Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 10

The Determination of Uncertainties in Creep Testing to European Standard prEN 10291

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CODE of PRACTICE FOR THE DETERMINATION OF UNCERTAINTIES IN CREEP TESTING TO ASTM E139 - 95

1. SCOPE

This procedure covers the evaluation of uncertainties in the determination of the results of creep tests carried out according to the following testing standard [2]:

European Standard prEN 10291: Metallic Materials - Uniaxial Creep Testing In Tension - Method of Test (Jan. 1998)

This test method is also covered by ASTM E193–95 [3], which has more detailed instructions and discussion of procedure and calculation. Recommendations of European Creep Collaborative Committee Working Group 1 [4] have also been incorporated.

This Code of Practice is aimed at determination of uncertainties in the results of a single creep test. The effect of repeating a test under nominally the same conditions, or at a series of stresses or temperatures, is discussed in [1] Section 4. Repeatability is also discussed in [5].

2. SYMBOLS

a_0, b_0	initial width & thickness of rectangular specimen
a_u, b_u	minimum width & thickness of ruptured rectangular specimen
d_0	initial diameter of parallel length
d_u	minimum diameter after rupture
e	extensometer elongation reading
ε	strain
$\dot{\varepsilon}$	strain rate
L_0, L_u	initial and after-rupture gauge lengths
L_c	initial parallel length
L_{e0} , L_{eu}	initial and after-rupture extensometer lengths
ΔL_e	during-test extensometer elongation
L_{r0}	initial reference length
S_0, S_u	initial, and minimum after-rupture, cross-sectional areas
s	machining tolerance for specimen diameter or width

Loads and Stresses

P	load on test piece
σ_0	initial stress on test piece

Test Results

A_u	percent elongation after rupture
$\dot{\varepsilon}_{min}$	minimum strain rate in secondary creep
t_u	rupture life of specimen
t_{fx}	time to x% creep strain
Z_u	percent reduction of area after rupture

Other Parameters

A	constant in strain rate vs. stress / temperature equation
n	stress exponent for creep
Q	creep activation energy
R	gas constant
t_0	time when rupture first recorded
T	test temperature

3. INTRODUCTION

There are requirements for test laboratories to evaluate and report the uncertainty associated with their test results. Such requirements may be demanded by a customer who wishes to know the bounds within which the reported result may be reasonably assumed to lie; or the laboratory itself may wish to understand which aspects of the test procedure have the greatest effect on results so that this may be monitored more closely or improved. This Code of Practice has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4 - CT97-2165, in order to simplify the way in which uncertainties are evaluated. It is hoped to avoid ambiguity and provide a common format readily understandable by customers, test laboratories and accreditation authorities.

This Code of Practice is one of seventeen prepared and tested by the UNCERT consortium for the estimation of uncertainties in mechanical tests on metallic materials. These are presented in [1] in the following Sections:

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. Nevertheless, for background information on measurement uncertainty and values of standard uncertainties of devices used commonly in material testing, the user may need to refer to the relevant section in [1]. Several sources of uncertainty, such as the reported tolerance of load cells, extensometers, micrometers and thermocouples are common to several mechanical tests and are included in Section 2 of [1]. These are not discussed here to avoid needless repetition. The individual procedures are kept as straightforward as possible by following the same structure:

The main procedure
Fundamental aspects for that test type
A worked example

This document guides the user through several steps to be carried out in order to estimate the uncertainties in creep test results. The general process for calculating uncertainty values is described in [1].

4. PROCEDURE FOR ESTIMATING UNCERTAINTIES IN CREEP TESTS

Step 1 - Identification of the Measurands for Which Uncertainty is to be Determined

The first Step consists of setting the measurands, i.e. the quantities which are to be presented as results of the test.

According to requirements, Table 1 lists the measurands and the intermediate quantities used in their derivation. Not all the quantities will be required in a particular test. For example strains during a stress-rupture test might not be measured.

Extensive investigations may be undertaken in addition to these results, to determine activation energy, stress exponent, and parameters in modelling strain, strain rate and time and ductility at rupture, as functions of stress, temperature, composition, heat-treatment etc. Evaluation of uncertainties of these parameters is beyond the scope of the present procedure.

Table 1. Intermediaste results and measurands, their units, and symbols within prEN 10291

Quantity	Unit	Symbol
Intermediate results		
initial stress	MPa	σ_0
original cross-sectional area of parallel section	mm^2	S_0
minimum cross-sectional area after creep rupture	mm^2	S_u
Test measurands		
percentage reduction of area after creep rupture		Z_u
percentage elongation after creep rupture		A_u
creep rupture time	h	t_u
creep elongation time (time to x% creep strain)	h	t_{fx}
minimum strain rate in secondary creep	h^{-1}	å $_{min}$

The quantities in Table 1 are calculated from the following formulae.

4.1.1 Initial Cross-Sectional Area S_0, and Stress σ_0

For cylindrical specimens

$$S_0 = \pi d_0^2 / 4 \qquad (1a)$$

and for rectangular

$$S_0 = a_0 * b_0 \qquad (1b)$$

In both cases

$$\sigma_0 = P / S_0 \qquad (1c)$$

4.1.2 Final Cross-Sectional Area S_u, and Percentage Reduction of Area Z_u

For a cylindrical specimen with minimum diameter d_u, S_u is given by the formula corresponding to Eq. (1a). In the case of rectangular specimens, a method may have to be devised for a fracture surface, which is not of an easily measurable shape. According to ASTM E139 paragraph 10.4, reduction of area is only reported for round specimens.

$$Z_u = 100(S_0 - S_u) / S_0 = 100(1 - (S_u / S_0)) \qquad (2)$$

4.1.3 Percentage Elongation After Creep Rupture, A_u

When initial and after-rupture lengths are measured between gauge length marks on the parallel section:

$$A_u = 100 (L_u - L_0) / L_0 = 100 (L_u / L_0 - 1) \qquad (3a)$$

In some cases the specimen has raised ridges (prEN 10291 Fig. 2 type c or e), or V-notches in enlarged ends (type d) for extensometer attachment. These can be used for rupture elongation measurement. The reference length is the parallel length plus a calculated addition for the radiused shoulders (see A1.3), and the elongation is the increase of length between the attachment points. Then:

$$A_u = 100 (L_{eu} - L_{e0}) / L_{r0} = 100 \Delta L_{eu} / L_{r0} \qquad (3b)$$

4.1.4 Creep Rupture Time, t_u

If specimen load or displacement is logged at intervals of time Δt, a rupture recorded at time t_0 is equally likely to have occurred at any time within the interval ($t_0 - \Delta t$, t_0),

with the expectation

$$t_u = t_0 - \Delta t/2 \qquad (4)$$

This calculation will not arise if the specimen load or extension is recorded continuously.

4.1.5 Creep Elongation Time, t_{fx}

t_{fx} represents the time to x% creep strain, e.g. $t_{f0.2}$ for 0.2% strain.

Provided the data sampling frequency is sufficiently high, extensometer readings near the required elongation will be obtained. Because of extensometer uncertainty, these will not lie exactly on a straight line, but linear regression analysis then gives the estimated time for the required strain, its standard error, and the strain rate. See Appendix B for an illustration of this method.

ASTM E139 paragraph 12.6.2 requires strain measurements at 0.1h intervals up to 100h, then at 1h intervals, if times at given percentage strains are to be taken from a single reading. Higher frequency of measurement may be needed in the initial stages.

For low strain rate tests, the target elongation may be shown at several times. The mean of these gives the required result.

4.1.6 Minimum Strain Rate in Secondary Creep, $\dot{\varepsilon}_{min}$

Recorded strain/time data will usually not lie on a smooth curve, and derived strain rate over each recording interval will be erratic. Strain rate has to be averaged over a period in which it appears to be constant, and this can be carried out manually from the strain/time or strain-rate/time curves.

