

***Manual of Codes of Practice for the Determination of Uncertainties in
Mechanical Tests on Metallic Materials***

Code of Practice No. 16

**The Determination of Uncertainties of Poisson's Ratio
(from a Tension Test)**

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1. SCOPE

This procedure covers the evaluation of uncertainty of Poisson's ratio from a tension test of structural materials at room temperature, carried out according to ASTM E 132. It is limited to specimens of rectangular section and to materials in which and stresses at which, creep is negligible compared to the strain produced immediately upon loading.

ASTM E 132 - 97: "*Standard Test Method for Poisson's Ratio at Room*

The Code of Practice is restricted to tests performed at ambient temperature with a digital acquisition of load and elongation. Loads shall be applied either by calibrated dead weights or in a testing machine that has been calibrated in accordance with Practices ASTM E 4 - 98, "*Standard Practices for Force Verification of Testing Machines*".

2. SYMBOLS AND DEFINITIONS

B_0	width of the specimen
e_L	axial displacement
e_T	transversal displacement
F	load
L_0	axial gauge length
m_L	slope of axial displacement versus load
m_T	slope of transversal displacement versus load
U	expanded uncertainty associated with y
$u_{(c)\mu}$	combined uncertainty of Poisson's ratio
$u_c(y)$	combined uncertainty on the mean result of a measurement
c_i	sensitivity coefficient associated with uncertainty on measurement x_i
$u(x_i)$	standard uncertainty
V	estimated value of measurand
y	test or measurement mean result
μ	Poisson's ratio

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

Definition: (ASTM E6 - 98, "*Standard Terminology Relating to Methods of Mechanical Testing*") Poisson's ratio, μ — the absolute value of the ratio of transverse strain to the corresponding axial strain resulting from uniformly distributed axial stress below the proportional limit of the material.

Discussion 1— Above the proportional limit, the ratio of transverse strain to axial strain will depend on the average stress and on the stress range for which it is measured and, hence should not be regarded as Poisson's ratio. If this is reported, nevertheless, as a value of "Poisson's ratio" for stresses beyond the proportional limit, the range of stress should be stated.

Discussion 2— Poisson's ratio will have more than one value if the material is not isotropic.

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be controlled more closely.

This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated.

The aim is to produce a series of documents in a common format which is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into 6 sections as follows, with all the individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in Reference 1. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure.
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)

- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty of Poisson's ratio from tension testing. The ASTM E 132 says:

“When uniaxial force is applied to a solid, it deforms in the direction of the applied force, but also expands or contracts laterally depending on whether the force is tensile or compressive. If the solid is homogeneous and isotropic, and the material remains elastic under the action of the

applied force, the lateral strain bears a constant relationship to the axial strain. This constant, called Poisson's ratio, after the French scientist who developed the concept, is a definite material property like Young's modulus and Shear modulus.

Poisson's ratio is used for design of structures where all dimensional changes resulting from application of force need to be taken into account, and in the application of the generalized theory of elasticity to structural analysis.

In ASTM E 132 the value of Poisson's ratio is obtained from strains resulting from uniaxial stress only.

The accuracy of the determination of Poisson's ratio is usually limited by the accuracy of the transverse strain measurements because the percentage errors in these measurements are usually greater than in the axial strain measurements. Since a ratio rather than an absolute quantity is measured, it is only necessary to know accurately the relative value of the calibration factors of the extensometers. Also, in general, the values of the applied loads need not be accurately known. It is frequently expedient to make the determination of Poisson's ratio concurrently with determinations of Young's modulus and the proportional limit.

Loads shall be applied either by calibrated dead weights or in a testing machine that has been calibrated in accordance with Practices ASTM E 4.

