Quantities, Units, and Ionising Radiation Fundamentals

Summary
The general concepts of quantities and units are introduced.

The advantages of the International System of Units (SI) are mentioned, and reference made to the realisation of units for selected quantities at Standards Laboratories.

Quantities and units for the measurement of ionising radiation are then discussed in detail, with particular reference to those developed for general use.

Relationships between fluence, kerma, dose and stopping power are given, with introductions to the physics of ionising radiation interactions, and including the Bragg-Gray and Spencer-Attix (small) cavity theories, and large cavity theory.

Contents
1. Fundamentals
2. Standards and calibration
3. Radioactivity
4. Radiation field
5. Radiation interactions
6. Stopping powers
7. Dosimetry
8. Cavity theory

1: Fundamentals: Quantities and units

It is important to distinguish a quantity from a unit. In everyday language the word quantity is understood to be some ‘amount’, but in the field of measurement a ‘quantity’ is a characterisation of a physical phenomenon in terms that are suitable for numerical expression.

A physical quantity is a phenomenon capable of expression as the product of a number and a unit.

A unit is a selected reference sample of a quantity.

There are seven base units: the kilogram (kg), metre (m), second (s), ampere (A), kelvin (K), mole (mol) and candela (cd).

Derived units are obtained from combinations of the base units. Derived units may have special names. However some of the special names are restricted to certain quantities, e.g. hertz (Hz, s⁻¹) is the unit of frequency, but becquerel (Bq, s⁻¹) is the unit of activity.

The Conférence Générale des Poids et Mesures (CGPM) set up by the Metre Convention is responsible for the International System of Units (SI). The International Commission on Radiation Units and Measurements (ICRU)
The International Commission on Radiological Protection (ICRP) recommends protection level quantities.

This table shows the relationship between SI base and derived units.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Type of unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>metre</td>
<td>SI base unit</td>
<td>m</td>
</tr>
<tr>
<td>Area</td>
<td>metre</td>
<td>SI derived unit</td>
<td>m²</td>
</tr>
<tr>
<td></td>
<td>square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>joule</td>
<td>SI derived unit with special name</td>
<td>J (= kg m² s⁻²)</td>
</tr>
<tr>
<td>Absorbed dose</td>
<td>gray</td>
<td>SI derived unit with special name (restricted use)</td>
<td>Gy (= m² s⁻²)</td>
</tr>
<tr>
<td>Absorbed dose</td>
<td>rad</td>
<td>Non-SI unit</td>
<td>rad (= 0.01 Gy)</td>
</tr>
</tbody>
</table>

Within SI, all derived units can be obtained from the base units without extra numerical factors.

**SI unit prefixes**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>10²⁴</td>
<td>yotta</td>
<td>Y</td>
<td>10⁻¹</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>10²¹</td>
<td>zetta</td>
<td>Z</td>
<td>10⁻²</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>10¹⁸</td>
<td>exa</td>
<td>E</td>
<td>10⁻³</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>10¹⁵</td>
<td>peta</td>
<td>P</td>
<td>10⁻⁶</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>10¹²</td>
<td>tera</td>
<td>T</td>
<td>10⁻⁹</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>10⁹</td>
<td>giga</td>
<td>G</td>
<td>10⁻¹²</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>10⁶</td>
<td>mega</td>
<td>M</td>
<td>10⁻¹⁵</td>
<td>femto</td>
<td>f</td>
</tr>
<tr>
<td>10³</td>
<td>kilo</td>
<td>k</td>
<td>10⁻¹⁸</td>
<td>atto</td>
<td>a</td>
</tr>
<tr>
<td>10²</td>
<td>hecto</td>
<td>h</td>
<td>10⁻²¹</td>
<td>zepto</td>
<td>z</td>
</tr>
<tr>
<td>10¹</td>
<td>deka</td>
<td>da</td>
<td>10⁻²⁴</td>
<td>yocto</td>
<td>y</td>
</tr>
</tbody>
</table>
2: **Standards and calibration**

A primary standard makes an absolute measurement, whereas secondary and other reference standard instruments must be calibrated so that the calibration is traceable to the primary standard.

A primary standard measures (or realises) the quantity of interest from first principles. Primary standards for derived units, like air kerma and absorbed dose, involve some calibration, but only to make the measurement traceable to standards for base SI units (so Gy is related to J and kg, etc.). For example, a primary standard calorimeter may work by measuring the temperature rise in a known mass of graphite: It would be important to know the mass of graphite present using calibrated measuring equipment, and the temperature measuring equipment would have to be calibrated against a standard temperature scale.

The National Measurement System is the organisation of these reference standards and the calibration process into a coherent infrastructure, designed to ensure that measurements across the country as a whole are compatible. Calibration of a dosimeter involves comparing its response with that of another, more trustworthy, instrument. This process may involve some internal adjustment in order to make the dosimeter “read correctly”. More often, we obtain the numerical coefficient (the ratio of what you want, divided by what you get) by which readings should be multiplied in order to give the “correct result”.

In this hierarchy of standards the primary standard sits at the apex, and everything depends on its accuracy. Compatibility of measurements in different countries depends on the consistency of their respective national standards, which is tested by comparing primary standards either directly or via the BIPM in Paris, which coordinates the International Measurement System. This is formalised in a treaty called the Mutual Recognition Arrangement.

