Technical Supporting Document

Optical Atomic Clocks for Space

Patrick Gill
Helen Margolis
Anne Curtis
Hugh Klein
Stephen Lea
Stephen Webster
Peter Whibberley

Version 1.7
November 2008

ESTEC / Contract No. 21641/08/NL/PA
CONTENTS

1 INTRODUCTION .................................................................................................................. 1

2 COLLECTION OF USER REQUIREMENTS ............................................................................... 2
  2.1 SCIENCE (FUNDAMENTAL PHYSICS) .............................................................................. 2
  2.1.1 Tests of the Einstein Equivalence Principle ............................................................ 3
  2.1.2 Exploring large-scale gravity: the Pioneer anomaly ............................................... 9
  2.1.3 Tests of post-Newtonian gravity ........................................................................... 11
  2.2 EARTH OBSERVATION (GEOSCIENCE) ........................................................................ 12
  2.2.1 Determination of the Earth’s geoid ....................................................................... 13
  2.2.2 CHAMP, GRACE and GOCE mission capabilities ............................................... 15
  2.2.3 Optical clock possibilities for gravity sensing ......................................................... 16
  2.3 OPTICAL MASTER CLOCK IN SPACE ......................................................................... 18
  2.4 GLOBAL NAVIGATION SATELLITE SYSTEMS (GNSS) ................................................ 20
  2.4.1 GNSS optical clock simulations ........................................................................... 21
  2.5 SUMMARY OF USER REQUIREMENTS ......................................................................... 24

3 REVIEW OF OPTICAL ATOMIC CLOCK TECHNOLOGY .................................................... 26
  3.1 OPTICAL LOCAL OSCILLATORS .................................................................................. 27
  3.1.1 Principles of operation .......................................................................................... 27
  3.1.2 State-of-the-art performance ................................................................................ 28
  3.1.3 Limitations to performance .................................................................................. 29
  3.2 ATOMIC REFERENCE: TRAPPED ION OPTICAL CLOCKS ........................................ 33
  3.2.1 Principles of operation ....................................................................................... 33
  3.2.2 Candidate systems and current performance ....................................................... 35
  3.2.3 Systematic frequency shifts ................................................................................ 37
  3.2.4 Technology considerations .................................................................................. 43
  3.3 ATOMIC REFERENCE: NEUTRAL ATOM OPTICAL LATTICE CLOCKS ....................... 48
  3.3.1 Principles of operation .......................................................................................... 48
  3.3.2 Candidate systems and current performance ....................................................... 49
  3.3.3 Systematic frequency shifts ................................................................................ 50
  3.3.4 Technology considerations .................................................................................. 54
  3.4 OPTICAL FREQUENCY COMBS ................................................................................ 57
  3.4.1 Principles of operation .......................................................................................... 57
  3.4.2 Accuracy and stability tests .................................................................................. 59
  3.4.3 Technology considerations .................................................................................. 63
  3.5 OPTICAL FREQUENCY COMPARISON ....................................................................... 69
  3.5.1 Satellite frequency transfer techniques ................................................................ 69
  3.5.2 Frequency comparison via optical fibres .............................................................. 74

4 SELECTION OF OPTICAL ATOMIC CLOCK HARDWARE ................................................... 80
  4.1 TECHNOLOGY SELECTION ......................................................................................... 80
  4.2 TECHNOLOGY DEVELOPMENTS NECESSARY FOR SPACE OPERATION .................. 85
  4.2.1 Sub-unit technology developments ..................................................................... 86
  4.2.2 Overall technology development guidelines ....................................................... 93
  4.3 SUBSYSTEM INTEGRATION ....................................................................................... 93

5 ENGINEERING MODEL DESIGN AND PLANNING ........................................................... 95

6 INFRASTRUCTURE PREPARATION .................................................................................. 97
  6.1 FREQUENCY COMPARISON INFRASTRUCTURE ....................................................... 97
  6.1.1 Optical fibre requirements ................................................................................... 97
  6.1.2 Enhanced satellite frequency transfer techniques ................................................. 101
  6.2 INTEGRATION CENTRE ............................................................................................ 102

7 MANPOWER ..................................................................................................................... 103
  7.1 DETAILED DEVELOPMENT PLAN ............................................................................. 103
7.2 MANPOWER .................................................................................................................. 105

8 SUMMARY OF RECOMMENDATIONS ............................................................................. 108

9 REFERENCES.................................................................................................................. 110
EXECUTIVE SUMMARY

With the overall objective of supporting future missions in the areas of Science, Earth Observation and Navigation, the technology development programme presented in this document aims to achieve space deployment of an optical atomic clock by 2020.

Since the development of the first caesium atomic clock in 1955, microwave frequency standards and clocks have made an enormous contribution to scientific research and development with substantial benefits to society. A new generation of high performance optical atomic clocks is now emerging. The frequency stability of these optical clocks already exceeds that of the best caesium microwave clocks for all averaging times, and with further development they are expected to achieve fractional accuracies at the $10^{-18}$ level. This substantial jump in performance opens up new possibilities for space missions, with optical clocks offering major measurement advantages for future mission payloads by the provision of highly stable and accurate on-board frequency references.

Within the area of Science (Fundamental Physics), space-borne optical atomic clocks offer very significant opportunities to test the limits of our current understanding of the universe. They could be used to search for violations of the Einstein Equivalence Principle by making measurements of the gravitational redshift with unprecedented precision, or to discriminate clearly between different theories of gravity through measurements of the Shapiro time delay of highly stable optical signals to and from a spacecraft as they pass close by the sun at conjunction. They would also be a useful addition to missions designed to resolve the causes of the Pioneer anomaly. These types of mission scenario could potentially lead to a major scientific discovery: a breakdown of Einstein’s general theory of relativity or the observation of variations in fundamental constants could provide the first experimental evidence to support emerging theories aimed at unifying gravitation and quantum mechanics. The development programme outlined here thus has a vital role to play in supporting future Cosmic Vision mission proposals in the area of fundamental physics by increasing the technological readiness level of the optical clock technology.

Within the area of Earth Observation, space-borne optical atomic clocks would open up new possibilities in geodesy and remote atmospheric sensing. The sensitivity of highly stable and accurate optical clocks means that they can be used for direct measurement of the earth’s geoid (gravitational equipotential surface) at the few centimetre level with very high spatial resolution. Such measurements would have applications in a wide range of sectors, including civil engineering and construction, oil and gas exploration, ordinance survey and even global navigation. Long-term measurements of the variability of the local gravitational potential will also be possible, with important perspectives in monitoring seasonal and long-term trends in ice sheet masses, ocean current transport and overall ocean mass changes.

Within the area of Navigation, optical atomic clocks could find application in future evolutions of global satellite navigation systems. Using optical clocks in both the ground and the space segments of a satellite navigation system offers the possibility of up to two to three orders of magnitude improvement in satellite clock prediction accuracy. This would lead to far less reliance on regular updates of the clock parameters and hence provide much higher autonomy in the space segment of the system. With further progress in the understanding of atmospheric and multi-path effects, improved satellite clocks would also lead to improvements in position determination for users of the system.

Finally, an optical master clock in space could be realised in a mission scenario offering advantages to all the above areas. Variations in the local gravitational potential will have an increasing impact on ground-based optical clocks as their fractional uncertainties approach the $10^{-18}$ level. To take maximum advantage of this technology it will therefore be necessary to
operate them in spacecraft orbiting the Earth at relatively high altitudes where spatial and temporal variations of the Earth’s gravitational field are smoothed out. A future ultra-precise reference timescale based on space-borne optical clocks, combined with advanced inter-satellite and satellite-ground links, would have significant impact in a range of areas including calibration of international atomic timescales, geodesy and very precise navigation systems.

With NASA focussing its efforts on space and planetary exploration, there is an excellent opportunity for ESA to take the lead in exploiting optical clock technology for space applications. The key challenge to be tackled is the engineering development required to move from the laboratory systems of today to space-qualified instruments maintaining the highest levels of performance.

Our recommendation is that advanced portable prototypes of four different optical atomic clocks should be developed in parallel:

1. A trapped ion optical clock based on $^{88}\text{Sr}^+$;
2. A strontium atom optical lattice clock;
3. A quantum-logic-based trapped ion optical clock using $^{27}\text{Al}^+$;
4. A mercury atom optical lattice clock.

This multiple-clock development approach will allow for the highest performance specifications to be achieved during the prototyping phase, whilst at the same time undertaking the engineering developments necessary to prepare the clocks for space integration. It will lead to a number of different clock options with individual advantages of compactness, stability or accuracy, from which the best option for any particular mission scenario can be selected with a high degree of confidence and minimum technological risk. Some elements of the clock are common to all four systems, in particular the optical frequency comb and many aspects of the optical local oscillator. In parallel, it will be necessary to develop an improved frequency comparison infrastructure that does not compromise the stability or accuracy of the high performance optical atomic clocks. Both ground links and ground-satellite links must be addressed.

The proposed programme is a partnership between ESA member states, including the UK, Germany, France, Austria, Italy and Switzerland. Europe has several world-leading groups in optical frequency metrology, and our recommendation is that, together with specialised space-oriented time and frequency companies, they should take the lead in the advanced prototyping phase. However the need for early inputs on the space integration side will require consultancy and outline platform design from leading space integration companies as a precursor to the engineering model development phase. One of these space integrators should then take on the prime contractor role for this second phase.

With suitable investment, Europe will be able to accelerate the development of high performance optical clocks for applications ranging from tests of fundamental physical theories to geodesy and satellite-based navigation systems, and focus efforts on achieving the goal of a space-borne optical clock by 2020.
1 Introduction

In this document we address the potential opportunities for the development of space-borne optical atomic clocks for application in a variety of mission areas.

We begin with a survey of user requirements (section 2), drawn up in consultation with key stakeholders and user groups. Planned and proposed ESA missions and possible future augmentations are reviewed, with the focus on establishing the level of precision that would accrue from the use of optical clocks in missions within the areas of Science (Fundamental Physics), Earth Observation and Navigation.

To lay the foundations for the technology development plan that follows, we then review the current state of development of optical clock technology in section 3. The main subcomponents of an optical clock, namely the atomic reference, the optical local oscillator and the optical frequency comb are discussed in detail, together with methods for comparing remotely located optical clocks.

In section 4, we look at the match between user requirements and anticipated optical clock performance. The trade-off between expected system performance and experimental complexity for the different atomic reference options are considered, together with the associated implications for development timescales and costs. As a result of this analysis, and bearing in mind the technology that is common to all approaches, recommendations are made as to the number and type of advanced portable prototypes that should be developed in parallel in order that the best option for any particular mission scenario can be selected with confidence. Guidelines are provided for mass, volume and power requirements for each subsystem, and a plan for subsystem integration into advanced portable prototypes presented.