$\dot{\varepsilon}_{min}$ could also be determined by computing a best-fit curve for the strain/time data.

Step 2 - Identification of all Sources of Uncertainty

In the second Step, the user identifies all possible sources of uncertainty which may have an effect on the test. This list cannot be exhaustively identified beforehand as it is uniquely linked to the laboratory's test method and the apparatus used. Therefore a new list should be drafted each time one test parameter changes (when a plotter is replaced by a computer and printer for example). To help the user list all sources of uncertainty, five categories have been defined. The following table (Table 2) gives the five categories and examples of sources.

It is important to note that this table is NOT exhaustive. Other sources can contribute to uncertainties depending on specific testing configurations. Users are encouraged to draft their own list corresponding to their own test facilities.

Table 2. Sources of uncertainty and their likely contribution
to uncertainties in measurands
(1 = major contribution, 2 = minor contribution, blank = insignificant or no contribution)

Source	**Affected Measurand**							
	S_0	S_u	s_0	Z_u	A_u	t_u	t_{fx}	å$_{min}$
Test Piece								
specimen initial dimensions	1		1	1	1	1	1	1
minimum diameter after rupture		1		1				
extmtr./gauge length after rupture					1			
Apparatus								
load cell or lever / weights			1	2	2	1	1	1
specimen temperature				2	2	1	1	1
bending stresses				2	2	2	2	2
extensometer							1	2
Environment								
control of ambient temperature						2	2	2
Method								
data logger time interval						2	2	2
estimation of creep strain time							2	2
estimation of å$_{min}$ (graphical/statistical)								2
reference length shoulder integral					2			
Operator								

Step 3 - Classification of all Sources According to Type A or B

In accordance with ISO TAG 4 'Guide to the Expression of Uncertainties in Measurement' [6], sources of uncertainty can be classified as *type A* or *B*, depending on the way their influence is quantified. If a source's influence is evaluated by statistical means (from a number of repeated observations), it is classified *type A*. If a source's influence is evaluated by any other mean (manufacturer's documents, certification, ...), it is classified *type B*.

Attention should be drawn to the fact that one same source can be classified as *type A* or *B* depending on the way it is estimated. For instance, if the diameter of a cylindrical specimen is measured once, or taken from the machining specification, that parameter is considered *type B*. If the mean value of ten consecutive measurements is taken, then the parameter is *type A*.

Step 4. Estimation of Sensitivity Coefficient and Standard Uncertainty for each Source of Uncertainty

In this step the standard uncertainty of the measurand, u_i, for each input source x_i, is calculated according its input quantity value, the probability distribution and the sensitivity coefficient, c_i.

Appendix A1 describes the derivation of the standard uncertainties for the primary measured quantities and sources of uncertainty in a creep test, as listed in Tables 1 and 2.

The standard uncertainty ($u(x_i)$) of the input quantity x_i is defined as one standard deviation and is derived from the uncertainty of the input quantity. For a *Type A* uncertainty, the uncertainty value is not modified. For *Type B*, it is divided by a number, d_v, associated with the assumed probability distribution of the parameter within its uncertainty range. The divisors for the distributions most likely to be encountered are given in Chapter 2 of [1].

The contribution (u_i) of the individual input quantity's standard uncertainty ($u(x_i)$) to the standard uncertainty of the measurand is found by multiplying $u(x_i)$ by the sensitivity coefficient c_i. This is derived from the relationship between output (measurand, y) and input quantities(x_i), and is equal to the partial derivative, i.e.

if	$y = F(x_1, x_2,...)$ where F denotes some function
then	$c_i = \partial y / \partial x_i$
and	$u_i = c_i\, u(x_i)$

Step 5: Calculation of Combined Uncertainties of Measurands

Assuming that the N individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed in a root sum squares manner:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} [c_i . u(x_i)]^2}$$

This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity.

It will be seen that, in some cases, the uncertainty of an input quantity is itself a combined uncertainty from an earlier stage in the calculations. For example, uncertainty of rupture time $u(t_u)$ depends on *(inter alia)* the uncertainty in stress $u(\sigma_0)$. This in turn depends on load and cross-sectional area, and the latter again on initial diameter, which is one of the primary measured quantities.

In the detailed procedure for calculating combined uncertainties (in Appendix A2) and the illustrative worked example (Appendix B), these successive uncertainties are calculated separately in sequence.

Step 6: Computation of the Expanded Uncertainty U_e

This Step is optional and depends on the requirements of the customer. The expanded uncertainty U_e is broader than the combined uncertainty U_c, but in return, the confidence level increases. The combined uncertainty U_c has a confidence level of 68.27%. Where a high confidence level is needed (aerospace and the electronics industries), the combined uncertainty U_c is broadened by a coverage factor k to obtain the expanded uncertainty U_e. The most common value for k is 2, which gives a confidence level of approximately 95%. If U_c is tripled the corresponding confidence level rises to 99.73%.

Standard worksheets can be used for calculation of uncertainties, and an example is shown in Table 3 below. In creep tests, the sensitivity coefficients giving the uncertainty in a measurand due to the uncertainties in the primary measured quantities (diameter, temperature, load etc.) are rather complicated, and there has to be a series of intermediate steps, as illustrated in the worked example in Appendix B.

The formula for calculating the combined uncertainty in Table 3 is given in Appendix A2 Eq. (13c).

Table 3. Typical worksheet for uncertainty budget calculations in estimating the uncertainty in percentage elongation A_u

source of uncertainty (type)	symbol	value	uncer-tainty	prob. distribn	divisor	c_i	u_i
parallel length (B)	L_c	u	a	rect.	√3	1	=a/√3
end correction		v	not included				
reference length	L_{r0}	u+v					=a/√3
initial extensometer length (A)	L_{e0}	x	b	normal	1	1	b
final extensometer length (A)	L_{eu}	y	c	normal	1	1	c
elongation	ΔL_{eu}	y-x					$=\sqrt{(b^2+c^2)}$
percentage elongation	A_u	$\Delta L_{eu}/L_{r0}$					
combined standard uncertainty	u_c			normal			Eq.(13c)
expanded uncertainty	U_e			normal			

Step 7: Presentation of results

Once the expanded uncertainty has been chosen, the final result can be given in the following format:

$$V = y \pm U \text{ with a confidence level of } X\%$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, and U is the expanded uncertainty. The results would therefore be presented in the form shown below. These are the basic test results, and the Standards list other information required in the test report.

> The results of a creep test conducted according to ASTM E139-95 on sample XYZ123, with a confidence interval of 95% are:

Test Conditions:	
Initial stress	σ_0
Temperature	T
Test Results:	
Rupture time	$t_u \pm U_e(t_u)$
Time to x% strain	$t_{fx} \pm U_e(t_{fx})$
Minimum strain rate	$\mathring{a}_{min} \pm U_e(\mathring{a}_{min})$
Percentage elongation	$A_u \pm U_e(A_u)$
Percentage reduction of area	$Z_u \pm U_e(Z_u)$

5. REFERENCES

(1) *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0 946754 41 1, Issue 1, September 2000.

(2) European Committee for Standardisation: European Standard prEN 10291: *Metallic Materials - Uniaxial Creep Testing In Tension - Method of Test* (Jan. 1998).

(3) American Society for Testing and Materials: Designation E193 - 95: *Standard Practice for Conducting Creep, Creep Rupture and Stress Rupture Tests of Metallic Materials*.

(4) Brite Euram BE5524: European Creep Collaborative Committee Working Group 1 Recommendations.

(5) DG Robertson *Development and Validation of a Code of Practice for a Low Cost Method of Strain Measurement from Interrupted Creep Testing*, Final Report, ERA Report 99-0084, February 1999, ERA Project 62-01-1466. European Commission Standards Measurements and Testing Programme, CEC No. MAT1-CT940065.