It is surprising (but reassuring!) that, despite four different methods being in use for establishing primary standards of absorbed dose to water they agree very well (within 1%). This results in a robust calibration network since it is unlikely that there are common systematic errors in the four different methods. This is in contrast to the situation with air kerma standards for high-energy photon beams, which has in the past suffered from systematic errors common to all primary standards worldwide.
3: **Radioactivity**

**Activity** $A$

\[
A = \frac{dN}{dt} = -\lambda N
\]  \hspace{1cm} 3.1

Where $dN$ is the expectation value of the number of nuclear transformations or decays in the time interval, $dt$; and $\lambda$ is the decay constant.

Mathematically $dN$ is understood to be the differential of an expectation value of the number of active nuclei $N$. The arguments of differential quotients are always non-stochastic quantities: There can of course only ever be a whole number of nuclei, and the number will change downwards only by unity with each decay.

Unit: s$^{-1}$

Special name for the unit of activity is the becquerel (Bq). The curie (Ci) is still often used; 1 Ci = 37 GBq.

Of course, integrating Eq. 3.1 with respect to time, one obtains the familiar

\[
N(t) = N_0 e^{-\lambda (t-t_0)}
\]  \hspace{1cm} 3.2

or equivalently,

\[
A(t) = A_0 e^{-\lambda (t-t_0)}
\]  \hspace{1cm} 3.3

where $A(t)$ is the activity at some time $t$, and $A_0$ is the initial activity at some time $t_0$.

The primary standard of activity is the $4\pi$ beta-gamma-coincidence counter at NPL. (For more information consult the references.)

**Related quantities**

**Half-life** $t_{1/2}$

\[
t_{1/2} = \frac{\ln 2}{\lambda}
\]  \hspace{1cm} 3.4

The half-life is the mean time taken for a radionuclide to decay to one half of its initial activity.

Unit: s

**Mean life** $\tau$

\[\tau = \frac{1}{\lambda} \]  \hspace{1cm} 3.5

The mean life is the mean time for a radionuclide to decay to $1/e$ of its original activity.
Almost all radioactive materials used in radiotherapy are artificially made either in reactors or in accelerators. For most materials such as $^{60}\text{Co}$ or $^{192}\text{Ir}$ the dose delivered to the patient is effectively constant through the treatment (the half-life of $^{60}\text{Co}$ is about 5.3 years while that of $^{192}\text{Ir}$ is 74 days), while for others the dose delivered is limited by the half-life of the radioactive species used. For example, $^{131}\text{I}$, used in the treatment of thyroid cancers, has a half-life of 8 days, but this is delivered by being chemically targeted to the thyroid tissue and so is outside the scope of this course. This course addresses issues concerning external beam radiotherapy or brachytherapy, where either long-lived radioactive sources or accelerators are used to deliver a known dose to the patient.

In terms of the physics of what happens for radioactive decay: First, the radioactive cobalt is made in a reactor,

$$^{59}\text{Co} + n \rightarrow ^{60}\text{Co}$$

Then in the radiation source, the radioactive disintegration takes place. In $\beta$-decays, this takes place initially via the weak interaction, with a half-life of 5.271 years:

$$^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^- + \nu$$

The excited nickel nucleus then decays via the electromagnetic interaction, which is much stronger than the weak interaction so the decay is much more rapid, with a half-life of about 3.3 picosecond:

$$^{60}\text{Ni}^* \rightarrow ^{60}\text{Ni} + 2\gamma$$

The two photons escape with energies of 1.17 and 1.33 MeV. It is these photons which are of interest to us.

Other sources of photons or fast electrons may be accelerators, where electrons are accelerated to very high energy and then either used to irradiate a patient directly in electron beam radiotherapy, or to generate high-energy bremsstrahlung photons which are then heavily filtered before irradiating the patient.
4: Radiation field

In order to calculate the radiation absorbed dose delivered to a medium, we need to know as a first step the number of particles or photons, or the amount of energy, passing through that medium. This requires a definition of the concept of fluence.

Fig 1 Illustration of particle fluence

Fluence Φ

A number of particles passing a surface, constitutes a fluence. It is defined as

\[ \Phi = \frac{dN}{da} \]  \hspace{1cm} 4.1

where \( dN \) is the number of particles incident on a sphere of cross-sectional area \( da \). The use of a sphere expresses the fact one considers the area perpendicular to the direction of each particle, and thus with the sphere all particles passing through area \( da \) around a point are considered. The fluence is independent of the incident angle of the radiation.

Unit: m\(^{-2}\)

Planar fluence is the number of particles crossing a plane per unit area and hence does depend on the angle of incidence of the particles.

Energy fluence Ψ

\[ \Psi = \frac{dE}{da} \]  \hspace{1cm} 4.2

where \( dE \) is the radiant energy incident on a sphere of cross-sectional area \( da \).

Unit: J m\(^{-2}\)
Note: Energy is often expressed in units of electron volts, symbol eV. 1 eV is equal to the energy gained by an electron in passing through a potential difference of 1 volt. This is not an SI unit, but is accepted for use with the SI. 1 eV is approximately $1.602 \times 10^{-19}$ joule.

For a monoenergetic beam, $\Psi = \Phi E$, where $E$ is the energy of the beam.