Guidelines on how to manage the transition from advanced portable prototypes to engineering models are presented in section 5, together with a high-level cost estimate for the engineering model development.

In section 6 we consider the extent to which existing European infrastructure is adequate to support the optical clock development programme and make recommendations for necessary enhancements.

A detailed plan for the development of advanced portable prototypes is presented in section 7. This identifies key partners together with the manpower, expertise and facilities they are able to bring to the programme. A schedule is given with key milestones against which progress may be monitored.

We conclude in section 8 with a summary of the key recommendations made in the report.
2 Collection of user requirements

Optical atomic clocks with fractional stabilities and accuracies at the $10^{-17}$ to $10^{-18}$ level will provide major benefits across a variety of future space missions. In this section we present a review of planned and proposed ESA missions and possible future augmentations, and identify those where high-performance optical atomic clock technology offers the potential to significantly enhance the scientific goals and measurement capability. This review draws on the outputs of a previous ESA-supported study [Gill 2008], in addition to consultation with ESA directorate personnel and other expert scientists in academia and industry.

The primary ESA directorates considered likely to benefit from optical atomic clock technology are Science (Fundamental Physics), Earth Observation and Navigation. The focus is therefore on establishing the level of precision that would accrue from the use of optical clocks in missions from these areas.

2.1 Science (Fundamental Physics)

Our current understanding of the fundamental laws of physics is described by the “Standard Model” of particle physics and Einstein’s general theory of relativity. General relativity, whilst it has been very successful to date in describing cosmological phenomena, is a classical theory and hence fundamentally incomplete because it does not include quantum effects. An extremely important goal in fundamental physics is therefore the development of a quantum theory of gravity that describes all particle interactions in a unified way (Figure 1).

A new theory of quantum gravity must be fundamentally different from standard general relativity and quantum theory, and must thus violate one or more of the principles underlying these theories. The status of experimental tests of general relativity and the theoretical frameworks for analysing them were reviewed recently by Will [Will 2006]. General relativistic effects are typically very small, and hence very challenging to measure with high precision. The space environment offers a number of advantages for such tests compared to the terrestrial environment, such as variable gravity potentials, large distances, high velocity and low acceleration regimes [Turyshev 2007]. Experiments based on space-borne optical clocks offer a very significant opportunity to search for manifestations of quantum gravity, as discussed in the following sections, and could potentially lead to a major discovery.

Figure 1. Theories of gravitation such as general relativity are fundamentally incomplete because they do not include quantum effects. A unified theory of interactions would combine the theory of gravity with the standard model of particle physics.
2.1.1 Tests of the Einstein Equivalence Principle

A key concept in the formulation of gravitational theory is the principle of equivalence (Figure 2). The Einstein equivalence principle (EEP) states that:

1. The trajectory of a freely falling “test” body (one not acted upon by forces such as electromagnetism and too small to be affected by tidal gravitational forces) is independent of its internal structure and composition. This is known as the weak equivalence principle (WEP). In the simplest case of dropping two different test bodies in a gravitational field, the WEP states that the bodies fall with the same acceleration. This is known as the Universality of Free Fall.

2. The outcome of any local non-gravitational experiment is independent of the velocity of the freely falling reference frame in which it is performed. This is termed Local Lorentz invariance (LLI).

3. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed. This is referred to as Local Position Invariance (LPI).

The EEP is the foundation of the idea that space-time is curved. The only theories of gravity that satisfy the EEP are metric theories of gravity. One such theory is general relativity (GR), but there are other examples.

Searches for violations of the EEP are important because they are predicted by emerging theories aimed at unifying gravitation and quantum mechanics. Optical clocks have a particularly significant role to play in tests of LPI, but can also be used for tests of LLI (Figure 2).

Figure 2. Foundations of the theory of gravity, showing the relation between general relativity and the Einstein equivalence principle, and indicating the experiments that can be carried out to test certain aspects of the theory. Experiments that can be performed using optical clocks are highlighted in red.
2.1.1.1 Tests of Local Position Invariance

One important consequence of the principle of LPI is the universal redshift of clocks when subject to a gravitational potential. The principle can therefore be tested by gravitational redshift experiments.

**Absolute gravitational redshift experiments**

An absolute gravitational redshift experiment measures the frequency shift $Z$ between two identical frequency standards (clocks) placed at rest at different heights in a static gravitational field. Clocks run more slowly the closer they are to a massive body. If LPI is valid, then the structure of the clock plays no role and the result is a frequency shift

$$Z = \frac{\Delta \nu}{\nu} = \frac{\Delta U}{c^2},$$

(1)

where $\Delta U$ is the difference in the Newtonian gravitational potential between the two clocks and $c$ is the speed of light in vacuum. If LPI were not valid, then the formula for the shift would be modified to

$$Z = (1 + \beta) \frac{\Delta U}{c^2},$$

(2)

where the parameter $\beta$ would depend on the internal structure of the particular clock whose shift is being measured.

The most accurate measurement of the gravitational redshift was performed in 1976 by the Gravity Probe A experiment [Vessot 1980]. This compared the frequency of two hydrogen masers, one on the ground and one on board a spacecraft launched nearly vertically upwards to an altitude of 10,000 km. By observing the frequency variations at a level of precision consistent with the maser stability ($\sim 1 \times 10^{-14}$ for 100 s averaging time), a limit of $|\beta| < 7 \times 10^{-5}$ was deduced.

An improvement by a factor of 35 is expected from the ACES (Atomic Clock Ensemble in Space) mission, which is planned for launch in 2014 [Cacciapuoti 2007]. ACES will install a cold atom caesium clock (PHARAO) and a space hydrogen maser (SHM) on board the International Space Station (ISS). Rather than modulating the redshift by changing the altitude of the satellite as in the Gravity Probe A experiment (a measurement approach that relies on clock stability), ACES will use the high ($10^{-16}$) accuracy of PHARAO and of ground-based clocks ($10^{-16}$ or better) to make an absolute measurement of the frequency difference. For the ISS orbit, the gravitational redshift $Z$ is approximately $4.5 \times 10^{-11}$ (Figure 3) and so with clock accuracy of $10^{-16}$ a measurement precision of $2 \times 10^{-6}$ could be achieved.

Optical clocks could give very significant further gains in precision. The accuracy achievable depends both on the accuracy of the clocks used and on the gravitational potential difference $\Delta U$ between them.
Figure 3. Solar gravitational redshift as a function of distance from the sun. Also shown at 1 AU are the gravitational redshift changes experienced in the Gravity Probe A, ACES and EGE missions (blue circles), where the dominant contribution to the redshift comes from the earth’s gravitational potential, as well as the variation in redshift experienced on the surface of the earth due to its daily rotation and orbit around the Sun (red circles). (Figure adapted from [Prestage 2004].)

One proposed mission whose primary task is to measure the gravitational redshift with unprecedented precision is the Einstein Gravity Explorer (EGE) mission [Schiller 2007]. Several different measurement approaches are proposed within this mission, which uses a highly elliptical Earth orbit (Figure 4). The first is to make repeated measurements of the satellite clock frequency variation between apogee and perigee by comparison with a ground clock. The ground-satellite clock comparison would be carried out using an enhanced version of the microwave link developed for the ACES mission. The large variation in $\Delta U/c^2$ of approximately $4 \times 10^{-10}$ (Figure 3) enhances the sensitivity of the test, which relies on the high stability of the optical clock on board the satellite on the timescale of the visibility of the clock at perigee (approximately 1000 s) and at apogee (approximately 10 000 s). Systematic shifts, as long as they are uncorrelated with the orbital motion, are expected to average out, giving a sensitivity gain of up to $N^{1/2}$, where $N$ is the number of orbits. For an optical clock stability of $3 \times 10^{-16}$ at 1000 s (already achieved in the laboratory) and $N = 1000$, a measurement of the gravitational redshift with $2.5 \times 10^{-8}$ fractional uncertainty would be achievable. With anticipated future improvements in optical clock stability, a reduced uncertainty of $8 \times 10^{-9}$ could be achieved, limited by the projected link stability of $10^{-16}$ at 1000 s. Improvements beyond this level would require improved ground–satellite frequency transfer techniques. The second measurement approach proposed for the EGE mission is to make an absolute measurement of the frequency difference between the ground and the satellite clocks when the satellite is at apogee. Here the terrestrial gravitational potential
difference is maximum ($\Delta U/c^2 \sim 6.5 \times 10^{-10}$) and the link inaccuracy is smaller because of the longer averaging time, although the measurement does not take advantage of the repetitive nature of the orbit. However this measurement approach is complementary to the first method, because it relies on the accuracy rather than the stability of the optical clock, and for a clock accuracy of $2 \times 10^{-17}$ it is expected to provide a measurement of the gravitational redshift with $3 \times 10^{-8}$ relative uncertainty. Once again this uncertainty will reduce as the accuracy of optical clocks improves, up to the point where the link inaccuracy becomes the limit.

In both the above approaches, the dominant contribution to the redshift is from the earth’s gravitational potential. A third approach, which instead addresses the solar gravitational frequency shift, is to compare terrestrial clocks with a satellite clock in a geostationary orbit. The gravitational potential difference between a geostationary satellite and the earth’s surface is modulated with solar day period and amplitude with a peak-to-peak variation of $\Delta U/c^2$ of about $5 \times 10^{-12}$ (Figure 3). Although this is significantly lower than for a highly elliptical orbit, it is approximately six times higher than the daily modulation for the comparison of ground-based clocks via a satellite transponder, and the measurement is less affected by potential variations induced by geophysical effects. The terrestrial–space clock comparison could also be technically simpler for a geostationary orbit as compared to a highly elliptic orbit, thus avoiding potential systematic errors in the comparison.

The SAGAS (Search for Anomalous Gravitation using Atomic Sensors) mission proposal, which is directed at flying highly sensitive atomic sensors (an optical clock and a cold atom accelerometer) on a Solar System escape trajectory, offers the prospect of a very high accuracy measurement of the solar gravitational redshift [Wolf 2007]. For this trajectory, the change in gravitational potential is $\Delta U/c^2 \sim 10^{-8}$ (Figure 3), with the dominant contribution arising in the first few years after launch. For an optical clock of $10^{-17}$ accuracy, the gravitational redshift can therefore be tested with $10^{-9}$ fractional uncertainty. An optical link is proposed for ranging, frequency comparison and communication. As before, the uncertainty in the gravitational redshift measurement will reduce as the accuracy of optical clocks improves, assuming that the link to ground is sufficiently accurate.