(6) BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML: *Guide to the Expression of Uncertainty in Measurement*. (ISO 1993) (Known as the "TAG4 Guide").

(7) American Society for Metals: *Atlas of Creep and Stress-Rupture Curves* (ASM International1988).

APPENDIX A1

Calculation of Uncertainties in Measured Quantities

A1.1 Introduction

The reason for classifying sources as *type A* or *B* is that each type has its own method of quantification. By definition, a *type A* source of uncertainty is already a product of statistical computation. The calculated influence is thus left as is.

Type B sources of uncertainty can have various origins: a manufacturer's indication, a certification, an expert's estimation or any other mean of evaluation. For *type B* sources, it is necessary for the user to estimate for each source the most appropriate (most probable) distribution (further details are given in Section 2 of [1]). According to the distribution model chosen, a correction factor is required in order to compute the standard uncertainty u_s for each *type B* source.

TYPE A : Statistically computed influence

A *Type A* uncertainty is most often computed from a set of N repeated measurements of the required quantity x. This then gives a mean value μ for x, and *Type A* uncertainty estimate:

standard uncertainty $u_s(x)$ = (standard deviation) / $\sqrt{N}$,

$$= \frac{1}{\sqrt{N}} \sqrt{\sum_{i=1}^{N} \frac{(x_i - \mu)^2}{N-1}}$$

The standard deviation is denoted by STDEV in Microsoft Excel, and often by σ_{n-1} on hand calculators. (Note the use of σ here for standard deviation, and elsewhere for stress).

A *Type A* uncertainty can also be calculated when the value of a dependent variable y is measured at a series of values of an independent variable x, and an estimate of Y, the value of y when x = X, is derived by regression analysis. In the worked example in Appendix B, the time at 1% strain and its uncertainty are calculated from a series of strain/time data points.

TYPE B : Standard uncertainty u_s made of the given uncertainty u_j divided by a factor d_v given in Section 2 of [1] : $u_s = u / d_v$

The most common distribution model for *Type B* uncertainties is the rectangular, which means that the "true" value of a measured quantity is equally likely to have any value within the range ($\mu \pm u$), where μ is the mean value. The probability of the value lying between x and ($x+\delta x$) is $\delta x/(2u)$, i.e. independent of x, and from the definition of standard deviation for a continuous variable, it can be shown that:

$$u_s = u/\sqrt{3}$$

i.e. in this case, $d_v = \sqrt{3}$.

Turning now to the Sources of Uncertainty listed in Table 2 (page 7), we demonstrate how the typical uncertainties in individual measured quantities, differences in apparatus etc. lead to each source's influence. It is assumed that measuring equipment has been appropriately calibrated, and that a test procedure has been written following the standard which minimises typical measurement errors, and that the procedure is being applied by a trained operator

Important Note: Some *Type B* errors have been calculated in the following discussion and the worked example in Appendix B. It will be self-evident to the capable laboratory how *Type A* errors on individual measurements can lead to smaller uncertainties for an individual test piece, or how repeat measurements on several test pieces can lead to the calculation of *Type A* errors on the evaluated quantities directly (see also Section 4 of [1]).

A1.2 Uncertainties in Test Specimen Initial Diameter / Width & Thickness

The specimen diameter d_0 , or width and thickness a_0, b_0, and their uncertainties, may be taken from the specimen drawing if the specimen has been checked and certified to be within the machining tolerance of $\pm$ s. For example, in prEN 10292, Table 2.1 specifies a maximum difference between any two transverse dimensions, i.e. 2s, of 0.03mm for specimens 6 to 10 mm in diameter. The uncertainty is then *Type B* with rectangular distribution, i.e. standard uncertainty, $u(d_0)$, $u(a_0)$ or $u(b_0)$, is equal to $s / \sqrt{3}$.

Alternatively, specimen dimensions can be measured at different positions with a micrometer or calliper and a *Type A* estimate of uncertainty $u_1(x)$ made as shown on the previous page (x denotes d_0, a_0 or b_0).

The diameter values must also comply with the requirements of the Standard for the difference between maximum and minmum value.

The measuring instrument will also have its own uncertainty u_m, available from the manufacturer's specification or periodic calibrations. If this is given as a maximum error, a rectangular distribution may be assumed, giving a *Type B* standard uncertainty:

$$u_{sm} = u_m / \sqrt{3}$$

This is combined with the standard uncertainty of the set of measurements ($u_1(x)$) by quadratic summation:

$$u(x) = \sqrt{[\ (u_1(x))^2 + (u_{sm})^2\ ]} \qquad (5)$$

Dimensional uncertainties affect the uncertainty of initial stress through Eqs. (1a), (1b) and (1c), and reduction of area through Eq. (2). Uncertainty in stress also contributes to the uncertainties in rupture time, time to given creep strain, and strain rate.

A1.3 Uncertainties in Measuring Initial Gauge, Extensometer and Reference Lengths

The length may be between gauge length marks, as on a stress-rupture specimen. The uncertainty for diameter can be taken from the machining specification tolerance *(Type B)*, or measured several times *(Type A*) and combined with the standard uncertainty of the calliper.

When an extensometer length between raised ridges or machined grooves is used as the basis for elongation calculation, the reference length is the sum of parallel length and a correction for the radiused shoulders of the form:

$$L_s = \int \left(\frac{d_0}{d(x)} \right)^{2n} dx$$

where n = stress exponent for creep rate, d(x) = diameter at position x, and the integral is taken for x over the radiused length. When n is unknown, it is assumed to be 5 ([2] pp. 22, 23). Shoulder strain corrections are also discussed in ASTM E139 10.2.3 [3].

The ends of the parallel section may be difficult to locate for measurement purposes, and parallel length is best taken from the machining specification. Its uncertainty is *Type B*, with value = tolerance/√3, assuming rectangular distribution within the permitted range.

The uncertainty of the end correction is difficult to assess, particularly if an assumed value for stress exponent is used. However, calculations with typical specimen sizes have indicated that a ± 1 variation in n gives ± 0.2 mm change in the integral, or 0.5% of a 50mm parallel length. The worked example has a 4.7% uncertainty in the extension to rupture (ΔL_{eu}). This is therefore the main factor in uncertainty of percentage elongation. Even though the validity of the expression integrated is also open to question, it is suggested that the end correction uncertainty contribution be considered negligible.

Several measurements should be made, at different circumferential positions *(Type A)* for an extensometer length between raised ridges or machined grooves. Alternatively, a *type B* estimate can be made from the machining specification.

A1.4 Uncertainty in Measuring Load *(Type B)*

This is a *Type B* estimate derived from the calibration certificate of the load cell or lever system and weights. A rectangular distribution is assumed again, so the standard uncertainty u(P) is $s / \sqrt{3}$, where $\pm s$ is the certified maximum error.

If the error is given as a percentage of the applied load, it is a relative uncertainty. For a load cell, it may be presented as a percentage of the maximum load which is an absolute uncertainty in the applied load.

Load uncertainty affects uncertainties in stress (Eq. (1)) and times to rupture or given creep strain and minimum strain rate (Appendix D).

A1.5 Uncertainty in Measuring After-Rupture Minimum Cross-section *(Type A)*

The uncertainty in after-rupture minimum diameter of cylindrical specimens is *type A*, equal to (standard deviation of n measurements)/$\sqrt{n}$ (n ideally about 10). For each measurement, the specimen should be separated and re-assembled, and measured on a different diameter. This procedure may be judged to be too time-consuming, and also will cause damage to fracture surfaces, which might be needed for scanning electron microscopy. Mounting the two pieces in rotating centres will speed up the operation. One or two measurements will not allow an estimate of uncertainty in reduction of area. If the fracture is not circular, several measurements of minimum diameter must be made (ASTM E139 9.8.2).