**Fluence differential in energy $\Phi_E(E)$**

For a beam with a spectrum of energies it is useful to extend the concepts to fluence differential in energy, or the distribution of fluence with respect to energy, $\Phi_\Phi(E)$.

$$\Phi_\phi(E) = \frac{d\Phi}{dE}(E)$$

where $d\Phi$ is the fluence of particles with energy between $E$ and $E + dE$.

Unit: m$^{-2}$ J$^{-1}$

Similarly the energy fluence differential in energy $\Psi_E(E)$ can be defined

$$\Psi_\phi(E) = \frac{d\Psi}{dE}(E) = \frac{d\Phi}{dE}(E)E$$

Figure 2 illustrates the relationship between particle fluence $\Phi_E$ and energy fluence $\Psi_E$, both as functions of energy, for a particular spectrum of X-rays.

![Figure 2](image-url)  
*Fig. 2* Photon fluence and energy fluence spectra at 1 m from the target of an x-ray machine with tube potential 250 kV and added filtration of 1 mm Al and 1.8 mm Cu (target material: W; inherent filtration 2 mm Be).
A complete description of a radiation field requires the fluence distribution as a function of: (i) particle type e.g. electrons, photons, neutrons (this may include any relevant quantum state, e.g. spin), (ii) spatial position, (iii) direction, (iv) energy and (v) time.

The rate quantities e.g. fluence rate $\frac{d\Phi}{dt}$ tend to have their own symbols. Up to now we have described only scalar quantities; it is possible to define and use vector quantities, e.g. vectorial fluence $\vec{\Phi}$.

5: **Dosimetry**

Photon energy is delivered to materials as absorbed dose in a two-stage process. In the first stage the photon energy from a photon fluence is transferred to charged particles in the medium; in the second stage energy is transferred to the medium through ionisations and atomic excitations.

**Kerma** $K$ (from the acronym *Kinetic Energy Released per unit Mass*) quantifies the first stage, where the energy is transferred from indirectly ionising radiation to directly ionising radiation.

For most radiotherapy applications this happens through Compton scattering interactions, where the photon scatters off atomic electrons leading to a photon of reduced energy scattered away at some angle, and an energetic electron slowing down in the medium resulting in an absorbed dose to that medium.

![Fig 3 Primary standard of air kerma for 50 kV X-rays](image)

At higher energies (as may be delivered from high-energy bremsstrahlung X-ray treatments from accelerators), a significant fraction of the interactions will be through pair production, where the photon passes close enough to the cell nucleus that it interacts with the nuclear electric field, leading to the production of an electron-positron pair. These energetic particles slow down in the medium, thus delivering the dose, and at the end of the track the positron will annihilate with an electron with the emission of two back-to-back photons with energy 511 keV.
The main photon interactions in materials may be illustrated thus:

**Photoelectric effect:**

\[ \gamma - e^- \]

Low-energy photons
Ejects a single atomic electron

\[ E_e = E_\gamma - \text{binding energy} \]

This effect dominates at low energies, up to a few tens of keV.

**Compton scattering:**

\[ \gamma - e^- \]

Medium-energy photons
Ejects a single atomic electron

\[ E'_\gamma = \frac{E_\gamma}{1 + \alpha(1 - \cos \theta)} \]

\[ E_e = E_\gamma \frac{m_0c^2(1 - \cos \theta)}{1 + m_0c^2(1 - \cos \theta)} \]

where \( \alpha = E_\gamma/m_0c^2 \) (\( m_0c^2 \) is the mass-energy of the electron, 0.511 MeV) and \( \theta \) is the photon scattering angle. The minimum scattered photon energy occurs at \( \theta = 180^\circ \), with the maximum scattered electron kinetic energy at \( 0^\circ \); these energies are given by

\[ E_{\gamma,\min} = E_\gamma \frac{1}{1 + 2\alpha} \quad \text{and} \quad E_{e,\max} = E_\gamma \frac{2\alpha}{1 + 2\alpha} \]

The Compton effect dominates at medium energies from a few tens of keV up to several MeV in low-Z materials such as water, graphite, or tissue.

**Pair production:**

\[ \gamma - e^- - e^+ \]

Only for high-energy photons, with

\[ E_\gamma > 1.022 \text{ MeV} \]

\[ E_\gamma = E_e^- + E_e^+ + 1.022 \text{ MeV} \]

This is much more pronounced with high-Z material as the interaction is almost always with the nuclear electric field.

The positron slows down in the medium, and then annihilates with an atomic electron, with the resultant emission of two back-to-back 0.511 MeV photons.
Occasionally at higher energies, triplet production can occur, where the incoming high-energy photon interacts with an atomic electron instead of the nucleus. Also of course at higher energies (typically above about 10 MeV), photoactivation can occur where the incoming photon interacts with the nuclear electric field to produce an excited compound nucleus, which may decay by the emission of (most commonly) a neutron, leaving an unstable nucleus which will then break up via radioactive decay later on. This is rare in low-Z materials such as water or tissue, which at least means we are less likely to make the patient radioactive.

Figure 4 shows the relative interaction probabilities in water or tissue, for photons up to 10 MeV. Above 20 MeV, pair production becomes dominant.

**Figure 4** Relative interaction probabilities of photons in low-Z material such as water or tissue.

The kerma for a medium is defined as:

$$K = \frac{dE_{\nu}}{dm}$$  \hspace{1cm} (5.1)

where $dE_{\nu}$ is the mean kinetic energy transferred to charged particles via the interactions described above, here from photons, in a mass $dm$. The medium should always be specified as the kerma is medium-dependent. There are several primary standards to realise $K$ for different particle types (photons, electrons, protons and other light nuclei) and energies (a few tens of keV for X-rays, up to hundreds of MeV for protons and light nuclei).