![Figure 4. General concept of the EGE mission, reproduced from [Schiller 2007].](image-url)
“Null” redshift tests

A “null” redshift experiment tests whether the relative rates of two different clocks depends on position. Local position invariance implies that the frequencies $\nu_A$ and $\nu_B$ of two atomic clocks of different structure should suffer identical redshifts as they move together through a changing gravitational potential $\Delta U$. This is termed the universality of the gravitational redshift (UGR). If LPI were violated, the change in the fractional frequency ratio between the clocks would be

$$\frac{\Delta (\nu_A / \nu_B)}{(\nu_A / \nu_B)} = (\beta_A - \beta_B) \frac{\Delta U}{c^2}. \quad (3)$$

Such a violation may be described phenomenologically by a dependence of one or more fundamental constants on the gravitational potential. Such dependence is predicted by a number of theories aiming to unify gravitation with other interactions and is usually associated with the effect of massless (or very light) scalar fields [Flambaum 2007].

For ground-based clocks the dominant contribution to changes in the ambient gravitational potential is due to the annual elliptical orbit of the Earth about the Sun. This leads to variations in $U/c^2$ of $3.3 \times 10^{-10}$ (Figure 3). Current bounds on $|\beta_A - \beta_B|$ are at the few parts in $10^6$ level. For example, a seven year comparison of caesium fountain primary frequency standards and hydrogen masers showed that $|\beta_A - \beta_B| < 1.4 \times 10^{-6}$ [Ashby 2007] whilst an analysis of the frequency ratio between the $^{199}\text{Hg}$ optical clock and a caesium fountain primary frequency standard over a period of 6 years showed that $|\beta_{199\text{Hg}} - \beta_{\text{Cs}}| < 3.5 \times 10^{-6}$ [Fortier 2007].

Although some improvement in these limits can be anticipated as ground-based clocks improve in accuracy, much more sensitive tests can be carried out in space because of the larger values of $\Delta U/c^2$ that can be achieved (Figure 3). Null redshift experiments are technically simpler than absolute gravitational redshift measurements because there is no need for a highly accurate knowledge of the gravitational potential along the orbit and the measurement does not depend on a link to ground clocks.

For this type of mission the sensitivity of the clock transition frequency to changes in the fundamental constants is important. Comparisons between optical clocks depend mainly on the fine structure constant $\alpha$. Relativistic many-body calculations have been performed for the transitions of interest as optical frequency standards in a number of atomic species [Dzuba 1999, Dzuba 2003, Angstmann 2004], using a range of values for $\alpha$ to determine their sensitivity to changes in this fundamental constant. Defining a sensitivity coefficient $A$ such that a fractional change $\delta \alpha$ leads to a fractional shift $A(\delta \alpha \alpha)$ in the frequency of the clock transition, the results are shown in Table 1. For a null redshift test, the most sensitive test would be obtained by comparing two species with very different sensitivities to changes in $\alpha$.

If we consider for example a mission to the outer solar system (such as SAGAS) the change in $U/c^2$ is approximately $10^{-8}$. With clocks of $10^{-18}$ fractional accuracy, a LPI test at a level of $10^{-10}$ could therefore be performed. Similar accuracy could be achieved on a mission to Mercury, because the change in gravitational potential is similar. Even more stringent tests could be performed with a clock ensemble on a single close fly-by of the sun, such as in the SpaceTime mission proposal [Maleki 2004]. With a minimum distance of 6 solar radii, a maximum value of $U/c^2$ of $3.5 \times 10^{-7}$ was to be attained in this experiment (Figure 3). Replacing the original microwave clocks with optical clocks of $10^{-18}$ fractional accuracy would thus enable a LPI test at a level of $3 \times 10^{-12}$ to be performed.
Table 1. Sensitivity factor $A$ to variation in $\alpha$ of the clock transition frequency in a number of different optical frequency standards. (Note that the calculations are not isotope-specific.)

<table>
<thead>
<tr>
<th>Ion or atom</th>
<th>Clock transition</th>
<th>$A$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr$^+$</td>
<td>$^2S_{1/2} - ^4D_{5/2}$</td>
<td>0.43</td>
<td>[Dzuba 1999]</td>
</tr>
<tr>
<td>Yb$^+$</td>
<td>$^2S_{1/2} - ^4D_{3/2}$</td>
<td>0.88</td>
<td>[Dzuba 2003]</td>
</tr>
<tr>
<td>Yb$^+$</td>
<td>$^2S_{1/2} - ^4F_{7/2}$</td>
<td>−5.30</td>
<td>[Dzuba 2003]</td>
</tr>
<tr>
<td>Hg$^+$</td>
<td>$^2S_{1/2} - ^4D_{5/2}$</td>
<td>−3.19</td>
<td>[Dzuba 1999]</td>
</tr>
<tr>
<td>In$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>0.18</td>
<td>[Angstmann 2004]</td>
</tr>
<tr>
<td>Al$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>0.008</td>
<td>[Angstmann 2004]</td>
</tr>
<tr>
<td>Ca$^+$</td>
<td>$^1S_0 - ^3P_1$</td>
<td>0.02</td>
<td>[Angstmann 2004]</td>
</tr>
<tr>
<td>Sr$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>0.06</td>
<td>[Angstmann 2004]</td>
</tr>
<tr>
<td>Yb$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>0.31</td>
<td>[Angstmann 2004]</td>
</tr>
<tr>
<td>Hg$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>0.81</td>
<td>[Angstmann 2004]</td>
</tr>
</tbody>
</table>

2.1.1.2 Tests of Local Lorentz Invariance

Space missions including optical clocks in their payloads also offer a number of possibilities for testing special relativity, and hence local Lorentz invariance.

**Ives-Stilwell experiments**

The frequency difference between a space-borne clock and a terrestrial clock will in general contain a term that depends on the coordinate velocities of the two clocks in addition to the gravitational redshift. A precise measurement of this term amounts to a test of time dilation (Ives-Stilwell experiment). Considering the SAGAS mission as an example, this term is about $4 \times 10^{-9}$ towards the end of the mission duration, and hence with the proposed clock accuracy of $10^{-17}$ could be measured with a relative uncertainty of approximately $3 \times 10^{-9}$ [Wolf 2007]. The best current limit set by this type of experiment is $2.2 \times 10^{-7}$ [Saathoff 2003] and so this would represent an improvement by approximately a factor of 70.

**Kennedy-Thorndike experiments**

Space-borne optical clocks can also be used to test the independence of the speed of light on the velocity of the laboratory (Kennedy-Thorndike experiment) by comparing the frequency of a stable optical resonator with the frequency of an independent optical frequency standard, a technique introduced by Hils and Hall [Hils 1990].

According to the Mansouri-Sexl test theory, a possible dependence of the speed of light $c$ on the magnitude $v$ of the laboratory velocity relative to a hypothetical preferred reference frame $\Sigma$ and on the angle $\theta$ between the propagation direction of the light and the direction of $v$ can be parameterized as

$$\frac{c(v, \theta)}{c_0} = 1 + A \frac{v^2}{c_0^2} + B \frac{v^2}{c_0^2} \sin^2 \theta$$

where $c_0$ is the constant speed of light in the preferred frame $\Sigma$. The natural candidate for $\Sigma$ is the cosmic microwave background. If local Lorentz invariance holds then $A$ and $B$ are zero.

8
For a terrestrial experiment the laboratory velocity $v(t)$ is modulated daily by the rotation of the Earth about its axis (amplitude $\sim 300 \, \text{m s}^{-1}$, depending on latitude) and annually by the Earth’s orbital motion around the Sun (amplitude $\sim 30 \, \text{km s}^{-1}$). Since the resonance frequency of the optical cavity is proportional to $c(v)$, the frequency difference between the frequency standard and the cavity will be modulated in a corresponding way if Lorentz invariance is violated. Stringent limits on $B$ have been set by Michelson-Morley experiments, with the most precise leading to a limit of $|B| < 8.1 \times 10^{-11}$ [Stanwix 2006], so that Kennedy-Thorndike experiments test mainly the parameter $A$. The most sensitive terrestrial experiment to date set a limit of $|A| < 2.1 \times 10^{-5}$ [Braxmaier 2002].

Space experiments have the advantage of high orbital velocity and strongly reduced cavity deformation in the microgravity environment [Lämmerzahl 2001, Lämmerzahl 2004]. For example, for the elliptic orbit of the proposed EGE mission, the velocity $v$ of the laboratory varies between $+4 \, \text{km s}^{-1}$ and $-4 \, \text{km s}^{-1}$ over approximately one hour. This variation is an order of magnitude larger than the change of velocity of the Earth’s surface. The variation also occurs over a shorter timescale, which is beneficial for this type of experiment because the drift of the optical resonator is more predictable over shorter timescales. With averaging over a number of orbits, an improvement of a factor of 20 compared to the best terrestrial experiments is expected [Schiller 2007]. More generally, in any mission whose payload involves an optical clock, the difference between the atomic reference frequency and the frequency of the cavity used in the optical local oscillator will be monitored continuously and can therefore be analysed within this framework for violations of Lorentz invariance.

### 2.1.2 Exploring large-scale gravity: the Pioneer anomaly

Although experimental tests of gravity have so far shown good agreement with General Relativity over scales ranging from millimetres (laboratory experiments) to the size of planetary orbits (space experiments), most attempts to develop a quantum theory of gravity predict modifications at smaller and/or larger length scales.

There are already several phenomena that cannot be explained within the standard theory of general relativity [Lämmerzahl 2006]. One is the anomaly observed in the rotation curves of galaxies, where the outermost stars rotate faster than would be expected based on the gravitational field given by the visible stars in the centre of the galaxy. This phenomenon is usually ascribed to dark matter, the nature of which remains unknown, but could also be a consequence of a modification of general relativity at galactic or cosmological scales. A second anomaly appears in two-way Doppler tracking data from the Pioneer 10 and 11 spacecraft at distances greater than 20 astronomical units (AU) from the Sun. Analysis of this data indicates the presence of a small, anomalous, Doppler frequency drift (Figure 5), which can be interpreted as a constant excess acceleration towards the sun of $a_{PA} = 8.5 \times 10^{-10} \, \text{ms}^{-2}$ [Anderson 1998]. This signal is known as the Pioneer anomaly and its origins currently remain unexplained. Further tests of the laws of gravity at the largest possible distances are therefore extremely important.
Optical clocks have been proposed as a useful addition to future missions designed to resolve the causes of the Pioneer anomaly [Dittus 2006], with the scheme being to compare a stable clock on board the spacecraft with a reference clock located on earth. The clock comparison measures the difference in gravitational potential and thus probes the gravitational field at the satellite location. A Pioneer anomaly mission would be flown on a Solar System escape orbit to probe large distance scales at greater than 20 AU from the Sun, and hence the on-board clock must exhibit extremely good long term stability over timescales of 10 years. If the anomalous acceleration arises from a gravitational potential $U_{PA}$, then $dU_{PA}/dr = -a_{PA}$, and the anomalous satellite clock frequency shift as it moves between distances $r_1$ and $r_2$ from the Sun is given by

$$\frac{\Delta \nu}{\nu} = \frac{U_{PA}(r_1) - U_{PA}(r_2)}{c^2} = \frac{1}{c^2} \int_{r_1}^{r_2} dU_{PA} = \frac{1}{c^2} \int_{r_1}^{r_2} -a_{PA} dr = -\frac{a_{PA}(r_2 - r_1)}{c^2}. \quad (5)$$

For a travel distance of about 70 AU, this fractional frequency shift is $-1 \times 10^{-13}$. An optical clock would be able to measure anomalous accelerations significantly below $a_{PA}$ and would therefore be a good choice for such a mission.