The standard error of the measurements of d_u, if obtained, is combined with the standard error of the measuring instrument, for initial diameter as shown in Eq. (5). The calculation of area uncertainty from diameter uncertainty is shown below in A2.3.

This uncertainty affects reduction of area through Eq. (2).

After-rupture minimum cross-sectional area is not normally required on a rectangular specimen.

A1.6 Uncertainties in Measuring After-Rupture Gauge and Extensometer Lengths *(Type A)*

The gauge length between marks on the parallel section, or extensometer length between raised ridges, and its uncertainty, are measured on the re-assembled ruptured specimen. As for minimum diameter, it is preferable to separate and re-assemble the specimen for each measurement, to give independent estimates.

The uncertainty of the after-rupture length based on n measurements is (standard deviation)/$\sqrt{n}$. It is a major contributor to uncertainty in percentage elongation (Eqs. 3a, 3b).

A single measurement of after-rupture length will not allow calculation of percentage elongation uncertainty.

A1.7 Uncertainty in Measuring Rupture Time *(Type B)*

If specimen load or displacement are logged at intervals of time Δt, a rupture recorded at time t_0 is equally likely to have occurred at any time within the interval (t_0 - Δt, t_0), with expectation

$$t_u = t_0 - \Delta t/2$$

and the initial determination of standard uncertainty of t_u is

$$u_m (t_u) = \Delta t / (2\sqrt{3}) \qquad (6)$$

i.e. *type B*. The uncertainty in t_0 is assumed to be negligible.

The subscript m in $u_m(t_u)$, and in subsequent equations below, denotes that the expression is an initial estimate based on uncertainties in the data used in the direct measurement. The influences of uncertainties in load and temperature must then be added, as shown in Appendix A2.

A1.8 Uncertainty in Measuring Creep Elongation Time *(Type A* or *B)*

As noted in Section 4 Step 1.5, three situations are possible, depending on strain rate and data recording frequency.

(a) interpolation between data points above and below the target elongation
(b) the target elongation may be recorded at more than one time
with case (b) tending to occur in long-term tests with very low strain rates
(c) a single reading is taken where strain is nearest the target value, as per ASTM E139 12.6.2.

In case (a), a number of data points will normally cover a period of constant elongation rate and linear regression can be used to determine the time for the required elongation, the standard error of the estimate, and the strain rate.

In case (b) , suppose the elongation is indistinguishable from the target value over a time period t_1 . The “true” elongation time is more likely to be near the centre of the uncertainty band than at an edge.

If a normal distribution is assumed, with the range t_1 covering 2 standard deviations, the initial uncertainty is given by:

$$u_m(t_{fx}) = t_1 / 2 \qquad (7)$$

The value chosen for the divisor will not be important, since the uncertainty in case (a) or (b) is normally much less than that due to temperature and stress uncertainties.

In case (c), the uncertainty is *Type B*, with value equal to half the interval between strain measurements.

A1.9 Uncertainty in Temperature *(Type B)*

Temperature uncertainty u(T) will normally be *type B*, combining maximum errors of

$e_m(T)$ in the measurement thermocouple
$e_u(T)$ for along-specimen uniformity
$e_c(T)$ for the measuring system

by the usual quadratic summation method.

The standard uncertainty of temperature is then

$$u(T) = \sqrt{[(e_m(T)^2 + e_u(T)^2 + e_c(T)^2)]} / \sqrt{3} \qquad (8)$$

Temperature affects strain rate, rupture and creep times in a non-linear manner, and the derivation of resulting uncertainties in t_u is given in Appendix D.

A1.10 Uncertainty in Extensometer Reading of Elongation *(Type B)*

The extensometer should be calibrated, and certified to have a maximum readout error over its working range. If this is $\pm$s mm, the standard uncertainty u(e) of the extensometer reading is s/√3, assuming a rectangular distribution.

Depending on the system employed, the effect of long-term amplifier drift may have to be included in extensometer uncertainty.

A1.11 Effect of Environment

Variation of ambient temperature may affect specimen temperature control and extensometer amplifiers. The extent of these should be determined, and added to the relevant uncertainty if significant.

Close control of laboratory temperature is essential for results of low uncertainty.

A1.12 Effect of Bending Stresses

Non-coaxiality of gauge length, threads and straining rods causes bending stresses. Laboratories should examine this factor in uncertainty of their own results. For each test machine, bending stresses can be measured on a set of strain gauges around the circumference of a specimen. The effect of bending stresses will be related to the inherent ductility of the material at that stress, temperature and time (the latter bringing microstructural changes into the equation). Therefore predicting their effect upon rupture time is extremely difficult, as the ductility (and hence capacity to accommodate bending) will change with time and temperature.

ECCC WG1 Issue 2 (Vol. 3 Annex 1 ch. 2.6) [4] suggests that bending stresses be less than 0.2 σ_0. The WG recommend that their influence be investigated, but assume that within this limit, rupture time and ductility will not be affected. ASTM E139 [3] requires that strain gauge measurements be made at room temperature, and bending strain should not exceed 10% of axial strain. A lower limit may be set for brittle materials.

For the purposes of this Code of Practice, bending stresses should be monitored and accurate axiality maintained, but no method is proposed to calculate the effect of bending on uncertainty of results.

A1.13 Effect of Method

The influence of data logging interval on rupture time uncertainty has been considered above. According to ASTM E139 9.6.2, the time interval between data records should be not more than 1% of the anticipated test duration, or 1 hour, whichever is longer.

Minimum strain rate may be estimated by laying a straightedge along a strain/time plot, and it will be possible to obtain an uncertainty from repeated measurements. Curve fitting, or regression analysis if linear, can give a standard error for the minimum slope.

APPENDIX A2

Formulae for the Calculation of the Measurands' Combined Uncertainties

A worked example is presented in Appendix B to illustrate the following operations.

A2.1 Calculation of Uncertainty in Initial Cross-Sectional Area

For a cylindrical specimen

relative uncertainty:
$$\frac{u(S_0)}{S_0} = 2\frac{u(d_0)}{d_0} \tag{9a}$$

absolute uncertainty:
$$u(S_0) = \frac{\eth d_0 \; u(d_0)}{2} \tag{9b}$$

and for a rectangular specimen

$$\frac{u(S_0)}{S_0} \approx \sqrt{\left(\frac{u(a_0)}{a_0}\right)^2 + \left(\frac{u(b_0)}{b_0}\right)^2} \quad \text{or} \quad u(S_0) = b_0 u(a_0) + a_0 u(b_0) \tag{9c}$$

A2.2 Calculation of Uncertainty in Stress

From Eq. (1c), this is the result of uncertainties in load and cross-sectional area:

$$\frac{u_c(\sigma_0)}{\sigma_0} = \sqrt{\left(\frac{u(P)}{P}\right)^2 + \left(\frac{u(S_0)}{S_0}\right)^2} \tag{10}$$

Relative area uncertainty $u(S_0)/S_0$, is derived from Eq. (9a) or (9c).

A2.3 Calculation of Uncertainty in Final Minimum Cross-Sectional Area

For a cylindrical specimen, the formula corresponds to Eqs. (9a, b) above:

$$\frac{u(S_u)}{S_u} = \frac{2\,u(d_u)}{d_u} \quad \text{or} \quad u(S_u) = \frac{\eth d_u \; u(d_u)}{2} \tag{11}$$

A2.4 Calculation of Uncertainty in Reduction of Area

The treatment below considers only uncertainties in specimen measurements. In general, increasing temperature or stress gives shorter rupture time and higher ductility, and in principle, an uncertainty component for reduction of area (or elongation) due to uncertainties in stress and temperature could be calculated. However, the effect coefficients $\partial Z_u/\partial\sigma_0$ etc. are not generally known or theoretically modelled, but are believed to be relatively small. This contrasts with creep rates and rupture times, which have a high stress exponent and exponential temperature dependence.