Unit: J kg⁻¹

The special name for the unit of kerma is the gray (Gy). There is a similar quantity for directly-ionising radiation: CEMA, the converted energy per unit
mass, for which the unit is also the gray. Most of what follows can be applied to this also.

**Kerma relationship to fluence**

For monoenergetic photons, the kerma is given by

$$K = \Psi \frac{\mu_r}{\rho} \tag{5.2}$$

The kerma is usually expressed in terms of the distribution $$\Psi(E)$$ of the uncharged energy fluence with respect to energy. The kerma $$K$$ is then given by

$$K = \int \Psi(E) \frac{\mu_r(E)}{\rho} dE \tag{5.3}$$

where $$\frac{\mu_r(E)}{\rho}$$ is the mass energy transfer coefficient of the material for uncharged particles of energy $$E$$ (usually tabulated). From this, the ratio of kerma in two materials where the fluence ratio is the same (through proper scaling of dimensions) is equal simply to the ratio of average mass energy transfer coefficients (the scaling theorem).

Total kerma is usually split into two parts: **collisional kerma** and **radiative kerma**. Collisional kerma $$K_{\text{coll}}$$ leads to the production of electrons that dissipate their energy as ionisation near electron tracks in the medium, and it is this component that delivers the absorbed dose within a medium. Radiative kerma $$K_{\text{rad}}$$ leads to the production of bremsstrahlung as the charged particles are decelerated in the medium. This process results in energy carried significantly far away from the region of the interaction and therefore does not contribute to the absorbed dose in that region.

The collisional kerma $$K_{\text{coll}}$$ is given by

$$K_{\text{coll}} = \int \Psi(E) \frac{\mu_m(E)}{\rho} dE \tag{5.4}$$

where $$\frac{\mu_m(E)}{\rho}$$ is the mass energy absorption coefficient of the material for uncharged particles of energy $$E$$, again usually tabulated.

This may be rewritten in a form similar to eq. 5.2:

$$K_{\text{coll}} = \Psi \frac{\mu_m}{\rho} \tag{5.5}$$

where

$$\Psi = \int_0^{E_{\text{max}}} \Psi(E) dE \tag{5.6}$$

is the total (integrated) energy fluence and
\[
\frac{\bar{\mu}_m}{\rho} = \frac{1}{\Psi} \int_0^{E_{\text{max}}} \Psi_E(E) \frac{\mu_m(E)}{\rho} dE
\]

is the mass energy absorption coefficient averaged over the energy fluence spectrum.

For selected energy fluence spectra, \( \frac{\bar{\mu}_m}{\rho} \) may be tabulated.

**Absorbed dose, \( D \)**

Electrons travel through the medium and slow down, depositing energy along their tracks. Therefore the absorption of energy described by absorbed dose does not take place at the same location as the transfer of energy to charged particles described by kerma. The absorbed dose is defined as

\[
D = \frac{d\bar{\varepsilon}}{dm}
\]

where \( d\bar{\varepsilon} \) is the mean energy imparted to matter of mass \( dm \). Energy imparted is the energy incident minus the energy leaving the mass, minus the energy released in any nuclear transformations (to stop the dose becoming negative when, for example, the mass contains a radioactive source).

Unit: J kg\(^{-1}\). The absorbing medium should always be specified.

The special name for the unit of absorbed dose is the gray (Gy).

There are different primary standards to realise the Gy for various particle types and energies. NPL currently maintains primary standard therapy level absorbed dose calorimeters for photon beams and electron beams, and is currently building new primary standard therapy level absorbed dose graphite calorimeters for photon and electron beams, for proton beams and for brachytherapy sources. For a more detailed review of calorimeters consult the references.

**Fig 5** Primary standards of absorbed dose: The photon microcalorimeter (left) and the simpler electron calorimeter (right), showing core inside jacket
Kerma and dose (charged particle equilibrium)

Generally, the transfer of energy (kerma) from a photon beam to charged particles at a particular location does not lead to the absorption of energy by the medium (absorbed dose) at the same location. This is due to the finite range of the secondary electrons released through photon interactions: A 10 MeV electron for example, has a range in water of about 5 cm.

Since photons (from Compton scattering or from pair annihilation) will almost always escape from the volume of interest, one relates absorbed dose to collisional kerma. In general, the ratio of dose and collisional kerma will be denoted as:

\[ \beta = \frac{D}{K_{\text{coll}}} \quad 5.9 \]

If the photons from radiative kerma escape the volume of interest, it is assumed that \( \beta \approx 1 \).

Figure 6 illustrates the relation between collisional kerma and absorbed dose for a high energy photon beam under build-up conditions; (a) under conditions of charged particle equilibrium (CPE), and (b) under conditions of transient charged particle equilibrium (TCPE). As a high-energy photon beam penetrates the medium, collisional kerma is maximal at the surface of the irradiated material because photon fluence is greatest at the surface. Initially, the charged particle fluence, and hence the absorbed dose, increases as a function of depth until the depth of dose maximum (\( z_{\text{max}} \)) is attained.