One such mission that aims to study the Pioneer anomaly is SAGAS, where the payload includes not only an optical clock but also an absolute accelerometer based on atom interferometry and a laser link for ranging, frequency comparison and communications. This integrated package of complementary instruments provides a highly versatile payload that will allow the Pioneer anomaly to be explored through the different effects of gravity on clocks, light and the free fall of test bodies, and hence to discriminate between the different hypotheses that have been put forward to explain the anomaly [Wolf 2007].
2.1.3 Tests of post-Newtonian gravity

In the weak-field, low-velocity limit, which is sufficiently accurate to describe most solar system tests that can be performed in the foreseeable future, theories of gravity are classified using the parameterised post-Newtonian (PPN) formalism [Will 2006]. In the most general form, ten different parameters are required to characterise a metric theory. However in the simplest case, in which it is assumed that the universe is isotropic and that conservation laws for total momentum and angular momentum are satisfied, this reduces to the two Eddington-Robertson-Schiff parameters, \( \beta \) and \( \gamma \), both of which have the value of unity in general relativity (GR). Accurate measurements of these parameters can thus help to distinguish GR from other metric theories of gravity.

Optical clocks are mainly of interest for improved measurements of the parameter \( \gamma \) which characterizes the amount of space-time curvature produced by unit rest mass. This parameter was first determined by measuring the apparent change in the angular position of nearly occulted stars during the 1919 solar eclipse [Dyson 1920]. However more accurate studies have been performed with interplanetary spacecraft by measuring the time delay (Shapiro delay) of radio signals to and from the spacecraft as they pass close by the sun at conjunction. The gravitational field of the Sun causes an increase \( \Delta t \) in the time taken for electromagnetic radiation to travel the round trip between the ground antenna and the spacecraft, given by

\[
\Delta t = 2(1 + \gamma) \frac{GM_{\odot}}{c^2} \ln \left( \frac{4r_1r_2}{b^2} \right),
\]

where \( G \) is the Newtonian constant of gravitation, \( M_{\odot} \) is the mass of the sun, \( c \) is the speed of light, \( r_1 \) and \( r_2 \) are the respective distances of the ground antenna and the spacecraft from the Sun, and \( b \) is the impact parameter. The most accurate experiments to date do not measure this delay directly but rather measure the fractional frequency shift \( \Delta \nu/\nu \) of the radiation, which is the time derivative of the relativistic delay \( \Delta t \):

\[
\frac{\Delta \nu}{\nu} = \frac{\mathrm{d}(\Delta t)}{\mathrm{d}t} = -4(1 + \gamma) \frac{GM_{\odot}}{c^3b} \frac{\mathrm{d}b}{\mathrm{d}t}.
\]

For a spacecraft much further away from the Sun than the Earth, \( \mathrm{d}b/\mathrm{d}t \) is approximately equal to the orbital velocity of the Earth (30 km s\(^{-1}\)) and at grazing incidence \( b = 7 \times 10^8 \) m. The maximum size of this effect for a two-way signal is therefore \( 1.7 \times 10^{-9} \) and for a one-way signal it is half this.

The most stringent limit on \( \gamma \) to date comes from Doppler ranging to the Cassini mission during solar occultation, and yields a result \( \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5} \), in agreement with the predictions of GR [Bertotti 2003]. The Mercury Orbiter Radioscience Experiment (MORE) on BepiColombo, which is scheduled for launch in 2013, aims to measure \( \gamma \) with ten times higher accuracy [BepiColombo 2000], but optical clocks offer the prospect of substantial further improvements.

One example is the proposed SAGAS mission, whose payload includes a trapped ion optical clock with \( 10^{-17} \) fractional accuracy. This would carry out measurements similar to the Cassini mission, during one or several solar conjunctions (depending on detailed trajectory) but with significantly improved sensitivity and at optical rather than radio frequencies, which significantly minimises effects from the solar corona and the Earth’s ionosphere. Assuming only one occultation over the complete mission duration, a somewhat pessimistic estimate of the uncertainty achievable suggests that SAGAS will be able to measure \( \gamma \) with a relative
uncertainty of about $10^{-7}$. However it is more likely that uncertainty in the $10^{-8}$ to $10^{-9}$ region will be achievable by modelling the non-gravitational accelerations of the spacecraft over the short timescales of the occultation [Wolf 2007].

ASTROD I (Astrodynamical Space Test of Relativity using Optical Devices I) is another mission concept that uses lasers rather than radio waves for ranging a spacecraft close to occultation [Appourchaux 2008]. This single spacecraft mission aims to determine $\gamma$ with a relative uncertainty of about $10^{-7}$, and uses a caesium microwave clock to provide timing information for two-way pulse laser ranging between Earth and the spacecraft. However a second mission, ASTROD II is envisaged as a three-spacecraft mission that would test $\gamma$ to $10^{-9}$, and for this the much higher stability and accuracy of an optical clock would be required.

Another interesting proposal aims to measure the gravitational time delay between two drag-free spacecraft [Ashby 2008]. This mission consists of two dedicated spacecraft, one near the Lagrange point L1, and another that passes behind the Sun three times during the mission lifetime. The proposal is for the ranging to be done using a laser that is modulated at a microwave frequency provided by an ultrastable clock on board the L1 spacecraft. The second spacecraft would contain a laser transponder to detect the incoming laser modulation and send it back to the L1 spacecraft, where its phase would be measured in order to determine the time delay. Using a cold atom microwave clock of the PHARAO type, the estimated uncertainty in $\gamma$ would be $10^{-8}$. However there is clear potential to further improve the precision using an optical clock and optical time delay measurements.

The parameter $\gamma$ is affected by most types of modifications of GR, with some of these theories predicting deviations from unity (the GR value) in the $10^{-7}$ to $10^{-5}$ range [Damour 2002a, Damour 2002b]. Its measurement to the accuracy promised by optical clocks would therefore make it possible to discriminate clearly between different theories of gravity.

2.2 **Earth observation (geoscience)**

As discussed in the previous section, it is clear that the space environment offers a number of advantages for high precision tests of fundamental physical theories through the availability of reduced seismic noise, large gravitational field changes, large distances and high velocities. It also provides an excellent medium for Earth observation and geoscience, offering opportunities to examine and characterize atmospheric and climate properties, surface topologies and vegetation, and Earth gravity changes and plate tectonics on the global scale. A relatively straightforward example of an earlier ESA mission proposal is WALES (Water vapour lidar experiment in space), which sought to examine from space the water vapour content in the atmosphere. This type of idea led on to the A-SCOPE (Advanced Space Carbon and Climate Observation of Planet Earth) proposal for monitoring carbon dioxide concentrations by satellite. This remote atmospheric sensing by satellite relies primarily on laser induced differential absorption measurement (LIDAR) by monitoring reflection signals on and off particular absorption species such as water or carbon dioxide. The use of filtered optical frequency combs to accurately monitor the absorption profile is one idea that has been suggested for this task [Gill 2008]. In addition to atmospheric absorption studies, and arguably of greater significance, is a larger amount of geoscience and technology that derives from Earth gravity effects. This includes the dynamics of the planetary interior (plate tectonics, earthquakes and oil stratification), the dynamics of water flow on and atmospheric currents above the Earth’s surface (see, for example, [Zlotnicki 2008]), and the orbitology of satellite systems.
2.2.1 Determination of the Earth’s geoid

The acceleration due to gravity is not constant over the surface of the Earth, varying by several tenths of a percent. This is primarily due to the rotation of the Earth causing a flattening at the poles with a resulting increase in gravity there compared to the equator. Additional local variation is caused by the influence of high mountain ranges and deep ocean trenches, and of non-uniform mass and material distributions within the Earth’s mantle such as rock, oil and water deposits. These influences also change with time, for example through earthquakes, volcanic activity, tectonic plate movement and ice-sheet movements. In order to take account of these spatial and temporal variations in gravity field, the concept of the geoid has been introduced to define a surface of equal gravitational potential, which corresponds to an ocean surface at rest, and is critical for example in indicating the direction of water flow. Thus the geoid can provide a means to monitor sea-level changes on a global scale and provide an absolute global height reference system.

2.2.1.1 Oceanography

Oceanography has a direct effect on global climate dynamics, through the oceans’ heat capacity, convection and evaporative interchange with the atmosphere, combined with direct solar radiation effects. Understanding of these processes directly affects the precision of predictive weather forecasting on the global scale. Whilst satellite altimetry allows the measurement of sea-surface height with high spatial resolution over global distances, providing direct input into ocean climate models, it does not provide an absolute measurement of the ocean dynamical topography and resultant circulation. This requires a reference to the ocean mass at rest as typified by the geoid. Uncertainties in the geoid value on the metre scale limit the precision with which climate circulation models can be applied. This is significant for several reasons. Firstly, it is desirable to be able to correlate actual and theoretical oceanographic data at a sufficiently small spatial resolution by combining both altimetric and gravitational data, in order to better understand oceanic mass and heat transport properties in the presence of local eddy current perturbations. This in turn will lead to a better understanding of the influence on ocean circulation of jet streams and weather fronts at high spatial resolution, as well as heat transport budgets and uncertainties. Finally, dynamic altimetric – geoid difference data with sufficient spatial resolution will also provide insights into the variation of ocean transport as a function of depth.

