensitivity coefficient ⇒ Eqn. 46 and 51 leads to -1.82×10^{-8}
1st term (not squared) of Eqn. 52 ⇒ $8.66\times10^{-4}\times1.25\times10^{-2} = 1.083\times10^{-5}$
2nd term (not squared) of Eqn. 52 ⇒ $2.31\times10^{-1}\times2.51\times10^{-5} = 5.798\times10^{-6}$
3rd term (not squared) of Eqn. 52 ⇒ $0.337\times2.02\times10^{-7} = 6.81\times10^{-8}$

4^{th} term (not squared) of Eqn. 52 $\Rightarrow 33.2 \times 2.02\times10^{-7} = 6.71\times10^{-6}$
5^{th} term (not squared) of Eqn. 52 $\Rightarrow 99.1 \times 1.82\times10^{-8} = 1.8\times10^{-6}$
Eqn. 52 $\Rightarrow$ square root of 1^{st} term^2 + 2^{nd} term^2 + **...** +5^{th} term^2= 1.41×10^{-5}

TABLE B4: Uncertainty Budget Calculations for the Proof Stress
(sensitivity coefficient is not dimensionless - see appendix A)

Source of uncertainty									
Symbol	**Measurands or evaluated quantities**	**Symbol of uncertainty**	**Value**	**Type**	**Probability Distribution**	**Divisor dv**	**Sensitivity coefficient c_i**	**u(Xi)**	**v_i of v_{eff}**
F_{ε_p}	Force at ε_p = 2.00E-3	$u_{F_{C(\varepsilon_p)}}$	±33.49N	A+B	normal	1	4.2E-2	±1.41MPa	∞
S_0	Original cross-sectional area	u_{S_0}	±5.82E-2mm^2	B	rectan.	1	10.14	±0.59MPa	∞
$R_{P0.2}$	Proof stress	$u_{c(Rp)}$	**Combined uncertainty**	A+B	normal	±0.63%		**±1.53MPa**	∞
		$u_{e(Rp)}$	**Expanded uncertainty**	A+B	normal	**k = 2**	±1.27%	**±3.06MPa**	∞

Steps:

$F_{e_p} = -6.59\times10^7 e_p^2 + 3.19\times10^5 e_p + 5370$ (Eqn. 53) obtained from the recorded ASCII-file

$u_{F_{e_p}} = \sqrt{(2(-6.59\times10^7)0.002 + 3.19\times10^5)^2 (1.41\times10^{-5})^2} = 4.5N$ (Eqn.54 and Eqn.55)

$u_{F_{C(\varepsilon_p)}} = \sqrt{4.5^2 + u_F^2}$; $u_F = \sqrt{\frac{d^2}{3}} = \sqrt{\frac{(0.01\times 5749)^2}{3}} = 33.19N$ (class 1)

$u_{F_{C(\varepsilon_p)}} = \sqrt{4.5^2 + 33.19^2} = 33.49\text{N}$

Sensitivity coefficient $\Rightarrow$ Eqn. 3 and 19 leads to 4.2×10^{-2}
Sensitivity coefficient $\Rightarrow$ Eqn. 3 and 20 leads to 10.14

1st term (not squared) of Eqn. 58 $\Rightarrow$ $33.49 \times 4.2\times10^{-2} = 1.41$
2nd term (not squared) of Eqn. 58 $\Rightarrow$ $5.82\times10^{-2} \times 10.14 = 0.59$

Eqn. 58 $\Rightarrow$ square root of 1st term^2 + 2nd term^2 = 1.53

B4. Reported Results

E	=	207.5 GPa	± 1.71 GPa	(± 0.82 %)
Rp0.2%	=	241.45 MPa	± 3.06 MPa	(± 1.27 %)

The above reported expanded uncertainties are based on standard uncertainties multiplied by a coverage factor k=2, providing a level of confidence of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT recommendations.