This build-up of absorbed dose is responsible for the skin sparing effect in the case of high energy photon beams. However, in practice the surface dose is small but does not equal zero, because of the electron contamination in the beam due to photon interactions in the media upstream from the phantom or due to charged particles generated in the accelerator head and beam modifying devices.
If there were no photon attenuation or scattering in the medium, but yet production of electrons, the hypothetical situation would occur of a build-up region followed by a region of complete CPE where $D = K_{\text{coll}}$. 

In the more realistic situation, however, due to photon attenuation and scattering in the medium, a region of TCPE occurs, where there exists an essentially constant relation between collisional kerma and absorbed dose. This relation is practically constant since, in high energy photon beams, the average energy of the generated electrons, and hence their range, do not change appreciably with depth in the medium.

In the special case where true charged particle equilibrium does exist, at the depth of maximum dose in the medium $z_{\text{max}}$, the relation between absorbed dose $D$ and total kerma $K$ is given by:

$$D = K_{\text{coll}} = K(1 - \bar{g})$$

where $\bar{g}$ is the bremsstrahlung fraction, depending on the electron kinetic energy; the higher the energy, the larger is $\bar{g}$. The bremsstrahlung fraction also depends on the material considered, with higher values of $\bar{g}$ for higher $Z$ materials. For electrons produced by cobalt-60 gamma rays in air the bremsstrahlung fraction is 0.0032.
6: Charged particles – Stopping powers

'Stopping power' is actually a misnomer because dimensionally, it is a force. However, the term “stopping power” is universally recognised and used. It tells you how much energy is lost to a medium, by a charged particle of given properties (mass, charge) in crossing that medium. Very clearly, this is at the heart of the physics of the delivery of absorbed dose to a medium.

Stopping powers are calculated for electrons and positrons using the Bethe theory for “soft” collisions, with the stopping power as a result of “hard” collisions calculated using Møller cross-sections for electrons and Bhabha cross-sections for positrons.

A “soft” collision occurs when a charged particle passes an atom at a considerable distance, i.e. \( b \gg a \) where \( b \) is the impact parameter and \( a \) is the atomic radius. Only a very small amount of energy is transferred to an atom of the absorbing medium in a single collision.

In a “hard” collision where \( b \approx a \), a secondary electron (often referred to as a delta electron) with considerable energy is ejected and forms a separate track.

According to ICRU Report 37, the complete mass collisional stopping power for electrons and photons is

\[
\frac{S_{\text{coll}}}{\rho} = \frac{N_A Z}{A} \pi \rho^2 \frac{2m_e c^2}{\beta^2} \left[ \ln \left( \frac{E_K}{l} \right)^2 + \ln \left( 1 + \frac{\tau}{2} \right) + F^{-}(\tau) - \delta \right] \tag{6.1}
\]

with \( F^{-} \) for electrons given as

\[
F^{-}(\tau) = (1 - \beta^2) \left[ 1 + \frac{\tau^2}{8} - (2\tau + 1) \ln 2 \right]
\]

and \( F^{+} \) for positrons given as

\[
F^{+}(\tau) = 2 \ln 2 - \frac{\beta^2}{12} \left[ 23 + \frac{14}{\tau + 2} + \frac{10}{(\tau + 2)^2} + \frac{4}{(\tau + 2)^3} \right]
\]

where \( \tau = E_K/m_e c^2 \) and \( \beta = v/c \).

The density effect correction \( \delta \) in Eq. 6.1 accounts for the polarisation of the medium caused by the passing of a charged particle, which reduces the effective Coulomb force exerted on that particle. This affects the soft collision component of the stopping power. It is significant in the calculation of the ratio of stopping powers between media of different densities, such as that between water and air or water and graphite, and several models have been developed for it. The references contain more information on this.

The linear stopping power is defined as the expectation value of the rate of energy loss per unit path length \( dE/dx \) of the charged particle. The mass stopping power is defined as the linear stopping power divided by the density of the absorbing medium. Convenient units for the linear and the mass stopping powers are MeV/cm and MeV·cm²/g, respectively.
There are two types of stopping powers: *collisional* resulting from interactions of charged particles with atomic orbital electrons; and *radiative* resulting from interactions of charged particles with atomic nuclei, giving rise to the production of bremsstrahlung. These may be illustrated thus:

**Collisional stopping power:**

Energy loss by ionisation and excitation

Dominates for lower energies and low-\(Z\) material

*Absorbed dose* is delivered to material via this process

**Radiative stopping power:**

Scattering, mainly by nuclei

Energy loss by photon emission (bremsstrahlung)

Dominates for higher energies and high-\(Z\) material

Radioactivation can also occur with high-energy electrons but the probability is significantly lower than with photons; in essence, a bremsstrahlung photon has to be generated in the nuclear field which then may interact with the nucleus as described for photoactivation (page 10) to give an excited compound nucleus.

**Restricted and unrestricted stopping powers**

The *unrestricted mass collisional stopping power* expresses the average rate of energy loss by a charged particle in all (hard as well as soft) collisions. It is used for example in Bragg-Gray cavity theory, where the assumption is made that there is no significant contribution to the charged particle fluence from high-energy scattered electrons arising from “hard” collisions.

The concept of the *restricted mass collisional stopping power* is introduced to calculate the energy transferred to a localised region of interest. By limiting the energy transfer to secondary charged (delta) particles to a threshold \(\Delta\), highly energetic secondary particles are allowed to escape the region of interest. This is addressed in the modified cavity theory due to Spencer and Attix.