Extensometers - Class B1 as described in Practice ASTM E 83 - 96, "*Standard Practice for Verification and Classification of Extensometers*", shall be used except as otherwise stated in the product specifications. It is recommended that at least two pairs of extensometers be used - one pair for measuring axial strain and the other for transverse strain, with the extensometers of each pair parallel to each other and on opposite sides of the specimen. Additional extensometers may be used to check on alignment or to obtain better average strains in the case of unavoidable variations in thickness. The extensometers should be placed on the specimen with a free distance of at least one specimen width between any extensometer and the nearest fillet, and at least two specimen widths between any extensometer and the nearest grip.

Applying the method of least squares can reduce the errors that may be introduced by drawing a straight line through the points. The value of Poisson's ratio thus obtained should coincide with that obtained for a single large load increment for stresses below the proportional limit.

For the method of least squares, random variations in the data are considered as variations in strain. In determining the stress range (load range) for which data should be used in the calculations, it is helpful to examine the data using the strain deviation method described in Test Method ASTM E 111 (determination of Young's modulus). Due to possible small offsets at zero load and small variations in establishing the load path in the specimen during the first small increment of loading, the readings at zero and the first small increment of load are typically not included in the calculations, and the line is not constrained to pass through zero."

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTY OF POISSON'S RATIO FROM TENSION TESTING

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are reported.

Table 1 Measurand, its unit and symbol

Measurand	Unit	Symbol
Poisson's ratio	Dimensionless	μ

Table 2 Measurements, their units and symbols

Measurements	Units	Symbol
Load applied during the test	N	F
Axial displacement	mm	e_L
Transverse displacement	mm	e_T
Axial gauge length	mm	L_0
Transverse gauge length	mm	B_0

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2, the user must identify all possible sources of uncertainty which may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand, as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (for example when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined. Table 3 lists the four categories and gives some examples of sources of uncertainty in each category. It is important to note that Table 3 is NOT exhaustive and is for GUIDANCE only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 3 Sources of uncertainty and their likely contribution to uncertainty of Poisson’s ratio from tensile testing

[1 = major contribution, 2 = minor contribution, 0 = no contribution (zero effect), ? = unknown]

Source of uncertainty	Type	m
1. Test specimen		
Dimensional compliance	B	2
Surface finish	B	2
Residual stress	B	?
2. Test system		
Original gauge length	B	1
Extensometer angular positioning	B	1
Alignment	B	2
Test machine stiffness	B	2
Uncertainty in force measurement	B	2
Uncertainty in displacement measurement	B	1
3. Environment		
Ambient temperature and humidity	B	2
4. Test Procedure		
Zeroing	B	1
Uncertainty in readings	A	1
Uncertainty in stress rate (strain rate)	B	1
Sampling frequency	B	1
Dead weights or uninterrupted loading	B	1-2
Control mode (force or strain control)	B	?
Choice of proportional limits	B	1

To simplify the uncertainty calculations it is advisable to regroup the significant sources affecting Poisson’s ratio in Table 3 in the following categories:

- Uncertainty due to errors in the measurement of displacement
- Uncertainty due to errors in the gauge length (axial, transversal)

The worked example in Appendix B uses the above categorisation when assessing uncertainties.

Step 3. Classifying the Uncertainty According to Type A or B

In this third step, which is in accordance with Reference 2, 'Guide to the Expression of Uncertainties in Measurement', the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical

means (from a number of repeated observations), it is classified Type A, if it is evaluated by any other means it should be classified as Type B.

The values associated with Type B uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For Type B uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of Reference 1).

It should be noted that, in some cases, an uncertainty can be classified as either Type A or Type B depending on how it is estimated.

Step 4. Estimating the Standard Uncertainty for Each Source of Uncertainty

In this step the standard uncertainty, u , for each input source is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d , associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of Reference 1.

In many cases the input quantity to the measurement may not be in the same units as the output quantity. In such a case, a sensitivity coefficient, c_T , is used to convert from input quantity to the output quantity (for more information see Appendix A).

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^N [c_i \cdot u(x_i)]^2} \quad (1)$$

where c_i is the sensitivity coefficient associated with x_i . This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.26%.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, U , is defined in Reference 2 as “the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand”.

