Measurement At Home

Report on measuring the speed of sound with toilet rolls

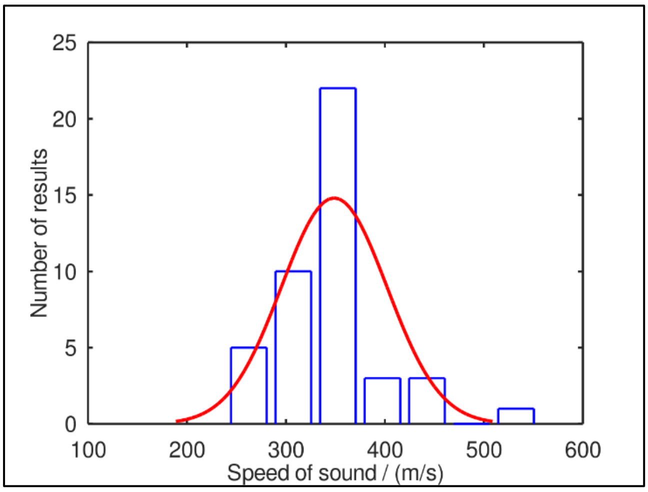
1. Overview

This report uses 44 results received in a few days of issuing the challenge, several from France. **Thank you and merci for results and comments:** Aubin, Benjamin, Billon, Clement, Connor, Druault, Edward, Ewan, Felix, Harry, Hector, Hugo, Jacob, Jim, John, Joshua, Juhel, Karl, M, Manny, Merlin, Oscar, Poj, Ruby, Sas, Warichet and Zhuoxuan.

The overall finding was that the mean measured speed of sound was 341 m/s with a spread (standard deviation) of about 34 m/s. This agrees extremely well with an expected value (343 m/s for value at 20 °C).

2. What was the spread in results?

This histogram shows the spread in results.



3. Why did people get different answers?

Measurements usually give different results when repeated.

Measurement scientists (metrologists) consider the factors that contribute to these variations, or uncertainty, and try to limit them. There are two types of uncertainty.

The first, **random uncertainties** lead to randomly varying results. For example, using a ruler to measure a length of something that's tricky to keep still – like a puppy. A solution is to take many measurements and use the average. More measurements increase confidence in the result.

The second type, **systematic uncertainties**, provide the same result each time though can be shown to differ from the 'true' value. For example, due to the ruler's markings being in the wrong place. Systematic uncertainties are trickier to deal with as the 'true' value is not usually known, so clever investigation is needed to determine them. In this example, you could compare your ruler with another ruler.

This particular investigation had many people using different set ups, approaches (like different frequencies) in various environmental conditions (temperatures and humidity), which overall lead to a good average value.

Metrologists create ***uncertainty budgets*** to describe all the contributions, and that's what we will do here. Our budget lists uncertainty contributions and an estimate of how much they affect the result.

Uncertainty budget for measuring the speed of sound using toilet rolls		
Uncertainty contribution	**Notes - values given cover 68% of cases. (1)**	**Size of effect on final answer**
Signal frequency	We assume signal generators provide signals good to 1/1000th, e.g. 3 Hz in 3,000 Hz.	Variation on (3 in 3000) Hz leads to difference of **0.35 m/s**
Finding quiet spot	***Juhel*** said: "I first heard the record all along, and then, couldn't give an accurate value of the wavelength with such a rustic way." **7 mm**	14-42 m/s varying with frequency used. Higher frequency means smaller wavelength so bigger effect. We use average of **26 m/s**
Moving from listening to marking quiet spot	The headphone cable might move in between hearing and measuring by **5 mm**	10-30 m/s varying with frequency used. Higher frequency means smaller wavelength so bigger effect. We use average of **19 m/s**
Placing and reading the mark	***Manny*** said: "The blue tack was at least a half a centimetre long which means my measurement could be a centimetre out." **3 mm**	6-18 m/s varying with frequency used. Higher frequency means smaller wavelength so bigger effect. We use average of **11 m/s**
Temperature (2)	Could be between 15-25 °C	**6 m/s**
Humidity (3)	Assuming range of 40-50 %RH	**1 m/s**
	Total uncertainty 'sum' of above (4)	**34 m/s**

Quite detailed notes on the above table:

1. It is standard practice in uncertainty budgets to quote at the '68% level'. This means values for effects cover 68% of the expected spread.
2. Temperature can be thought of as a measurement of the speed which atoms and molecules move. Faster speeds are measured as higher temperatures. The faster atoms and molecules in air move, the faster they 'carry' sound waves. For normal atmospheric temperatures, equations can describe the speed of sound in air for temperature θ (theta) in Celsius.

   If you are doing this 'properly', you would use the equation $c = \sqrt{\gamma RT/M} \approx 20.047 \times \sqrt{\theta + 273.15}$ where γ is the adiabatic index, R is the ideal gas constant, T is temperature and M is molar mass. For experiments near room temperature we can use a simpler equation $c \approx 331.3 + (0.606 \times \theta)$ m/s.
3. The temperature discussion above assumes completely dry air. Water molecules are less massive (so move faster) than average air molecules, so increasing water content of air makes sound travel faster through it. However, even in the most humid conditions on earth, water molecules only constitute a few percent of the total air mass, so the effect is small. At sea level, changes between 0 and 100% humidity causes a change in speed of about 0.35%. The range in your house will likely be between 40-50%.
4. You will notice that the 'Total Uncertainty' is less than the sum of the individual components. This is because we use the standard practice of adding all separate components in quadrature (the square root of the sum of each component squared). This mathematical approach manages the fact that, since they are independent and random, components can go in different directions (e.g. a temperature effect could increase the value while humidity could reduce it).

One component in the table (finding quietest spot) has the biggest uncertainty contribution. Budgets like this help experimental design as they show how much care needs to be taken in different areas.

People used sound frequencies from 2000 to 6000 Hz. Higher frequencies gave longer wavelengths that were easier to mark, but harder to hear, and people ran out of headphone cable. Science usually involves compromise.

Finally, some results were outside the budget's calculated total uncertainty. We expect this as we described contributions (note 1) at the 68% confidence level, so about one third of results will be outside that range. Also, it may be that estimates of contributions are too low, we have missed out contributions or other issues. We noticed that when we could find three quiet spots for one frequency, distance between spots 1-2 and 2-3 were different, suggesting an effect we have not considered.

4. Final thoughts

The average of 40 results from 27 people agreed very well with an expected value. Many people, all taking different approaches, can increase the likelihood of getting a 'good' answer as their variations cover both systematic and random uncertainties. The spread in values can indicate how well the experiment is designed and how well experimenters carried out the task.

Anyone can do a measurement, but evaluating how 'good' the result is, and trying to improve its accuracy are difficult tasks that keep many people busy at NPL.

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