Since Eq. (2) is not a simple sequence of terms added or multiplied together the formula for uncertainty is a little more complex (see Appendix D):

$$\frac{u_c(Z_u)}{100} = \sqrt{\left(\frac{S_u u(S_0)}{S_0^{\,2}}\right)^2 + \left(\frac{u(S_u)}{S_0}\right)^2} \qquad (12)$$

The cross-sectional area uncertainties are given in Eqs. (9) and (11).

If the information is available, the effects of uncertainties in stress and temperature can be added as terms $(u_\sigma(Z_u))^2$ and $(u_T(Z_u))^2$.

A2.5 Calculation of Uncertainty in Percentage Elongation

Uncertainties of the various lengths are covered in Appendix A1.3 and A1.6.

Based on L_0 and L_u from Gauge Length Marks

The expression for uncertainty in elongation is equivalent to Eq. (12):

$$\frac{u_c(A_u)}{100} = \sqrt{\left(\frac{L_u u(L_0)}{L_0^{\,2}}\right)^2 + \left(\frac{u(L_u)}{L_0}\right)^2} \qquad (13a)$$

Based on Extensometer Length and Reference Length

The elongation and its uncertainty are

$$\Delta L_{eu} = L_{eu} - L_{e0}$$
$$[u(\Delta L_{eu})]^2 = [u(L_{eu})]^2 + [u(L_{e0})]^2 \qquad (13b)$$

Percentage elongation and its uncertainty are then given by

$$A_u = 100\, \Delta L_{eu} / L_{r0}$$

$$\frac{u_c(A_u)}{A_u} = \sqrt{\left(\frac{u(\Delta L_{eu})}{\Delta L_{eu}}\right)^2 + \left(\frac{u(L_{r0})}{L_{r0}}\right)^2} \qquad (13c)$$

The remarks in A.2.4 on including effects of uncertainties in stress and temperature on uncertainty of reduction of area also apply here.

A2.6 Calculation of Uncertainty in Rupture Time

The uncertainty $u_m(t_u)$ due to the length of the data logging period Δt, if applicable, has been given by Eq. (6).

$$u_m\,(t_u) = \Delta t / (2\sqrt{3})$$

The rupture time t_u is related to stress σ_0 and absolute temperature T (see Appendix D), and the additional components of uncertainty in t_u are then given by:

$$\frac{u_s(t_u)}{t_u} = \frac{n\,u(\sigma_0)}{\sigma_0} \qquad \text{for stress} \qquad (14a)$$

$$\frac{u_T(t_u)}{t_u} = \frac{Q\,u(T)}{RT^2} \qquad \text{for temperature} \qquad (14b)$$

The values of n and Q can be obtained from a series of rupture tests, or taken from existing data on a metallurgically similar material. Appendix C lists typical values for four classes of alloys, and these may be used if no other data are available.

n and Q can be determined from a series of rupture tests at different stresses and temperatures, and performing linear regression analysis of $\log(t_u)$ against $\log(\sigma_{net})$, and $\log(t_u)$ against (1/T).

The sensitivity coefficients for stress and temperature can be found by performing four extra tests, two at temperature T and stresses above and below σ_0, and two at σ_0 and temperatures above and below T. The varied parameter should differ from the central value by at least five times its standard uncertainty.

The ratio of the change in rupture time to change of parameter (stress or temperature) gives the sensitivity coefficient c() for the parameter, and

$$u_s(t_u) = c(\sigma_0) * u(\sigma_0) \qquad (14c)$$

$$u_T(t_u) = c(T) * u(T) \qquad (14d)$$

The combined uncertainty in rupture life is then the combination of the three components:

$$u_c(t_u) = \sqrt{\{ [u_m(t_u)]^2 + [u_s(t_u)]^2 + [u_T(t_u)]^2 \}} \qquad (15)$$

Normally, the uncertainty of the initial rupture time value, i.e. $u_m(t_u)$, will be much less than $u_s(t_u)$ and $u_T(t_u)$.

A2.7 Calculation of Uncertainty in Creep Elongation Time t_{fx}

This receives the same treatment as rupture time. An initial value $u_m(t_{fx})$ was obtained above (A1.8), reflecting uncertainty in the time measurement which is usually negligible.

The additional uncertainties due to stress and temperature uncertainties are

$$u_s(t_{fx}) = \frac{n\,u(\acute{o}_0)}{\acute{o}_0} t_{fx} \qquad \text{for stress} \qquad (16a)$$

and

$$u_T(t_{fx}) = \frac{Q\,u(T)}{RT^2} t_{fx} \qquad \text{for temperature} \qquad (16b)$$

Note that the factors multiplying t_{fx} on the right hand sides of these expressions are the same as in Eqs. (14a, b).

Finally there is a component due to uncertainty in strain measurement. In A1.10, the uncertainty in extensometer reading u(e) was obtained. Extension ΔL_e is the difference between initial and current values, and has uncertainty:

$$u(\Delta L_e) = \sqrt{2}\, u(e).$$

Strain ε is given by:

$$\varepsilon = \Delta L_e \,/\, L_{r0}$$

and its uncertainty by

$$\frac{u(\mathring{a})}{\mathring{a}} = \sqrt{\left(\frac{u(\Delta L_e)}{\Delta L_e}\right)^2 + \left(\frac{u(L_{r0})}{L_{r0}}\right)^2}$$

These expressions give:

$$u(\varepsilon) = \sqrt{} \, [2\, u(e)^2 + (\varepsilon\, u(L_{r0}))^2] \,/\, L_{r0} \qquad (17)$$

If the strain rate in h^{-1} at the target strain x is $\mathring{a}$, then the uncertainty in time at this point due to strain uncertainty is

$$u_e(t_{fx}) = u(\varepsilon) \,/\, \mathring{a} \qquad (17a)$$

and finally

$$u_c(t_{fx}) = \sqrt{[\, (u_m(t_{fx}))^2 + (u_S(t_{fx}))^2 + (u_T(t_{fx}))^2 + (u_e(t_{fx}))^2]} \qquad (18)$$

A2.8 Calculation of Uncertainty in Minimum Strain Rate $\mathring{a}_{min}$

Graphical estimation of $\mathring{a}_{min}$ does not readily provide an uncertainty $u_m(\mathring{a}_{min})$, but a *Type A* estimate may be made from repeated measurements, or analysis of strain/time data. As for t_u and t_{fx}, this will be much less than the effects of uncertainties in temperature and stress. As noted in Appendix D, the same expression applies for all three of these parameters, i.e. uncertainty contributions are:

$$u_s(\mathring{a}_{min}) = \frac{n\,u(\sigma_0)}{\sigma_0}\mathring{a}_{min} \qquad \text{for stress} \qquad (19a)$$

and

$$u_T(\mathring{a}_{min}) = \frac{Q\,u(T)}{RT^2}\mathring{a}_{min} \qquad \text{for temperature} \qquad (19b)$$

and then

$$u_c(\mathring{a}_{min}) = \sqrt{[(u_m(\mathring{a}_{min}))^2 + (u_s(\mathring{a}_{min}))^2 + (u_T(\mathring{a}_{min}))^2]} \qquad (19c)$$

APPENDIX B

Worked Example of Creep Test Uncertainties Calculation

B1. Test Data

This example is based on the results of a test on 2.25%Cr 1% Mo weld metal. The specimen was machined according to a drawing with the following dimensions and tolerances (units mm):

diameter	8.98/9.00
parallel length	50 ± 0.5
extensometer length	58 ± 0.5

The diameter of the extensometer attachment position and blending radius to the parallel section gave a calculated shoulder correction of 2.8mm.