It is obtained by multiplying the combined uncertainty, u , by a coverage factor, k , which is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2 which corresponds to a confidence interval of

95.4% (effectively 95% for most practical purposes). The expanded uncertainty, U , is, therefore, broader than the combined uncertainty, u_c . Where a higher confidence level is demanded by the customer (such as for aerospace and the electronics industries), a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in Type A analysis is small), the value of k should be calculated from the degrees of freedom given by the Welch-Satterthwaite method (see Reference 1, Section 4 for more details).

Table B1 in Appendix B shows the recommended format of the calculation worksheets for estimating the uncertainty of Poisson's ratio for a rectangular test piece. Appendix A presents the mathematical formulae for calculating uncertainty contributions.

Step 7. Reporting of results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U \quad (2)$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, U is the expanded uncertainty associated with y . An explanatory note, such as that given in the following example should be added (change when appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, $k = 2$, which for a normal distribution corresponds to a coverage probability, p , of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 02:2000.

5. REFERENCES

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APPENDIX A

**Mathematical Formulae for Calculating Uncertainties of Poisson’s Ratio
(from a Tension Test)**

The average longitudinal strain , ϵ_L , indicated by the longitudinal extensometers and the average transverse strain, ϵ_T , indicated by the transverse extensometers, are plotted against the applied load, F, straight line is drawn through each set of points, and the slopes, $d\epsilon_L/dF$, and $d\epsilon_T/dF$, of these lines are determined. Poisson’s ratio is then calculated as follows:

$$m = \frac{\left(\frac{de_T}{dF}\right)}{\left(\frac{de_L}{dF}\right)} \tag{3}$$

or

$$m = \frac{\left(\frac{de_T}{dF}\right) L_0}{\left(\frac{de_L}{dF}\right) B_0} \tag{4}$$

The uncertainty estimation starts with m_T and m_L . Both have been determined by linear regression.

$$e_L = m_L F + b_L \tag{5}$$

and

$$e_T = m_T F + b_T \tag{6}$$

$$m_L = \frac{de_L}{dF} \tag{7}$$

and

$$m_T = \frac{de_T}{dF} \tag{8}$$

Formulae of linear regression:

$$y = mx + b \tag{9}$$

Slope:

$$m = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i\right)^2} \quad (10)$$

Intercept equation:

$$b = \frac{\sum_{i=1}^n y_i - m \sum_{i=1}^n x_i}{n} \quad (11)$$

Empirical covariance:

$$S_{xy} = \frac{1}{n-1} \left[\sum_{i=1}^n x_i y_i - \frac{\sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n} \right] \quad (12)$$

Standard deviation of x-values:

$$S_x = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n} \right]} \quad (13)$$

Standard deviation of y-values:

$$S_y = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^n y_i^2 - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n} \right]} \quad (14)$$

Correlation coefficient:

$$r = \frac{S_{xy}}{S_x S_y} \quad (15)$$

Standard deviation of the slope:

$$S_m = \sqrt{\frac{(1-r^2)S_y^2}{(n-2)S_x^2}} \quad (16)$$

Standard deviation of the intercept:

$$S_b = \sqrt{S_m^2 \frac{(n-1)S_x^2 + \left(\sum_{i=1}^n x_i\right)^2}{n}} \quad (17)$$

Bound regarding the upper proportional limit for the determination of Young's modulus:

$$S_{m(rel)} = \frac{S_m}{m} \rightarrow \text{minimum} \quad (18)$$

The data pair at the minimum of $S_{m(rel)}$ means the upper proportional limit.