The restricted stopping power is therefore lower than the unrestricted stopping power. The choice of the energy threshold depends on the problem. For ionization chambers a frequently used threshold value is 10 keV (the range of a 10 keV electron in air is on the order of 2 mm).

The *restricted linear collisional stopping power* (also referred to as linear energy transfer) \(L_\Delta\) of a material, for charged particles, is the quotient of \(dE_\Delta\) by...
\[ dx, \text{ where } dE_\Delta \text{ is the energy lost by a charged particle due to soft and hard collisions in traversing a distance } dx, \text{ minus the total kinetic energy of the charged particles released with kinetic energies higher than a threshold } \Delta: \]

\[ L_\Delta = \frac{dE_\Delta}{dx} \]

The restricted mass collisional stopping power is the restricted linear collisional stopping power divided by the density \( \rho \) of the material.

The total mass stopping power is the sum of the collisional mass stopping power and the radiative mass stopping power. Figure 7 shows the total unrestricted and restricted (\( \Delta = 10 \) keV and 100 keV) electron mass stopping powers for graphite based on the data in ICRU Report 37. As the threshold for maximum energy transfer in the restricted stopping power increases, the restricted mass stopping power approaches the unrestricted mass stopping power for \( \Delta \to E_K/2 \), where \( E_K \) represents the electron kinetic energy. Note also that, since energy transfers to secondary electrons are limited to \( E_K/2 \), unrestricted and restricted electron mass stopping powers are identical for kinetic energies lower than or equal to \( 2\Delta \). This is indicated in Fig. 7 with short vertical lines at 20 keV and 200 keV.

\[ \text{FIG. 7 Unrestricted } S/\rho \text{ and restricted } ((S/\rho) \text{ with } \Delta = 10 \text{ keV and } 100 \text{ keV}) \text{ total mass stopping powers for carbon, based on data published in the ICRU Report 37. Vertical lines indicate the points at which restricted and unrestricted mass stopping powers begin to diverge as the kinetic energy increases.} \]

**Relationships between fluence and dose (electrons)**

Under the conditions that (1) radiative photons escape the volume of interest and (2) secondary electrons are absorbed on the spot (or there is charged particle equilibrium of secondary electrons), the absorbed dose to a medium \( D_{med} \) is related to the electron fluence \( \Phi_{med} \) in the medium, as follows:
where \( \frac{S_{\text{coll}}}{\rho} \) is the *unrestricted mass collisional stopping power* of the medium at the energy of the electron.

Owing to electron slowing down in the medium, even for a mono-energetic starting electron of kinetic energy \( E_0 \) there is always a primary electron fluence spectrum in the medium denoted by \( \Phi_{\text{med},E}(E) \), which is differential in energy and ranges from \( E_0 \) down to zero. In this case, the absorbed dose to the medium can be obtained by an integration of Eq. 6.3:

\[
D_{\text{med}} = \int_{E_0}^{E_{\text{max}}} \Phi_{\text{med},E}(E) \left( \frac{S_{\text{coll}}(E)}{\rho} \right)_{\text{med}} dE = \Phi_{\text{med}} \left( \frac{\bar{S}_{\text{coll}}}{\rho} \right)_{\text{med}}
\]

6.4

The right hand side of Eq. 6.4 shows that absorbed dose can be calculated using an equation formally similar to Eq. 6.3, by making use of spectrum-averaged collisional stopping power and total fluence.

The full, realistic electron fluence spectrum consists of primary charged particles that are, for example, the result of a multienergetic photon beam or electron beam interacting in the medium. These primary charged particles are slowed down and result in a secondary particle fluence. This fluence thus contains charged particles resulting from the slowing down through soft collisions of the primary charged particles, as well as delta electrons resulting from hard, knock-on collisions.

### 7: Cavity theory

**The Bragg-Gray cavity theory**

The Bragg-Gray cavity theory was the first cavity theory developed to provide a relationship between absorbed dose in a dosimeter, and the absorbed dose in the medium containing the dosimeter.

The conditions for application of the Bragg-Gray cavity theory are:

1. the cavity must be *small when compared with the range of charged particles incident on it* so that its presence does not perturb the fluence of charged particles in the medium;
2. the absorbed dose in the cavity is deposited *solely by charged particles crossing it*, i.e., photon interactions in the cavity are assumed negligible and thus ignored.

The result of condition (1) is that the electron fluences in Eq. 6.4 are identical, and are equal to the equilibrium fluence established in the surrounding medium. This condition can only be valid in regions of CPE or TCPE. In addition, the presence of a cavity always causes some degree of fluence...
perturbation, which requires the introduction of a fluence perturbation correction factor.

Condition (2) implies that all electrons depositing the dose inside the cavity are produced outside the cavity and completely cross the cavity. Therefore, no secondary electrons are produced inside the cavity and no electrons stop within the cavity.

Under these two conditions, according to Bragg-Gray cavity theory, the dose to the medium $D_{med}$ is related to the dose in the cavity $D_{cav}$ as follows:

$$ D_{med} = D_{cav} \left( \frac{\bar{S}}{\rho} \right)_{cav} ^{med} \tag{7.1} $$

where $\left( \frac{\bar{S}}{\rho} \right)_{cav} ^{med}$ is the ratio of the average unrestricted mass collisional stopping powers of the medium and cavity. The use of unrestricted stopping powers implies that the production of secondary high-energy charged particles (delta electrons) in the cavity and the medium, is ignored.