Test temperature 565°C

main thermocouple error	$e_m(T)$	= 0.5°C
specimen uniformity error	$e_u(T)$	= 1.5°C
measuring system error	$e_c(T)$	= 2°C

Load 10790 N (stress = 170MPa).
Weights and lever arm certified to give specified load ±1%.

Extensometer accuracy ± 0.02mm

After the initial period of primary creep, specimen parameters were recorded by computer at intervals of 8 hours. The specimen had failed at the 2208h recording time.

Time for 1% strain was required. Strain at 6 times around this point are given:

Time h	1162	1170	1178	1186	1194	1202
Strain %	0.98229	0.99009	0.99503	1.00563	1.01206	1.02331

Linear regression gives the results:
time to 1% strain = 1181h (standard error 2h): strain rate = 0.00102%/h = 1.02E-5h^{-1}

Minimum strain rate was determined graphically to be 9.7E-6h^{-1} at around 1200h.

The broken specimen gave the following 10 measurements of minimum diameter d_u (mm)
7.50, 7.65, 8.26, 7.79, 8.11, 8.02, 7.87, 8.11, 7.95, 7.99
Mean = 7.92 Standard deviation = 0.23
Standard uncertainty of mean d_u = 0.23 / $\sqrt{10}$ = 0.07mm

Digital calliper error = 0.02mm standard error = $0.02/\sqrt{3}$ = 0.012mm
Total standard uncertainty of $d_u = \sqrt{(0.07^2 + 0.012^2)} = 0.07$ mm.

The following are the 10 measurements of final extensometer length L_{eu} (mm)
64.25, 64.24, 64.42, 64.20, 64.53, 64.36, 64.82, 64.56, 64.54, 64.45
Mean = 64.43
Standard deviation = 0.19
Standard uncertainty of mean $L_{eu} = 0.19 / \sqrt{10} = 0.06$ mm

For this material and temperature, Appendix C gives values of 5.4 for the creep exponent n and 50000K for Q/R.

These data and the calculation results are shown in the calculation sheets Tables B1 to B9, at the end of this Appendix, together with references to the equations used in each uncertainty evaluation. A copy of a spreadsheet is provided in Table B10, which does most of the calculating automatically.

B2. Presentation of Results

To obtain the usual 95% confidence, a coverage factor of 2 should be applied to the standard uncertainties in the table, as follows.

2.25% Cr 1% Mo steel weld metal tested at 565°C and 170MPa.

Reduction of area	22.4 ± 2.8 %
Elongation	12.1 ± 1.2 %
Rupture time	2204 ± 482 h
Time to 1% strain	1181 ± 266 h
Minimum strain rate	(9.7 ± 2.2)E-6 h^{-1}

B3. Notes

Absolute and Relative Uncertainties

The left hand side of some equations for uncertainties of values consists of the ratio of the uncertainty to the value, i.e. u(x) / x = *expression*. In some cases, this ratio can be used in a subsequent calculation without evaluating the uncertainty itself.

For example, uncertainty in stress (Eq. (10)) uses $u(S_0) / S_0$ derived in Eq. (9a) or (9c). (S_0 = original cross-sectional area).

In the worksheet table, uncertainties are given as absolute values or ratios (%) as appropriate, or both.

In the final results table, uncertainties should be given in the same absolute units as the result.

Relative Importance of Uncertainty Components

It can be seen that the uncertainty of initial time measurement for rupture or given creep strain is insignificant compared with uncertainties due to stress and temperature variation.

In this example, *type B* uncertainties for parallel length and extensometer length (both 0.29mm) were the main factors in uncertainty of % elongation. *Type A* evaluation would have given a figure similar to the uncertainty of final extensometer length (0.06mm). The lower uncertainty will be particularly valuable for low-ductility materials.

Linear Regression Hand Calculation

Many hand calculators have programmed procedures to obtain the intercept and slope parameters (a and b) of the "best fit" straight line through a set of n (x, y) data points (for example ($\log(\sigma_0)$, $\log(t_u)$) values).

The estimated value of y when x = X, and its standard uncertainty, are given by

$$Y = a + bX \qquad u(Y) = \sqrt{\frac{1}{n(n-2)}\left(n\Sigma\left(y^2\right) - (\Sigma y)^2 - \frac{[n\Sigma(xy) - \Sigma x\Sigma y]^2}{n\Sigma\left(x^2\right) - (\Sigma x)^2}\right)}$$

The latter function is available in Microsoft Excel as STEYX.

Spreadsheet Calculation Aid

Tables B1 to B9 have been incorporated into an Excel 5.0 spreadsheet, which does most of the calculation automatically. It has been pasted into a Word Table containing the data of the worked example, and is given in Table B10. A working spreadsheet can be obtained from the author, or made up by entering the text and formulae into a blank spreadsheet. All formatting has to be set as required.

The spreadsheet formulae are given on the page following Table B10. New data needs to be entered only in the highlighted cells.

In some places, an option is given for uncertainty as *Type A* or *B*, and in relative or absolute values. In some situations this is not available, for example temperature errors will usually be absolute and *Type B*. However, the sheet can easily be modified as required. Uncertainties for initial measured creep strain time and minimum creep rate, if available, can be entered in G51 and G61. If not, zeros must be entered. This version only provides for cylindrical specimens.

Creep Test Uncertainties Calculation Sheets

Some quantities which are used later are represented by upper case letters, e.g. *value* (Z) or (= Z). The symbols are used later in explanatory comments in the right hand column.

A % sign following an uncertainty value indicates that this is a relative uncertainty, as a percentage of the value to which it applies. Otherwise, uncertainties are absolute. The equations referred to in the last column are in Appendix A2.

Table B1. Uncertainty in Initial Cross-Sectional Area

Source of Uncertainty			**Uncertainty in Source**					**Affected Measurand**			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
initial diameter	d_0 (mm)	8.99	B	0.01	rect.	√3	0.06% (A) 0.006	S_0 (mm^2)	63.47	0.12% (B) 0.08	B=2A (9a)

Table B2. Uncertainty in Initial Stress

Source of Uncertainty			**Uncertainty in Source**					**Affected Measurand**			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
load	P (N)	10790	B	1%	rect.	√3	0.6% (C)				
initial cross-sectional area	S_0 (mm^2)	63.47	B		-		0.12% (=B)	σ_0 (MPa)	170	0.6% (D)	$D=\sqrt{(B^2 + C^2)}$ (10)

Table B3. Uncertainties in Final Minimum Cross-Sectional Area and Percentage Reduction of Area

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
final minimum diam.	d_u mm	7.92	A	0.07		1	0.9% (E) 0.07	final minimum cross-sectional area			
								S_u (mm²)	49.26	1.8% (F) 0.9	F=2E (11a)
								reduction in area			
								Z_u (%)	22.4	1.4	(12)

Table B4. Uncertainty in Percentage Elongation

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
parallel length	L_c (mm)	50	B	0.5	rect.	√3	0.58% 0.29 (G)	initial reference length			
end correction		2.8		not included				L_{r0} (mm)	52.8	0.58% 0.29 (=G)	
extensometer length	L_{e0} (mm)	58	B	0.5	rect.	√3	0.29 (H)	ref. length elongation			
final ext'meter length	L_{eu} (mm)	64.43	A	0.06			0.06 (I)	ΔL_{eu} (mm)	6.43	0.30 (J) (4.7%)	$J=\sqrt{(H^2+I^2)}$ (13b)
								percent elongation			
								A_u (%)	12.1	0.6	(13c)

Table B 5. Uncertainty in Temperature

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
measurement thermocouple	T_m (K)		B	0.5							
specimen uniformity	T_u (K)		B	1.5							
controller error	T_c (K)		B	2.0							
total temperature error	(K)		B	2.55	rect.	$\sqrt{3}$	1.47	T (K)	838	1.47	

Table B6. Data from Appendix C, and Factors for Effects of Stress and Temperature Uncertainties on Uncertainties of Rupture Time, Creep Time and Strain Rate.