2.2.1.2 Geodesy

Geodesy is primarily concerned with the mapping of the Earth’s shape, both globally and locally. It provides a data set that feeds directly into many different branches of Earth science, civil engineering and construction, oil and gas exploration, ordnance survey, and even global navigation. In this latter respect, the determination of position on the earth’s surface is made on the basis of a geometric ellipsoidal reference, which in effect has no absolute reference frame. To define a gravitational potential surface or mean surface level (for example identifying the unambiguous direction of water flow) that can be applied across the globe, reference to the geoid is necessary. This geoid will be subject to variations over time and spatial location due to the changing gravitational potential, with a consequent need for routine mapping of the geoid on a regular basis as well as with increased accuracy and improved spatial resolution. Seasonal variations in geoid levels of approximately 10 mm are not uncommon [Tapley 2004], with fluctuations of up to 20 cm in some cases [Kleppner 2006]. Typical uncertainties for geoid determination, via extended periods of combined terrestrial gravity and satellite mapping over many years, are at the 30–50 cm level [Svehla 2005]. The difference between the geoid and ellipsoid can be as large as several tens of metres in mountainous regions due to the granularity of the mass distributions there (Figure 6).
Unified height systems

Traditional methodologies for geoid determination make use of a combination of geodetic levelling (by ground-based triangulation and survey) and gravimetry. In developed countries, this is achieved by reference to a series of gravity points with measured high accuracy values, and use of subsidiary levelling survey. Whilst this can provide local geoid height information at the sub-cm level in some cases, it is a laborious process, requiring long and inefficient measurement survey times, and is targeted on specific areas. Additionally, there is often little correspondence between data maintained within different countries, especially where they are located on landmasses separated by seas or oceans. Even within Europe, biases of ~ 50 cm between national height reference systems are not uncommon. Applications where this can be significant include pipeline control across continents, and multi-km tunnels and bridges such as that between Sweden and Denmark.

Survey and levelling with GNSS

A specific case of height determination is concerned with levelling for civil engineering purposes, particularly for projects involving water flow. Accuracies in the sub-centimetre range, and distributed over baselines up to several kilometres in some cases, remain the target for civil engineering applications. The ability to use GNSS-generated height data for this could offer a very significant improvement in levelling and survey time, provided the data is accurate enough. Here, the key requirement is to be able to relate GNSS height data, which is referenced to the GNSS-adopted ellipsoid, to the geoid. As already discussed, this requires accurate knowledge of the difference between the geoid and the ellipsoid. GNSS receivers for field use capable of this level of accuracy are not available as yet. However, the ability to convert from GNSS height data to levelled height data by incorporating the ellipsoid – geoid difference on baselines that are short enough to be relevant to the particular project remains a highly desirable goal on grounds of increased efficiency and reduced cost. The extent of the contributions to these targets that should be met by geoid determinations following the forthcoming GOCE mission is discussed in section 2.2.2, while further opportunities for optical clock-based measurements are described in section 2.2.3.
**Inertial navigation and orbit determination**

Inertial navigation relies on the use of accelerometers and gyroscopes on board the vehicle or moving platform to monitor acceleration and orientation respectively, with double integration of the measured acceleration leading to position data. These devices are commonplace in aircraft, missiles, submarines and some land-based vehicles as well as in oil drilling and pipeline control and maintenance. One difficulty, however, is that the measured acceleration consists of contributions from both vehicle and gravitational acceleration. The latter contribution tends to be computed using an approximated ellipsoidal gravity field that is devoid of local field knowledge. With the advantage of geoid determinations complete with local field perturbations added in with high spatial resolution, a more accurate sensing of vehicle acceleration data can be achieved, with a corresponding increase in positional accuracy.

This type of approach can also contribute to improved orbit determination of satellites. Here a variety of perturbing accelerations can arise, including those due to the gravity field, atmospheric drag, solar radiation, ocean tides and Earth plate movement. This is particularly true of low Earth orbits, such as those used for altimetric studies (e.g. GEOSAT, Envisat). Improved geoid measurements with higher spatial resolution can help to reduce gravity-based radial orbit errors for these altimetric satellites, and also contribute to the reduction of satellite flight and orthogonal direction errors, leading to better orbit prediction in close-to-real time processing. This is especially useful for oceanographic and atmospheric studies using satellites with on-board GNSS receivers.

### 2.2.2 CHAMP, GRACE and GOCE mission capabilities

To date, there have been two Earth gravity monitoring space missions: the German CHAMP (Challenging Minisatellite Payload) mission, launched in 2000, and the NASA GRACE (Gravity Recovery and Climate Experiment) mission, launched in 2002. The latest gravity field mapping mission GOCE (Gravity field and steady-state Ocean Circulation Explorer) is due for launch in September 2008. The planned synergies between these three missions involve a sequential improvement in our knowledge of the Earth’s gravity field.

The CHAMP mission [Reigber 2000] comprised a 3-axis accelerometer together with a GPS receiver for satellite-to-satellite tracking, together with a sensitive fluxgate magnetometer for magnetic field tracking. The GPS receiver enabled orbit tracking of the CHAMP spacecraft to 1 mm accuracy, leading to a factor of 100 improvement in long-wavelength gravity field measurements. This served as a starting point for the GRACE mission data analysis. The GRACE concept [Tapley 2004] centred on two spacecraft, one following the other at a separation of approximately 220 km and at an altitude of a few hundred kilometres. With GPS receivers and 3-axis accelerometers to provide positional referencing, an additional microwave ranging system allowed for the accurate monitoring of changes in the inter-satellite separation. These changes in separation result from the differential gravity field experienced by the leading craft relative to the following one, and allow improvements in medium wavelength gravity field mapping sensitivity by a factor of ten to a hundred.

Data from the GRACE mission provides the basis for the GOCE mission to improve gravity field measurement accuracy on the high resolution short-wavelength scale. The single GOCE spacecraft will orbit at an altitude of approximately 250 km, and houses a 3-axis gradiometer comprising 3 pairs of 3-axis accelerometers with each pair separated by 0.5 m, a geodetic-quality GPS receiver, and a laser retro-reflector to allow ground-based laser tracking. The primary mission objectives are to determine gravity field anomalies to an accuracy of $10^{-5}$ ms$^{-2}$, to determine the geoid to an accuracy of 1–2 cm, and to achieve these goals with a spatial resolution of better than 100 km. Data gathering over 18 months is planned. The
specific GOCE mission targets in respect of time-averaged (static) gravity field for the different application areas are itemised in Table 2 [Koop 2008, Rummel 2005], where the Earth strata applications are sub-divided into the lithosphere (solid crust and upper mantle), hydrosphere and asthenosphere (molten rock layer beneath the lithosphere).

<table>
<thead>
<tr>
<th>Application</th>
<th>Geoid accuracy (cm)</th>
<th>Gravity accuracy ($\times 10^5$ ms$^{-2}$)</th>
<th>Spatial resolution (half-wavelength) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solid Earth lithosphere:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper mantle density</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continental plate motions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i) sedimentary basins</td>
<td>1 – 2</td>
<td>50 – 100</td>
<td></td>
</tr>
<tr>
<td>ii) rifts</td>
<td>1 – 2</td>
<td>20 – 100</td>
<td></td>
</tr>
<tr>
<td>iii) tectonic motions</td>
<td>1 – 2</td>
<td>100 – 500</td>
<td></td>
</tr>
<tr>
<td>Seismic hazards</td>
<td>1</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Ocean lithosphere/ asthenosphere</td>
<td>0.5 – 1</td>
<td>100 – 200</td>
<td></td>
</tr>
<tr>
<td><strong>Oceanography:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short scale</td>
<td>1 – 2</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Basin scale</td>
<td>~ 0.1</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td><strong>Ice sheets:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock basement</td>
<td>1 – 5</td>
<td>50 – 100</td>
<td></td>
</tr>
<tr>
<td>Ice vertical movements</td>
<td>2</td>
<td>100 – 1000</td>
<td></td>
</tr>
<tr>
<td><strong>Geodesy:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GNSS levelling</td>
<td>1</td>
<td>100 – 1000</td>
<td></td>
</tr>
<tr>
<td>Unification of height references</td>
<td>1</td>
<td>100 – 20 000</td>
<td></td>
</tr>
<tr>
<td>Inertial navigation</td>
<td>~ 1 – 5</td>
<td>100 – 1000</td>
<td></td>
</tr>
<tr>
<td>Orbit determination</td>
<td>~ 1 – 3</td>
<td>100 – 1000</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Scientific requirements for the GOCE mission [Koop 2008, Rummel 2005].

Improved measurements of the time-varying gravity field are useful for monitoring seasonal and long-term trends in ice sheet masses, ocean current transport and overall ocean mass changes. Here the effects are best quantified in terms of mass changes (expressed as the thickness of a layer of water over the Earth’s surface) that cause the gravity changes. The most significant examples of the value of time-varying gravity field data are the estimated ice mass losses in Greenland and Antarctica derived from GRACE measurements [Velicogna 2006, Chen 2006].

2.2.3 Optical clock possibilities for gravity sensing

Before considering possibilities for future gravity-sensing missions that might include optical clock technology to sense Earth gravitational field changes, it is instructive to briefly review the range of existing and developing technologies available for gravity-sensing missions [Bruinsma 2008]. This includes inter-spacecraft ranging by microwave interferometry, laser interferometry (both by dual master-slave laser arrangements, and by single laser plus retroreflector arrangements), and gradiometry (using electrostatic devices, drag-free inertial sensing such as that being developed within the LISA Pathfinder mission, or cold atom interferometric gradiometers).
A K-band microwave ranging system between spacecraft is used in the GRACE mission. The precision of the range-rate measurements is 0.1 μm s⁻¹. This instrument is sufficiently precise for a GRACE follow-on mission that would primarily serve as a continuation of the current capabilities.

The laser interferometer, using the master-slave concept, has a demonstrated precision of 1 nm s⁻¹ and the potential to measure spacecraft-relative attitudes via beam alignment with 100 nrad Hz⁻¹/² noise. It requires low-power lasers (10 – 30 mW), which are already flight qualified, with a very high level of frequency stabilization (not space qualified yet, but under development for the LISA mission). Overall, this is a complicated solution, but could benefit from the LISA technology development.

The single laser plus retro-reflector concept, compared to a two-laser master-slave arrangement, has the disadvantage of requiring a larger optical power for the master laser for a given operating distance, though approximately 100 mW is sufficient for an inter-satellite distance of 10 km. The short inter-satellite distance may prohibit monitoring of the very long wavelength temporal variations. The primary advantages of this approach are simplicity (one laser, one interferometer and phase meters), robustness (use of a retro-reflector removes any need for active pointing on the second satellite), and reliability (no simultaneous operation of multiple lasers and interferometers).

The GOCE gradiometer uses six 3-axis electrostatic accelerometers, the sensitive axes of which have a sensitivity of $2 \times 10^{-12}$ ms² Hz⁻¹/² (compared to $10^{-10}$ ms² Hz⁻¹/² for the GRACE instruments) in a given measurement bandwidth. The capacitive readout has a noise level of about 6 pm Hz⁻¹/². Attitude control, thermal control and the stiffness of material used to construct and mount the instrument are extremely demanding, but they are critical for enhanced instrument sensitivity.