The dose to the cavity gas $D_{cav}$ is simply related to the ionization produced in the cavity by:

$$ D_{cav} = Q \left( \frac{W}{m} \right) _{cav} \tag{7.2} $$

Here, it may be noted that $W/e$ for air is 33.97 eV/ion pair, or 33.97 J/C.

**The Spencer-Attix cavity theory**

The Bragg-Gray cavity theory does not take into account the creation of secondary high-energy (delta) electrons generated as a result of the slowing down of the primary electrons via hard collisions in the sensitive volume of the dosimeter. The Spencer-Attix cavity theory is a more general formulation that accounts for the production of these electrons that themselves have sufficient energy to produce further ionisation. Some of these electrons released inside the cavity will have sufficient energy to escape from the cavity, carrying some of their energy with them. This reduces the energy absorbed in the cavity, and requires modification to the stopping power of the gas.

The Spencer-Attix cavity theory operates under the two Bragg-Gray conditions; however, these conditions now also apply to the secondary charged particle fluence, in addition to the primary charged particle fluence.

The secondary electron fluence in the Spencer-Attix theory is divided into two components based on the user-defined energy threshold $\Delta$. Secondary electrons with kinetic energies $E$ less than $\Delta$ are considered slow electrons that deposit their energy locally; secondary electrons with energies larger than or equal to $\Delta$ are considered fast (slowing down) electrons and are part of the electron slowing-down spectrum. Consequently, this spectrum has a low
energy threshold of $\Delta$ and a high energy threshold of $E_0$. Hence, energy deposition is calculated as the product of $L_\Delta(E)/\rho$ (the restricted mass collisional stopping power with threshold $\Delta$), and the fast electron fluence with electrons ranging in energy from $\Delta$ to $E_0$, $\Phi_{med,E}^\delta$, where the $\delta$ here indicates the inclusion of the contribution of the delta electrons in the slowing-down spectrum.

Because of the second Bragg-Gray condition which stipulates that there must not be electron production in the cavity, electrons with energy $\Delta$ must be capable of crossing the cavity. Hence, the threshold value $\Delta$ is related to the cavity size and is usually defined as the energy of an electron with range equal to the mean chord length across the cavity. (Usually for ion chamber problems, $\Delta$ is set at a nominal value of 10 keV.)

The Spencer-Attix relation between the dose to the medium $D_{med}$ and the dose in the cavity $D_{cav}$ is thus written as:

$$D_{med} = D_{cav} \left( \frac{L_\Delta}{\rho} \right)_{med}^{med}$$

where $\left( \frac{L_\Delta}{\rho} \right)_{cav}^{med}$ is the ratio of the mean restricted mass collisional stopping power of the medium to that the cavity. (This may be compared to Eq. 7.1, which is the Bragg-Gray relation.)

Using the electron fluence spectrum in the medium $\Phi_{med,E}^\delta(E)$, the full expression is:

$$\left( \frac{L_\Delta}{\rho} \right)_{cav}^{med} = \frac{\int_{\Delta}^{E_0} \Phi_{med,E}^\delta(E) \left( \frac{L_\Delta}{\rho} \right)_{med}^{med} dE + TE_{med}}{\int_{\Delta}^{E_0} \Phi_{med,E}^\delta(E) \left( \frac{L_\Delta}{\rho} \right)_{cav}^{cav} dE + TE_{cav}}$$

The terms $TE_{med}$ and $TE_{cav}$ are track-end terms and account for the energy deposited by electrons with kinetic energies between $\Delta$ and $2\Delta$.

The track-end electrons have an energy loss that brings their kinetic energy to lower than $\Delta$. Their residual energy after such events is therefore deposited on the spot, and these electrons are then removed from the spectrum. The track-end terms are approximated by Nahum as:

$$TE_{med} = \Phi_{med,E}^\delta(\Delta) \left( \frac{S_{col}(\Delta)}{\rho} \right)_{med}$$

and

$$TE_{cav} = \Phi_{med,E}^\delta(\Delta) \left( \frac{S_{col}(\Delta)}{\rho} \right)_{cav}$$
Note that the unrestricted collisional stopping powers are used here because the maximum energy transfer for an electron with energy less than $2\Delta$ is less than $\Delta$.

Monte Carlo calculations show that the difference between the Spencer-Attix and Bragg-Gray cavity theories is non-negligible, yet generally not very significant. Since collisional stopping powers for different media show similar trends as a function of particle energy, their ratio for two media is a very slowly varying function with energy. (For ionisation chambers in water, the energy dependence arises mainly from the difference in the density effect correction between the two materials, water and air.)

**Large cavity theory**

Bragg-Gray and Spencer-Attix cavity theories are applied to small cavities (such as ion chambers or other small dosimeters) in a medium. The fluence of charged particles in the cavity is largely unaffected by the presence of the cavity.

At the opposite extreme we have large cavities; a large cavity has dimensions such that the dose contribution from photon interactions outside the cavity may be ignored, compared with the contribution from electrons liberated by photon interactions inside the cavity.

Under these conditions there exists a condition of charged-particle equilibrium in the central regions of the cavity far from the walls. Given this, it is clear that (a) $D_{cav} = K_{coll}$ within the cavity (see Figure 6 and Eq. 5.1, 5.9), and that (b) $\Psi_{med} = \Psi_{cav}$.