	$n = 5.4$	$Q / R = 50000K$	$n\, u(\sigma_0) / \sigma_0 = n\, D = 0.032$ (L)	$u(T)\, Q / RT^2 = 0.105$ (M)		(14a, b)

Table B7. Uncertainty in Rupture Time

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
rupture time	t_u (h)	2204	B	4	rect.	$\sqrt{3}$	2.3 (N)			$u_m(t_u)=2.3$	= N
initial stress	σ_0 (MPa)	170					0.6%			$u_s(t_u)=70$	$= P = L. * t_u$
temperature	T (K)	838					1.47	rupture time		$u_T(t_u)=231$	$= S = M * t_u$
								t_u (h)	2204	241	$= \sqrt{(N^2 + P^2 + S^2)}$ (15)

Table B8. Uncertainty in Time at 1% Strain

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
time at 1% strain	t_{f1} (h)	1181	A	2		1	2			$u_m(t_{f1})$=2	= U
initial stress	σ_0 (Mpa)	170					0.6%			$u_s(t_{f1})$=38	= V = L.* t_{f1} (16a)
temperature	T (K)	838					1.2			$u_T(t_{f1})$=124	= W = M * t_{f1} (16b)
extensometer	e (mm)		B	0.02	rect.	√3	0.012	ε	0.01 (1%)	u(ε) = 3E-4	(17)
									$\dot{e}$ =1E-5h^{-1}	$u_e(t_{f1})$=30	= X (17a)
								time at 1% strain			
								t_{f1} (h)	1181	133	$=\sqrt{(U^2+V^2+W^2+X^2)}$ (18)

Table B9. Uncertainty in Minimum Strain Rate

Source of Uncertainty			Uncertainty in Source					Affected Measurand			
Source quantity	symbol (unit)	value	type	value	prob. distrbn.	divisor	standard uncertainty	measurand	value	standard uncertainty	equation
Minimum strain rate	$\dot{e}_{min}$ (h^{-1})	9.7E-6	not det-ermined								
initial stress	σ_0 (MPa)	170					0.6%			3.1E-7 (Y)	= L * $\dot{e}_{min}$ (19a)
temperature	T (K)	838					1.2	Minimum creep rate		1E-6 (Z)	= M * $\dot{e}_{min}$ (19b)
								$\dot{e}_{min}$ (h^{-1})	9.7E-6	1.1E-6	$= \sqrt{(Y^2+Z^2)}$ (19c)

Table B10. Copy of Excel Spreadsheet for Calculation of Measurands and Uncertainties

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	Measurand for uncertainty evaluation	**Source of Uncertainty**			**Uncertainty in Source**						**Affected Measurand**			
		Source quantity	symbol (unit)	value	type	value absolute	relative	prob. distrbn.	divisor	standard uncer'ty	measurand	value	standard uncertainty	
4														
5	Initial cross-	initial diameter	d_0 (mm)	8.990	B	0.01		rect	1.73	0.006				
6	sectional area	micrometer error	u_m (mm)		B	0		rect.	1.73	0.000	d_0 (mm)	8.990	0.0058	0.064%
7											S_0 (mm^2)	63.48	0.13%	
8														
9														
10	Initial stress	Load	P (N)	10790	B		1%	rect.	1.73	0.58%				
11		initial cross-sect. area	S_0 (mm^2)	63.48	A		0.13%	-	1	0.13%	σ_0 (MPa)	170.0	0.59%	
12														
13														
14	Final minimum	final min. diam.	d_u (mm)	7.92	A	0.07			1	0.070				
15	cross section &	micrometer error	u_m (mm)		B	0.02		rect.	1.73	0.0116	d_u (mm)	7.92	0.071	0.90%
16	Reduction of										S_u (mm²)	49.27	1.79%	0.88
17	area	initial cross-sect. area	S_0 (mm^2)	63.48						0.082				
18											Z_u (%)	22.4	1.4	
19														
20														
21		*from gauge length marks*												
22	Percentage	initial gauge length	L_0 (mm)											
23	elongation	final gauge length	L_u (mm)											

Table continues

Table B10 continuation

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	Measurand for uncertainty evaluation	**Source of Uncertainty**			**Uncertainty in Source**						**Affected Measurand**			
		Source quantity	symbol (unit)	value	type	value absolute	relative	prob. distrbn.	divisor	standard uncer'ty	measurand	value	standard uncertainty	
24														
25	Enter data in	*from extensometer length*												
26	one place only	parallel length	Lc (mm)	50	B	0.5		rect.	1.73	0.289	initial reference length			
27		end correction		2.8		0					L_{r0} (mm)	52.8	0.289	
28		extensometer length	L_{e0} (mm)	58	B	0.5		rect.	1.73	0.289	ref. length elongation			
29		final ext'meter length	L_{eu} (mm)	64.43	A	0.06			1	0.06	ΔL_{eu} (mm)	6.43	0.295	
30														
31											percent elongation			
32											A_u (%)	12.2	0.6	
33														
34														
35	Temperature	measuring th'couple	T_m (K)		B	0.5								
36		specimen uniformity	T_u (K)		B	1.5								
37		controller	T_c (K)		B	2								
38		total temp. error	T (K)	838	B	2.55		rect.	1.73	1.47	T (K)	838	1.47	
39														
40														
41		*Data from Appendix C, and Factors for Effects of Stress and Temperature Uncertainties on Uncertainty of Rupture Time*												
42		stress exponent n	5.4		Q/R (K)	50000			$nu(\sigma_0)/\sigma_0$	0.032	$u(T)\ Q / RT^2$	0.105		
43														

Table continues

Table B10 final section

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	Measurand for uncertainty evaluation	**Source of Uncertainty**			**Uncertainty in Source**						**Affected Measurand**			
		Source quantity	symbol (unit)	value	type	value absolute	relative	prob. distrbn.	divisor	standard uncer'ty	measurand	value	standard uncertainty	
44														
45	Rupture time	rupture time	t_u (h)	2204	B	4		rect.	1.73	2.3			2.3	$u_1(t_u)$
46		initial stress	σ_0 (MPa)	170.0						0.59%			70.5	$u_2(t_u)$
47		temperature	T (K)	838						1.47			231.3	$u_3(t_u)$
48											t_u (h)	2204	241.8	
49														
50														
51	Time at x% strain	time at x% strain	t_{fx} (h)	1181	A	2			1	2.0		$u_m(t_{fx})$=	2.0	
52		initial stress	σ_0 (MPa)	170.0						0.59%		$u_s(t_{fx})$=	37.8	
53		temperature	T (K)	838						1.47		$u_T(t_{fx})$=	123.9	
54		extensometer	e (mm)		B	0.02	rect.		1.73	0.012		$u_e(t_{fx})$=	31.4	
55		strain rate at x% strain	å (h^{-1})	0.00001										
56		reference length	L_{r0} (mm)	52.8						0.289				
57		strain	ε	0.01						0.00031	t_{fx} (h)	1181	133.3	
58														
59														
60														
61	Minimum strain rate	minimum strain rate	å $_{min}$ (h^{-1})	9.7E-06		0.0			1	0		u_m(å $_{min}$)	0	
62		initial stress	σ_0 (MPa)	170.0						0.59%		u_s(å $_{min}$)	3.1E-07	
63		temperature	T (K)	838						1.47		u_T(å $_{min}$)	1.0E-06	
64											å $_{min}$ (h^{-1})	9.7E-06	1.1E-06	

Formulae for the Spreadsheet in Table B10.