Technology developed for LISA Pathfinder (to be launched in 2010) includes a drag-free system with inertial sensors around the $10^{-14}$ ms² level and a local interferometric test mass readout with 10 pm Hz⁻¹/² noise, but designed for a very low perturbation orbit environment. Both of these instrument noise levels are about a factor of 100 lower than would be needed for a drag-free two-spacecraft GRACE-type mission. One main advantage of drag-free operation is being able to fly a two-spacecraft mission at a lower altitude, as for GOCE. The other is the reduction of stability requirements associated with accelerometers. A problem may be the associated shorter mission lifetime due to the lower altitude.

A quantum interferometer gravity gradiometer uses atoms that are laser-cooled to a few μK as free-fall test masses. Its building blocks are atom-interferometer-based accelerometers. The projected achievable precision is comparable to that of the GOCE gradiometer, but the measurement bandwidth of such an instrument is not yet known (for GOCE the bandwidth is 5 mHz to 0.1 Hz).

Given the sensitivity of high precision clocks to their location within a gravitational potential, the opportunities for space-clock-based gravimetry should be examined. Optical clocks offer the possibility to determine the difference in gravitational potential between different locations and therefore of establishing a unified global height datum (Figure 6). Such a system is required for several geodetic applications such as the global height synchronization of tide gauges for global sea level monitoring. This approach has the advantage of being independent of satellite acceleration. However, a height change of 1 cm at the Earth’s surface results in a clock frequency gravitational redshift of $10^{-18}$. Currently, the best stabilities and accuracies achieved with optical clocks on the ground approach $10^{-17}$ after averaging for several hours, though it is expected that these will improve to the $10^{-18}$ level during the next
A redshift magnitude of $10^{-18}$ corresponds to an orbit altitude uncertainty of 1 cm and 40 cm respectively for satellites in LEO and geostationary orbits, showing a much reduced sensitivity in the geostationary case. Nevertheless, the extended averaging necessary for $10^{-18}$ accuracy needed to sense geoid changes at the 1 cm level is too long (days) at present to be useful for direct sensing of the surface gravitational potential difference from LEO or geostationary orbits. A methodology that could be useful, however, is to operate with high accuracy optical clocks at fixed locations on the ground and/or on mobile platforms such as aircraft or low Earth orbit satellites, all of which would be referenced to a “master” clock operating in a medium earth or geostationary orbit (Figure 7). The ground clocks and airborne clocks could provide averaging capability over extended intervals at fixed points, or with high spatial resolution (e.g. 10 km or less) respectively, leading to high sensitivity local mapping. The LEO satellites would have lower spatial resolution at ground level (e.g. 10 – 50 km) with adequate gravitational sensitivity being built up over many orbits, but could provide more global coverage along tightly-defined orbital trajectories, with stepped trajectory changes at appropriate intervals allowing gravitational mapping to be constructed. Monitored frequency differences between geostationary (and/or MEO master clocks) and fixed ground reference clocks would allow the global height referencing datum to be established at the centimetre level.

Figure 7. An optical “master” clock on board a satellite could be used as a reference for measurements of the geopotential on the Earth’s surface, for frequency distribution to other spacecraft, or as a transponder for clock comparisons. (Figure reproduced from [Gill 2008].)

2.3 Optical master clock in space

The master clock in space concept offers wider opportunities than just gravitational mapping (Figure 7). With the advent of the high performance optical clocks capable of achieving accuracies in the $10^{-17}$ to $10^{-18}$ range on the ground in fixed laboratory environments, there arises the increasingly challenging problem of comparing such clocks located in distant laboratories. This is necessary to provide the means to gather data from the highest performance clocks to create the international atomic time scale, and to provide confirmation that clock systems in any particular location are consistent with other remote clocks at the level of locally claimed uncertainties. Traditionally, global intercomparisons between caesium microwave atomic clocks has been achieved using satellite-based frequency and time transfer techniques between standards laboratories and other institutes such as the BIPM and USNO. The lowest instabilities for frequency comparisons are obtained using two-way satellite time and frequency transfer (TWSTFT) or GPS-based carrier phase techniques (section 3.5.1). Although these comparison techniques are well matched to the caesium fountain microwave
clock performance in respect of the frequency transfer stability and accuracy achievable, they are inadequate for high stability optical clocks performing at the $10^{-17} - 10^{-18}$ level.

There are a number of potential approaches to solving the problem of remote comparison of high-accuracy optical clocks. One possibility would be the use of transportable optical clocks travelling between standards laboratories. At this juncture, given the local environmental control conditions necessary to push the laboratory optical clock performance to the $10^{-18}$ level, trade-off between performance and transportability is likely to preclude the routine use of transportable systems for a number of years. The major issue with this approach would of course be the need to separate out the varying gravity field experienced by the transportable system between the remote locations, and here, the input from independent gravitational field determinations outlined in section 2.2.3 would be necessary.

One possibility for improved frequency comparisons via satellite should be demonstrated by an improved satellite to ground microwave link (MWL) being developed for the ACES mission, scheduled for operation on the international space station (section 3.5.1.2). This is expected to improve on current techniques by a factor of 20, allowing intercontinental frequency comparison accuracies at the $10^{-17}$ level after one week of averaging time [Cacciapuoti 2007]. This could be enhanced by the use of an optical clock, ideally in a geostationary orbit. With optical “master” clock performance on board the satellite providing stabilities in the $10^{-17} - 10^{-18}$ range for averaging times of one to a few days, direct use of the optical comb microwave outputs to generate the MWL frequencies, together with other enhancements, should further improve the MWL operation, allowing frequency comparison between remote clocks at the $10^{-18}$ level.

A more direct way to compare remote optical clocks is by optical fibre transfer of an optical carrier frequency related to the ground source clock, as described in section 3.5.2.2. This method of optical-optical frequency comparison has been shown to give frequency transfer instabilities below $10^{-18}$ for 100 s averaging times over fibre lengths of up to approximately 250 km, for a mixture of installed and spooled fibre [Newbury 2007b]. For a 172 km link consisting entirely of installed dark fibre, a frequency transfer stability of $10^{-18}$ has been achieved for an averaging time of 2000 s [Jiang 2008]. Transfer over greater distances with intermediate phase-maintaining optical amplifier repeater stations is considered feasible, with some experimental demonstrations soon to be attempted. The obvious difficulty with this approach is the requirement for dark fibre linkage between the optical clocks. Whilst proving experiments have been carried out using local networks, negotiating with telecommunications providers for routine access to dark fibre, for example between European standards laboratories such NPL, LNE-SYRTE and PTB is logistically more difficult. However, the opportunity to compare remote clocks in certain national measurement institutes both using optical fibre frequency transfer techniques and via satellite microwave links represents a powerful validation process for both techniques, underpinning estimated frequency comparison accuracies for remote ground clock comparison by satellite microwave link where no fibre routing exists.

Another potential remote clock comparison technique is that of all optical frequency transfer (section 3.5.1.3). Here, a ground-to-satellite link for time transfer by laser link (T2L2) has been installed as an auxiliary payload on board the Jason-2 mission, launched in June 2008 [Guillemot 2006], and is currently undergoing initial tests. The first bi-directional satellite-to-satellite optical communication linkage was demonstrated in 2005 between the ESA Artemis and Japanese OICETS satellites [Fujiwara 2007].

These opportunities for remote ground clock comparison by satellite microwave link, coupled with optical fibre frequency transfer between certain locations, point to the desirability for an
optical “master clock” in space scenario as an additional mission objective for a gravitational-based fundamental physics and/or geoscience mission incorporating space optical clocks. The preferred arrangement from this point of view would be an optical master clock in geostationary orbit, for ease of accurate orbit determination, reduction of tracking requirements and continuous common-view over certain Earth regions [Gill 2008]. In such an orbit, altitude determination to 40 cm uncertainty would be necessary to take account of the master clock gravitational red shift at the $10^{-18}$ level. This can be achieved using laser ranging. For intercontinental comparisons, more than one optical master clock would be needed, with satellite-to-satellite microwave links. Alternatively, a single optical master clock in an appropriate medium earth orbit (MEO) that accessed the major ground clock locations could be used, although at the expense of continual satellite-in-view operation.

2.4 Global Navigation Satellite Systems (GNSS)

The evolution of the US global positioning system (GPS) from a military satellite-based location system to a navigation system that is widely used by civilians has taken place over the last two decades, generating an explosion in location-based applications in air, sea and land transport sectors as well as people and object tracking and location. It is based on a constellation of some thirty satellites in a number of MEO orbits. A smaller GLONASS system (24 satellites) optimised primarily for the Russian sector was completed in 1995. More recently, the first satellites for the Galileo European satellite navigation system have been launched, and another 30-strong satellite constellation will be launched and brought into service for the Galileo system over the next three years. Galileo is a purely civil system, with funding planned to be a joint undertaking between the EU, ESA and private enterprise. There are also planned global satellite navigation systems for China and India.

The navigation system satellites house microwave atomic clocks. In the case of GPS, these are commercial caesium and rubidium clocks. For Galileo, the satellite payload includes rubidium and passive hydrogen maser clocks. The GPS ground control segment in Colorado uses microwave master clocks (caesium and hydrogen masers) with back-up links to USNO, who are also developing rubidium fountain clocks to provide additional high stability input to the timescale. Galileo has two equivalent control centres located in Germany and Italy, both of which will house primary timing facilities containing four caesium clocks and two hydrogen masers. Both GNSS systems have several worldwide monitoring stations.

The overall error budget for position determination using GNSS includes contributions from satellite orbit determination and signal propagation from satellite to ground receiver (including atmospheric propagation errors associated with the ionosphere and troposphere and multi-path effects), as well as clock uncertainties. Total uncertainties using the standard code transmission arrangement are in the region of a few metres. Propagation limitations mean that these uncertainties are not greatly reduced by substituting optical clocks for the microwave systems. The situation is better with satellite signal carrier phase transmission, with improvements to approximately 2 m for standard microwave clocks, reducing to around 1 m with optical clock substitution and a factor of four improvement in orbit determination. The analysis described below demonstrates the improvements in system frequency stability by substituting optical clocks for the microwave clocks in the space segment and / or the ground segment. Further improvements in propagation analysis are needed to take full advantage of the significant improvements shown in these simulations. However, additional benefits would also accrue with the deployment of optical clocks into both segments, resulting in a greater degree of autonomy in the space segment due to improved orbit prediction and on-board clock prediction. These considerations are important for critical “safety of life” applications.
2.4.1 GNSS optical clock simulations

A recent ESA study [Gill 2007] considered the potential enhancements to a GNSS system offered by optical clocks in particular in relation to the Galileo system. Two types of on-board microwave clocks are currently planned for the Galileo system: rubidium and passive hydrogen masers, while an active hydrogen maser will be used on the ground as a master clock in the Precise Timing Facility (PTF) to maintain Galileo System Time (GST). Errors in satellite clocks translate into position determination errors on the ground of the order of 1 m per 3 ns due to the speed of electromagnetic radiation, so clock accuracy is critical for optimum performance.