In this case the ratio of dose between the medium and the cavity is clearly given simply by the ratio of the collisional kerma in the medium to that in the cavity, and from Eq. 5.5 is also equal to the ratio of the average mass-energy absorption coefficients,

$$\frac{D_{med}}{D_{cav}} = \frac{K_{coll,med}}{K_{coll,cav}} = \left( \frac{\bar{\mu}_{en}}{\rho} \right)_{med}$$

7.7

This is used in the derivation of air-kerma standards, where the cavities used are quite large (Figure 3 for example).

**8: Protection**

**Equivalent dose $H_T$**

$$H_T = \sum w_R D_{T,R}$$

8.1

Where $D_{T,R}$ is the absorbed dose (averaged over a tissue or organ $T$) due to radiations of type $R$ and $w_R$ is the radiation weighting factor. $D_{T,R}$ can not be measured experimentally.
Type and energy of radiation $R$ & Radiation weighting factor $W_R$

<table>
<thead>
<tr>
<th>Type and energy of radiation $R$</th>
<th>Radiation weighting factor $W_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photons, all energies</td>
<td>1</td>
</tr>
<tr>
<td>Electrons and muons, all energies</td>
<td>1</td>
</tr>
<tr>
<td>Neutrons</td>
<td></td>
</tr>
<tr>
<td>$&lt; 10$ keV</td>
<td>5</td>
</tr>
<tr>
<td>$10$ to $100$ keV</td>
<td>10</td>
</tr>
<tr>
<td>$&gt; 0.1$ to $2$ MeV</td>
<td>20</td>
</tr>
<tr>
<td>$&gt; 2$ to $20$ MeV</td>
<td>10</td>
</tr>
<tr>
<td>$&gt; 20$ MeV</td>
<td>5</td>
</tr>
<tr>
<td>Protons, other than recoil protons, $&gt; 2$ MeV</td>
<td>5</td>
</tr>
<tr>
<td>Alpha particles, fission fragments, heavy nuclei</td>
<td>20</td>
</tr>
</tbody>
</table>

Unit: J kg\(^{-1}\)

Special name for the unit of equivalent dose is sievert (Sv).

**Effective dose $E$**

\[ E = \sum_I w_I H_I = \sum_I w_I \sum_R w_R D_I,R \]

where $D_I,R$ is as above and $w_T$ is a tissue weighting factor which reflects the total detriment to health.

<table>
<thead>
<tr>
<th>Tissue or organ</th>
<th>Tissue weighting factor $w_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gonads</td>
<td>0.20</td>
</tr>
<tr>
<td>Bone marrow (red)</td>
<td>0.12</td>
</tr>
<tr>
<td>Colon</td>
<td>0.12</td>
</tr>
<tr>
<td>Lung</td>
<td>0.12</td>
</tr>
<tr>
<td>Stomach</td>
<td>0.12</td>
</tr>
<tr>
<td>Bladder</td>
<td>0.05</td>
</tr>
<tr>
<td>Breast</td>
<td>0.05</td>
</tr>
<tr>
<td>Liver</td>
<td>0.05</td>
</tr>
<tr>
<td>Oesophagus</td>
<td>0.05</td>
</tr>
<tr>
<td>Thyroid</td>
<td>0.05</td>
</tr>
<tr>
<td>Skin</td>
<td>0.01</td>
</tr>
<tr>
<td>Bone surface</td>
<td>0.01</td>
</tr>
<tr>
<td>Remainder</td>
<td>0.05</td>
</tr>
<tr>
<td>Whole body total</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Unit: J kg\(^{-1}\)

Special name for the unit of effective dose is sievert (Sv).

**Protection: Operational quantities**

For measurement purposes the operational quantities *ambient dose equivalent* and *directional dose equivalent*, are defined.
**Ambient dose equivalent** $H^*(d)$

The ambient dose equivalent $H^*(d)$, at a point, is the dose equivalent that would be produced by the corresponding expanded and aligned field, in the ICRU sphere at a depth $d$ in millimetres on the radius opposing the direction of the aligned field. For measurement of strongly penetrating radiations the reference depth used is $10$ mm and the quantity denoted $H^*(10)$

Unit: J kg$^{-1}$

Special name for the unit of ambient dose equivalent is sievert (Sv).

**Directional dose equivalent** $H'(d, \Omega)$

The directional dose equivalent $H'(d, \Omega)$, at a point, is the dose equivalent that would be produced by the corresponding expanded field in the ICRU sphere at a depth $d$ on a radius in a specified direction $\Omega$. Directional dose equivalent is of particular use in the assessment of dose to the skin or eye lens.

Unit: J kg$^{-1}$

Special name for the unit of directional dose equivalent is sievert (Sv).

**References**

2. ICRU Report 85a Fundamental Quantities and Units for Ionising Radiation 2011 (http://www.icru.org/)
5. National Physical Laboratory (NPL) http://www.npl.co.uk/ionising-radiation/dosimetry/
7. Health Protection Agency (HPA) HPA - Radiation
9. ICRU Report 52 1994 Particle Counting in Radioactivity Measurement
10. NPL Calorimetry: http://www.npl.co.uk/ionising-radiation/dosimetry/research/calorimetry
11. A R S Marsh, T T Williams. 50 kV Primary Standard of Exposure 1978 Design of Free-Air Chamber; NPL Report RS(EXT) 54; April 1982