K5=IF(H5="", G5/J5, E5*H5/J5) K6=G6/J6 M6=E5
N6=IF(F5="A", SQRT(K5^2+K6^2), IF(F5="B", K5, "")) O6=N6/M6
M7=PI()*M6^2/4 N7=O6*2 K10=IF(G10="", H10/J10, G10/E10/J10) E11=M7
H11=N7 K11=H11/J11 M11=E10/E11 N11=SQRT(K10^2+K11^2)
K14=IF(H14="", G14/J14, E14*H14/J14) K15=G15/J15 M15=E14
N15=SQRT(K14^2+K15^2) O15=N15/M15 M16=PI()*E14^2/4 N16=O15*2
O16=N16*M16 E17=M7 K17=N7*M7 M18=100*(1-M16/E17)
N18=100*SQRT((M16*K17/E17^2)^2+(O16/E17)^2) K22=IF(E22<>"",G22/J22, "")
K23=IF(E23<>"",G23/J23,"") K26=IF(E26<>"", G26/J26, "")
M27=IF(E26<>"", E26+E27, "") N27=IF(E26<>"", SQRT(K26^2+K27^2), "")
K28=IF(E26<>"", G28/J28, "") K29=G29/J29 M29=IF(E26<>"", E29-E28, "")
N29=IF(E26<>"", SQRT(K28^2+K29^2), "")
M32=IF(E26<>"", 100*M29/M27, IF(E22<>"", 100*(E23/E22-1), ""))
N32=IF(E26<>"", M32*SQRT((N27/M27)^2+(N29/M29)^2), IF(E22<>"", 100*
SQRT((E23*K22/E22^2)^2+(K23/E22)^2), ""))
G38=SQRT(G35^2+G36^2+G37^2) K38=G38/J38 M38=E38 N38=K38
K42=D42*N11 M42=K38*G42/E38^2 K45=G45/J45 N45=K45 E46=M11
K46=N11 N46=E45*K42 E47=E38 K47=K38 N47=E45*M42 M48=E45
N48=SQRT(N45^2+N46^2+N47^2) K51=G51/J51 N51=K51 E52=M11
K52=N11 N52=E51*K42 E53=E38 K53=N38 N53=E51*M42 K54=G54/J54
N54=K57/E55 E56=M27 K56=N27 K57=SQRT(2*K54^2+(E57*K56)^2)/E56
M57=E51 N57=SQRT(N51^2+N52^2+N53^2+N54^2) K61=G61/J61 N61=K61
E62=M11 K62=N11 N62=K42*M64 E63=E38 K63=K38 N63=M42*M64
M64=E61 N64=SQRT(N61^2+N62^2+N63^2)

APPENDIX C

Values of Stress Exponent n, and (Activation Energy Q) / R

The creep rate in stage 2 varies with temperature as an activation-controlled process, and in proportion to a power of the stress. The rupture time is approximately that required to reach the failure elongation at this strain rate, and hence

$$t_u = A\, \sigma_0^{-n} \exp(Q / RT) \qquad \text{(C1)}$$

The values of the constant A, stress exponent n, and activation energy Q, could be obtained by regression analysis of the results of a series of rupture tests at different stresses and temperatures.

For uncertainty evaluation in a creep test it will be assumed that stress exponent n and activation energy Q have similar values to those for similar materials if the comprehensive data for the test material are not available.

The figures in the Tables below are derived from stress-rupture data in Atlas of Creep and Stress-Rupture Curves [7]. They give the values of n and Q/R in the Eq. (C1) above which best fit the data in the Atlas for a typical material in four classes. Least-squares regression analysis was used, after transforming Eq. (C1) to

$$\log\, t_u = \log A - n \log \sigma + 2.3\, Q / RT \qquad \text{(base 10 logs)}$$

Other materials in a class would give slightly different figures from the example chosen, but the figures given are representative, and sufficient for estimation of uncertainty arising from tolerance in stress and temperature.

Ferritic Pipe Steel	Temperature °C	500	540	580
2.25Cr 1Mo	n	7.1	5.4	4.3
Atlas p. 19.35	Q/R K	48000	50000	47000

Ferritic Rotor Steel	Temperature °C	480	530	580
0.5Cr 0.5Mo 0.25 V	n	6.7	4.7	4.0
Atlas p. 19.22	Q/R K	50000	43000	45000

Austenitic Stainless	Temperature °C	600	650	700
Type 316	n	6.9	4.7	3.7
Atlas p. 11.39	Q/R K	48000	49000	48000

Ni Base Superalloy	Temperature °C	700	815	870
Nimonic 90	n	4.9	4.3	4.1
Atlas p. 5.68	Q/R K	45000	52000	58000

APPENDIX D

Derivation of Formulae for Uncertainties

D.1

When a measurand Y is a function of independent measured quantities x_1 , x_2 , x_3......, and each x_i is subject to uncertainty $u(x_i)$, then the resulting combined uncertainty in Y is given by

$$u(Y) = \sqrt{\sum \left(u(x_i) \frac{\partial Y}{\partial x_i} \right)^2}$$

D.2 Reduction of Area

This is calculated from the final and initial cross-sectional areas S_u and S_0 using Eq. (2):

$$Z_u = 100\,[1 - (S_u / S_0)]$$

Then $$\frac{1}{100}\frac{\partial Z_u}{\partial S_0} = \frac{S_u}{S_0{}^2} \qquad \frac{1}{100}\frac{\partial Z_u}{\partial S_u} = -\frac{1}{S_0}$$

Hence $$\frac{u(Z_u)}{100} = \sqrt{\left(\frac{S_u\, u(S_0)}{S_0{}^2}\right)^2 + \left(\frac{u(S_u)}{S_0}\right)^2}$$

D.3 Rupture and Creep Elongation Times and Minimum Creep Rate
Effect of Stress and Temperature Uncertainties

Over a small temperature or stress range, stage 2 strain rate varies in the following manner:

$$d\varepsilon/dt = B\, \sigma_0{}^n \exp(-Q / RT)$$

where n is the stress exponent, Q the creep activation energy, and B a constant.

As a simplification for estimation of uncertainties, it is assumed that most of the test time in spent in stage 2 at this constant strain rate, and rupture occurs at a fixed strain. The times when strain = x% (t_{fx}) , or rupture (t_u), then vary as the reciprocal of the expression above, i.e.

$$t = A\, \sigma_0{}^{-n} \exp(Q / RT)$$

where $t = t_u$ or t_{fx}.

The partial derivatives are

$$\frac{\partial t}{\partial \sigma_0} = -A\, n\, \sigma^{-n-1} \exp(Q/RT) = -\frac{n}{\sigma_0} t$$

$$\frac{\partial t}{\partial T} = -\frac{AQ}{RT^2}\, \sigma^{-n} \exp(Q/RT) = -\frac{Q}{RT^2} t$$

Hence

$$u(t) = \sqrt{\left(\frac{nu(\sigma_0)}{\sigma_0} t\right)^2 + \left(\frac{Qu(T)}{RT^2} t\right)^2} \qquad (t = t_u \text{ or } t_{fx})$$

The same expression also applies to creep strain rate, thus

$$u(\varepsilon'_{min}) = \sqrt{\left(\frac{nu(\sigma_0)}{\sigma_0} \varepsilon'_{min}\right)^2 + \left(\frac{Qu(T)}{RT^2} \varepsilon'_{min}\right)^2}$$

In the procedure in Step 5 and Appendix A2, and the example of uncertainty calculations in Appendix B, the two components $u_s()$ and $u_T()$ (due to stress and temperature uncertainties) are kept separate to clarify the sources of uncertainties in the measurand.

Other terms may be included in the summation under the square root sign, namely:

for t_u, the uncertainty of the initial time measurement (Eq. (15));
for t_{fx}, the results of initial time measurement and strain uncertainties (Eq. (18));
for $\dot{\varepsilon}_{min}$, uncertainty in initial graphical or statistical evaluation (Section A2.7).