The ESA study referred to above presented simulated offsets between individual satellite clocks and GST, for both the planned Galileo microwave clocks and for four scenarios employing optical clocks. For the planned Galileo system, rubidium clocks onboard the satellites are used in one scenario, and in the second scenario passive hydrogen masers are assumed. These two scenarios are referred to as Galileo I and II (Table 3).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Satellite Time</th>
<th>System Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galileo I</td>
<td>rubidium clock</td>
<td>active hydrogen maser</td>
</tr>
<tr>
<td>Galileo II</td>
<td>passive hydrogen maser</td>
<td>active hydrogen maser</td>
</tr>
</tbody>
</table>

Table 3: Clocks used in simulations of different Galileo system scenarios.

Each scenario was simulated ten times for a time interval of one year with a sampling period of $\tau = 10$ minutes. The underlying stochastic model of the atomic clocks is described by a combination of different frequency noise types: white, flicker and random walk. Figure 8 presents the Allan deviation of Galileo I and II offset simulations. The curves are dominated by the Allan deviation of the corresponding satellite clocks.

![Figure 8. Allan deviation for Galileo I and II scenarios of simulated offsets between satellite clocks and Galileo system time (GST).](image-url)
The simulated use of optical clocks in a GNSS involved the four different scenarios listed in Table 4. Scenarios I and II generate system time using an optical clock and implement satellite time using rubidium and passive hydrogen maser clocks respectively. The third scenario considers an optical clock onboard the satellite with an active hydrogen maser used for GST. The final scenario investigates the combination of two optical clocks, one on board the satellite and the other on the ground for system time.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Satellite Time</th>
<th>System Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>rubidium clock</td>
<td>optical clock</td>
</tr>
<tr>
<td>II</td>
<td>passive hydrogen maser</td>
<td>optical clock</td>
</tr>
<tr>
<td>III</td>
<td>optical clock</td>
<td>active hydrogen maser</td>
</tr>
<tr>
<td>IV</td>
<td>optical clock</td>
<td>optical clock</td>
</tr>
</tbody>
</table>

Table 4: Clocks used in the simulations of GNSS scenarios including optical clocks.

As in the simulations of Galileo I and II, each optical scenario is simulated ten times, for a time interval of one year and with a sampling period of $\tau = 10$ minutes. The optical clock performance level assumed is white frequency noise of $10^{-14}$ at one second and flicker frequency noise of $10^{-16}$.

Since the stability of optical clocks is better than that of rubidium and passive hydrogen maser clocks, the microwave clock noise dominates the Allan deviations of scenarios I and II. Figure 9 shows how scenario I reaches a flicker frequency noise floor of about $5 \times 10^{-14}$, which is equal to the flicker floor of the rubidium clock. Scenario II shows a flicker floor of about $10^{-14}$. The Allan deviation of the satellite clocks thus dominates the stability of scenarios I and II.

![Figure 9. Allan deviation of the simulated offset between satellite clocks and system time for four scenarios employing optical clocks (see Table 4).](image-url)
Scenario III, which assumes an optical clock onboard the satellite and an active hydrogen maser on the ground, is characterised by the noise of the active hydrogen maser. The Allan deviation is dominated by flicker frequency noise for $\tau$ values from $10^3$ s to about $2 \times 10^4$ s (Figure 9). At longer averaging times, the random walk frequency noise is apparent. The Allan deviation figure of scenario IV is dominated by white frequency noise for $\tau$ values up to about $2 \times 10^5$ s and by flicker frequency noise at longer averaging times.

Using the data from the simulations the predicted size of satellite clock errors can be estimated. Figure 10 shows prediction errors for both 2 hour and 24 hour periods after synchronisation. It can be seen that the potential improvements offered by optical clocks are significant.

![Figure 10. Satellite clock prediction errors for 2 hour and 24 hour synchronisation periods. (RAFS: rubidium atomic frequency standard; SPHM: space passive hydrogen maser; AHM: active hydrogen maser; Opt: optical clock.)](image)

Despite the simplifications in these first simulations, it has been shown that optical clocks could provide a considerable improvement in terms of prediction errors over time. The use of such clocks would therefore permit one to have less frequent updates of the clock parameters, easing the operations of the satellite system and providing higher autonomy in the space segment of the system. It is important to note that the simulations presented above are limited since crucial error sources such as the frequency drift of the atomic clocks, measurement noise and noise of the time facilities, as well as the noise from onboard satellite equipment have not been considered. Furthermore, position determination on the ground using GNSS is not only limited by clock errors: multi-path and atmospheric effects are also significant and contribute a substantial part of the user range error in current systems. These issues would need to be addressed in the coming years in order to take maximum advantage of clock performance upgrades.
In summary, optical clocks offer the possibility of up to two to three orders of magnitude improvement in GNSS satellite clock prediction accuracy. This in turn would lead to far less reliance on regular uplinks, which could be important for coping with any malfunction in the ground segment. Assuming that multi-pass and atmospheric effects can be dealt with, better satellite clocks will lead to corresponding improvements in position determination for users.

As discussed in section 2.3, fluctuations in the height of the geoid have an increasing impact on ground clocks as their fractional uncertainties approach the $10^{-18}$ level [Kleppner 2006]. The resulting potential for a number of master clocks in space could be of significant importance for very precise navigation and position determination. A recent suggestion [Svehla 2008] pointed to optical master clocks in GEO with optical or microwave links to a MEO satellite constellation as a possible Galileo II navigation system.

2.5 Summary of user requirements

As discussed in the preceding sections of this document, future mission scenarios that would benefit from optical atomic clock technology range from experiments to test the fundamental laws of physics to applications with direct benefits to society.

Within the area of Science (Fundamental Physics), space-borne optical atomic clocks offer very significant opportunities to test the limits of our current understanding of the universe. Optical clocks could be used to search for violations of the Einstein Equivalence Principle by making measurements of the gravitational redshift with unprecedented precision, or to discriminate clearly between different theories of gravity through measurements of the Shapiro time delay of highly stable optical signals to and from a spacecraft as they pass close by the sun at conjunction. They would also be a useful addition to missions designed to resolve the causes of the Pioneer anomaly.

These types of mission scenario could potentially lead to a major scientific discovery: a breakdown of Einstein’s general theory of relativity or the observation of variations in fundamental constants could provide the first experimental evidence to support emerging theories aimed at unifying gravitation and quantum mechanics. The development programme outlined here thus has a vital role to play in supporting future Cosmic Vision mission proposals in the area of fundamental physics by increasing the technological readiness level of the optical clock technology.

Within the area of Earth Observation, space-borne optical atomic clocks would open up new possibilities in geodesy and remote atmospheric sensing. The sensitivity of highly stable and accurate optical clocks means that they can be used for direct measurement of the earth’s geopotential at the few centimetre level with very high spatial resolution. Such measurements would have applications in a wide range of sectors, including civil engineering and construction, oil and gas exploration, ordnance survey and even global navigation. Long-term measurements of the variability of the local gravitational potential will also be possible, with important perspectives in monitoring seasonal and long-term trends in ice sheet masses, ocean current transport and overall ocean mass changes. The International Panel on Climate Change has stressed the need for such data as critical input to models used to study and forecast the effects of climate change.

Additional opportunities arise from the sub-components of the optical clock. For example, the optical frequency comb could be used for accurate monitoring of absorption profiles in
satellite-based LIDAR experiments, leading to further benefits in the characterisation of atmospheric properties such as carbon dioxide concentrations.

Within the area of **Navigation**, optical atomic clocks could find application in future evolutions of global satellite navigation systems.

Using optical clocks in both the ground and the space segments of a satellite navigation system offers the possibility of up to two to three orders of magnitude improvement in satellite clock prediction accuracy. This would lead to far less reliance on regular updates of the clock parameters and hence provide much higher autonomy in the space segment of the system. With further progress in the understanding of atmospheric and multi-path effects, improved satellite clocks would also lead to improvements in position determination for users of the system.

Finally, an **optical master clock in space** could be realised in a mission scenario offering advantages to all the above areas.

Variations in the local gravitational potential will have an increasing impact on ground-based optical clocks as their fractional uncertainties approach the $10^{-18}$ level. To take full advantage of this technology it will therefore be necessary to operate them in spacecraft orbiting the Earth at relatively high altitudes where spatial and temporal variations of the Earth’s gravitational field are smoothed out.

A future ultra-precise reference timescale based on space-borne optical clocks, combined with advanced inter-satellite and satellite-ground links, would have significant impact in a range of areas including calibration of international atomic timescales, geodesy and very precise navigation systems.
3 Review of optical atomic clock technology

Any optical atomic clock consists of three main elements: an ultra-stable probe laser, a suitably narrow atomic reference to which the laser is stabilized, and an optical frequency comb that provides the means to transfer the frequency stability of the reference to other spectral regions. Candidates for the atomic reference can be grouped into two classes: transitions in single laser-cooled trapped ions and transitions in cold atoms. These technologies are rather different and are therefore discussed separately. The relation between the three elements of the clock for the illustrative case of an ion-based frequency standard is shown in Figure 11.

To set the scene for the technology development plan laid out in the following sections of this report, the current state of development of the individual subsystems of an optical clock are reviewed in turn, followed by methods for comparing remotely located optical clocks.

Figure 11: Elements of an optical atomic clock, and the relationship between them. (This illustration is for the case of a trapped ion optical clock, but a similar relation exists between the subcomponents of a clock based on cold neutral atoms.)
3.1 Optical local oscillators

The atomic transitions used as reference frequencies in optical clocks typically have $Q$-factors in the region of $10^{15}$, corresponding to linewidths of order 1 Hz or less. To probe such narrow transitions without degrading the potential stability achievable from the standard requires an optical local oscillator that has similar or narrower linewidth. Techniques for achieving such a narrow linewidth are described in this section, together with a review of state-of-the-art performance. The fundamental limits to the achievable linewidth and stability of the local oscillator are also discussed.

3.1.1 Principles of operation

Readily available solid-state and semiconductor diode lasers have linewidths ranging from tens of kHz to tens of MHz, far larger than the spectral resolution required for probing the reference transitions of optical clocks. Reduction of the laser linewidth to the necessary level is most commonly achieved by using the frequency modulation (FM) sideband stabilisation (or Pound-Drever-Hall) technique [Drever 1983] to lock the optical local oscillator to a high finesse ultra-stable optical cavity (Figure 12).