Assignment of the symbols:

$e_T, e_L = y$; see Equation. 9

$F = x$; see Equation. 9

$m_T, m_L = m$; see Equation. 9 and 10

$b_T, b_L = b$; see Equation. 9 and 11

$S_{m_T}, S_{m_L} = S_m$; see Equation. 16

$S_{b_T}, S_{b_L} = S_b$; see Equation. 17

$S_{e_T}, S_{e_L} = S_y$; see Equation. 14

$S_F = S_x$; see Equation. 13

$S_{F, e_T}, S_{F, e_L} = S_{xy}$; see Equation. 12

$r_T, r_L = r$; see Equation. 15

$S_{m_T(rel)}, S_{m_L(rel)} = S_{m(rel)}$; see Equation. 18

Combined uncertainty of m :

$$\mathbf{m} = \frac{m_T L_0}{m_L B_0} \quad (19)$$

$$\frac{\mathcal{J}m}{\mathcal{J}m_T} = \frac{L_0}{m_L * B_0} \quad (20)$$

$$\frac{\mathcal{J}m}{\mathcal{J}m_L} = -\frac{m_T * L_0}{m_L^2 * B_0} \quad (21)$$

$$\frac{\mathcal{J}m}{\mathcal{J}L_0} = \frac{m_T}{m_L * B_0} \quad (22)$$

$$\frac{\mathcal{J}m}{\mathcal{J}B_0} = -\frac{m_T * L_0}{m_L * B_0^2} \quad (23)$$

$$u_{C(m)} = \sqrt{\frac{L_0^2 * S_{m_T}^2 + m_T^2 * u_{L_0}^2}{m_L^2 * B_0^2} + \frac{m_T^2 * L_0^2 * S_{m_L}^2}{m_L^4 * B_0^2} + \frac{m_T^2 * L_0^2 * u_{B_0}^2}{m_L^2 * B_0^4}} \quad (24)$$

$$u_{e(m)} = k * u_{C(m)} \quad (25)$$

APPENDIX B

A Worked Example for Calculating Uncertainty of Poisson’s Ratio at Room Temperature (from a Tension Test)

B1. Introduction

The object of this worked example is a sheet type specimen of a cold rolled steel. It is an example for uncertainty study of a **single test**. The test machine is a servo-hydraulic test machine with a capacity of 100 kN. This machine is equipped with extensometers according EN 100002-4. The machine increments the load in 25N-steps and hold this load for 15 seconds. During this time 30 readings has been recorded. The average values at the load-steps are used for the evaluation of Poisson’s ratio.

B2. Testing conditions

Testing Means	
Load Cell (F)	Class 1 Cell ; 100kN nom. capacity
Extensometer (e)	Axial: Class 0.5 ; 4mm nom. Transversal: Class 0.5 ; 4mm nom.
Original gauge length L_0, B_0	80 mm ;19.94mm
Vernier calliper	$\pm 50\mu\text{m}$,
Tooling alignment (angular)	VASL-Equipment garanties compliance to standard
Tooling alignment (coaxiality)	VASL-Equipment garanties compliance to standard
Tooling stiffness	It is depend on clamping system
Test Method	
Zero Check Frequency	By hand
Calibration	once a year, it is calibrated at the same time (according EN 10002 series)
Formula (decimals)	could be interest for intensiv calculations
Digitizing	15 Bit
Sampling Frequency	500 Hz max.
Load Rate	1N/sec.
Holding time	15 sec.
Sytem	MTS
Test Environment	
Temperature	air conditioned lab. (23°C $\pm 2^\circ$)
Operator	
Extensometer angular positioning	precision positioning is given by hand
Specimen	
Thickness (a_0)	0.8 mm
Width ($b_0=B_0$)	19.94 mm
Section (S_0)	15.95 mm ²
Tolerance of shape	$\pm 0.05\text{mm}$; compliant to standard
Parallelism	$\pm 0.1\text{mm}$; compliant to standard
Cylindricity	not relevant
Surface aspect	Rz is less 6.3 μm ; compliant to standard

B3. Example of Uncertainty Calculations and Reporting of Results

All calculations based on the formulae in Appendix A. Every table is produced for a certain measurand or evaluated quantity. The worked example shows the procedure concerning Poisson's ratio.

Results of the linear regressions:

The proportional limits results from linear regression analysis.

$$\begin{aligned}m_L &= 2.673 \cdot 10^{-5} \text{ mm/N} \\S_{m_L} &= 9.994 \cdot 10^{-8} \text{ mm/N} \\S_{m_{L(rel)}} &= 3.739 \cdot 10^{-3} \text{ mm} \quad (= 0.37\%) \\n &= 41 \text{ (number of data pairs until upper proportional limit, 95.7MPa)} \\m_T &= 2.208 \cdot 10^{-6} \text{ mm/N} \\S_{m_T} &= 1.92 \cdot 10^{-8} \text{ mm/N} \\S_{m_{T(rel)}} &= 8.695 \cdot 10^{-3} \text{ mm/N} \quad (= 0.87\%) \\n &= 38 \text{ (number of data pairs until upper proportional limit, 90.9MPa)}\end{aligned}$$

TABLE B1 Uncertainty Budget Calculations of μ (Poisson's Ratio)
(sensitivity coefficient is not dimensionless - see appendix A)

Source of uncertainty											
Symbol	Measurands or evaluated quantities	Value	Symbol of uncertainty	Value	Type	Probability Distribution	Divisor dv	u(xi)	Sensitivity coefficient c_i	u(Xi)	ν_i of ν_{eff}
m_T	Slope of transversal displacement versus load	2.208E-6 mm/N	S_{m_T}	$\pm 1.92E-8$ mm/N	A	normal	1	$\pm 1.92E-8$ mm/N	1.501E+5	$\pm 2.88E-3$	∞
m_L	Slope of axial displacement versus load	2.673E-5 mm/N	S_{m_L}	$\pm 9.99E-8$ mm/N	A	normal	1	$\pm 9.99E-8$ mm/N	1.24E+4	$\pm 1.24E-3$	∞
L_0	Original gauge length (axial)	80mm	u_{L_0}	± 0.4 mm	B	rectangular	$\sqrt{3}$	$\pm 2.31E-1$ mm	4.14E-3	$\pm 9.56E-4$	∞
B_0	Original gauge length (transversal)	19.94mm	u_{B_0}	± 0.1 mm	B	rectangular	$\sqrt{3}$	$\pm 5.77E-2$ mm	1.66E-2	$\pm 9.58E-4$	∞
μ	Poisson's Ratio	0.331	$u_{C(\mu)}$	Combined uncertainty	A+B	normal	$\pm 1.03\%$			$\pm 3.42E-3$	∞
			$u_{e(\mu)}$	Expanded uncertainty	A+B	normal	$k = 2$		$\pm 2.07\%$	$\pm 6.84E-3$	∞

Steps:

$$d_{L_0} = 0.4 \text{ mm (class 0.5); } \quad u_{L_0} = \frac{d_{L_0}}{\sqrt{3}}$$

$$d_{B_0} = 0.1 \text{ mm (class 0.5); } \quad u_{B_0} = \frac{d_{B_0}}{\sqrt{3}}$$

Sensitive coefficient \Rightarrow Equation. 19 and 20 leads to 1.501×10^5

Sensitive coefficient \Rightarrow Equation. 19 and 21 leads to 1.24×10^4

Sensitive coefficient \Rightarrow Equation. 19 and 22 leads to 4.14×10^{-3}

Sensitive coefficient \Rightarrow Equation. 19 and 23 leads to 1.66×10^{-2}

1st term (not squared) of Equation. 24 $\Rightarrow 1.501 \times 10^5 \times 1.92 \times 10^{-8} = 2.88 \times 10^{-3}$

etc.

Equation. 24 \Rightarrow square root of 1st term² + 2nd term² ... = 3.42×10^{-3}

B4. Reported Results

$$\mathbf{m} = \mathbf{0.331} \pm 6.84 \cdot 10^{-3} (\pm 2.07 \%)$$

The above reported expanded uncertainties are based on standard uncertainties multiplied by a coverage factor $k=2$, providing a level of confidence of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT recommendations.