A typical reference cavity used for such an ultrastable optical local oscillator is shown in Figure 13, and consists of two concave mirrors that are optically contacted onto each end of a cylindrical spacer. Both the spacer and the mirror substrates are usually made from ultra-low expansion (ULE) glass, which is a titania-doped silicate glass designed to have a coefficient of thermal expansion of $0 \pm 3 \times 10^{-8}/^\circ$C over a temperature range of $5 – 35^\circ$C, with a zero crossing close to room temperature. With high reflectivity coatings on super-polished mirror substrates with an absorption and scatter loss of a few ppm, finesse values in the region of $10^5 – 10^6$ are achievable [Rempe 1992]. Typical cavities used for optical local oscillators are around 10 cm long with finesse values in the region of 200 000, corresponding to cavity resonance linewidths in the range 5 – 10 kHz. The characteristics of the frequency discriminant obtained using the Pound-Drever-Hall locking technique mean that a very tight lock to this cavity resonance can be achieved, yet with a comparatively wide capture range.

Figure 12. Schematic diagram of a ULE-cavity-stabilised laser. (APD: avalanche photodiode; PBS: polarizing beamsplitter; $\lambda/4$: quarter-wave plate.)
Assuming that the lock fidelity is sufficiently high, the frequency stability of the optical local oscillator is limited not by the noise of the laser source but rather by the stability of the optical cavity resonance. Since changes in the cavity length are directly related to changes in its resonance frequency, isolation of the cavity from sources of vibration is critically important. The ULE cavity is therefore housed in an evacuated enclosure and operated at a temperature where the coefficient of thermal expansion is close to its zero crossing. Under these conditions, the thermal drift rate of the ULE cavity can be minimized, ideally leaving just the isothermal creep of around 30 mHz s$^{-1}$. The vacuum chamber housing the ULE cavity and the optics for the Pound-Drever-Hall lock are usually supported on a vibration-isolation platform (either active or passive), and in many cases are also enclosed within an acoustic isolation housing. ULE cavities are not tunable, and in general there will be no cavity resonance coincident with the frequency of the atomic reference transition used in the optical clock. An acousto-optic modulator (AOM) is therefore normally used to bridge the difference. A feed-forward arrangement can be used to control the AOM drive frequency so as to compensate for the first-order residual drift of the ULE cavity.

### 3.1.2 State-of-the-art performance

The benchmark performance for optical local oscillators is that of a ULE-cavity-stabilized dye laser at NIST, which has demonstrated a linewidth of 0.16 Hz and a fractional frequency instability of $3 \times 10^{-16}$ at 1 s [Young 1999a, Young 1999b]. However dye lasers are cumbersome and require frequent user intervention, making them unsuitable for application in a space-borne optical clock.

Significant work has centred on stabilization of semiconductor diode and solid-state lasers (Table 5). The best results have been obtained with Nd:YAG non-planar ring oscillators stabilized to a ULE cavity, for which linewidths of 0.4 Hz and stabilities of $10^{-15}$ from 1 – 20 s have been achieved (with linear drift compensated) [Webster 2004a, Notcutt 2005, Webster 2008]. The main problem with these systems is their very limited tunability. This means that they are unable to access and probe most optical clock transitions directly, although if a femtosecond comb is locked to the stable Nd:YAG laser, the stability can in principle be transferred to any other frequency within the bandwidth of the comb (section 3.4.2.1). Local oscillators used to probe optical clock transitions are usually tunable sources such as laser diodes or Ti:sapphire lasers. For semiconductor diode lasers, linewidths in the range 0.4 – 1 Hz and fractional frequency instabilities of around $1 \times 10^{-15}$ for averaging times of a few seconds have been reported by a number of laboratories [Stoehr 2006, Barwood 2007, Ludlow 2007, Alnis 2008]. At NPL, the ULE-cavity-stabilized Ti:sapphire laser that is used as the optical local oscillator for the ytterbium ion optical frequency standard has demonstrated linewidths of 5 Hz at an averaging time of 1 s, broadening to 20 Hz over 100 s [Margolis 2006].
Table 5. State-of-the-art performance of optical local oscillators for different averaging times $\tau$ with linear drift compensated.

<table>
<thead>
<tr>
<th>Laser type</th>
<th>Linewidth</th>
<th>Fractional stability</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dye</td>
<td>0.6 Hz, $\tau &lt; 32$ s</td>
<td>$&lt; 5 \times 10^{-16}$, $0.3 , s &lt; \tau &lt; 100$ s</td>
<td>[Young 1999a]</td>
</tr>
<tr>
<td></td>
<td>0.16 Hz, $\tau &lt; 20$ s</td>
<td>$&lt; 5 \times 10^{-16}$, $0.3 , s &lt; \tau &lt; 100$ s</td>
<td>[Young 1999b]</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>0.46 Hz, $\tau = 4$ s</td>
<td>$&lt; 1 \times 10^{-15}$, $1 , s &lt; \tau &lt; 20$ s</td>
<td>[Webster 2004a]</td>
</tr>
<tr>
<td></td>
<td>0.5 Hz, $\tau = 4$ s</td>
<td>—</td>
<td>[Notcutt 2005]</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>$&lt; 2 \times 10^{-15}$, $0.5 , s &lt; \tau &lt; 100$ s</td>
<td>[Webster 2008]</td>
</tr>
<tr>
<td>Diode</td>
<td>1.5 Hz, $\tau = 4$ s</td>
<td>$&lt; 2 \times 10^{-15}$, $0.5 , s &lt; \tau &lt; 20$ s</td>
<td>[Stoehr 2006]</td>
</tr>
<tr>
<td></td>
<td>1.4 Hz, $\tau = 3$ s</td>
<td>$&lt; 2.5 \times 10^{-15}$, $1 , s &lt; \tau &lt; 10$ s</td>
<td>[Barwood 2007]</td>
</tr>
<tr>
<td></td>
<td>0.4 Hz, $\tau = 4$ s</td>
<td>$&lt; 1 \times 10^{-15}$, $0.2 , s &lt; \tau &lt; 20$ s</td>
<td>[Ludlow 2007]</td>
</tr>
<tr>
<td></td>
<td>0.5 Hz, $\tau = 4$ s</td>
<td>$&lt; 2 \times 10^{-15}$, $0.1 , s &lt; \tau &lt; 10$ s</td>
<td>[Alnis 2008]</td>
</tr>
<tr>
<td>Ti:sapphire</td>
<td>5 Hz, $\tau = 1$ s; 20 Hz, $\tau = 100$ s</td>
<td>—</td>
<td>[Margolis 2006]</td>
</tr>
</tbody>
</table>

3.1.3 Limitations to performance

A number of effects can potentially limit the stability of an optical local oscillator used as the reference for an optical clock. These are discussed in the following sections, together with methods used to reduce or eliminate the effects.

3.1.3.1 Optical and electronic effects

The accuracy of the Pound-Drever-Hall frequency lock is critically dependent on the detection of the relative phase of the carrier and the imposed sidebands in the reflected signal from the high finesse ULE cavity. Parasitic etalons can couple to the high finesse cavity, causing this phase to become dependent on the optical path length between the cavity and the back-reflecting (scattering) surface that is the source of the etalon. Noise on the lock can therefore arise from air currents in the beam path, which cause changes in optical path length and hence fluctuations in the lock point. This leads to characteristic random walk frequency noise and can be significant on timescales from 1–1000 s. Detection noise, on the other hand, shows up as white frequency noise, and can be significant for frequencies above 10 Hz.

3.1.3.2 Vibrations

The most serious fluctuations of the ULE cavity length are caused by low-frequency (below 100 Hz) seismic and acoustic vibrations that couple through the cavity support structure and lead to forces that deform the cavity. In the most commonly used design where a reference cavity is supported from below with its symmetry axis horizontal this coupling typically leads to sensitivities to acceleration of order $100 \, \text{kHz}/(\text{ms}^2)$. Improvements to performance levels have recently been sought by designing cavities and their support structures so as to greatly reduce the sensitivity to vibrations.

In this design process, it is assumed that the optical mode of the cavity coincides with its symmetry axis and has a small diameter at the mirror surface relative to the cavity dimensions. To achieve vibration insensitivity, it is necessary to ensure that the distance between the two points at the centres of the mirror surfaces remains invariant when a force is applied to the support. This may be achieved in a simple way by symmetrical mounting: a force applied through the mount will cause one half of the cavity to contract whilst the other half expands, with the result that there is no net change in the cavity length.
One approach to realizing a high degree of symmetry is to mount the cavity vertically with the support at the geometrical mid-plane, as shown in Figure 14. In this case it can intuitively be seen that the vibration sensitivities for the top and bottom halves of the cavity will tend to cancel each other out to first order, with the top section compressing under gravity while the bottom section expands by a similar amount. This approach has successfully been demonstrated for both Nd:YAG oscillators [Notcutt 2005] and diode lasers [Ludlow 2007]. In the first case a simple cylindrical cavity was used, while in the second case the cavity was wider in the middle and tapered at the ends, a design that gives a more rigid construction without introducing excess material. Measured vibration sensitivities were in the range $5 \div 30$ kHz/(ms$^{-2}$).

Alternative schemes have also been developed for reducing the vibration-sensitivity of horizontally mounted ULE cavities, based on concepts attributed to Till Rosenband at NIST. A cavity mounted with its axis horizontal sags asymmetrically under its own weight. However, through optimization of the support positions (achieved at the design stage by means of finite element analysis), the distance between the centres of the two mirrors can be made invariant, even though the cavity still deforms on the application of a vertical force. In one variant of this technique, PTB have shown that with four symmetrical horizontal support points located just below the cavity horizontal mid-plane, vibration sensitivities of $1.5$ kHz/(ms$^{-2}$) and $14$ kHz/(ms$^{-2}$) in the vertical and horizontal planes can be achieved [Nazarova 2006]. Work at NPL has resulted in an alternative design in which material is symmetrically cut out from a cylindrical cavity spacer along the complete horizontal length in the lower quadrants (Figure 14), and the position of four symmetrically placed supports is varied so as to minimize the vibration sensitivity. This cavity design has shown vertical and horizontal sensitivities to vibration of < $0.1$ kHz/(ms$^{-2}$) and $3.7$ kHz/(ms$^{-2}$) respectively [Webster 2007]. This represents a thousand-fold reduction in the sensitivity to vertical vibrations, with the horizontal response being reduced by around a factor of 30.

These type of schemes are only effective for a fixed orientation of the cavity with respect to gravity, and are not generally portable because the cavity simply rests on its supports under its own weight. For portability, the cavity must either be clamped in transit [Alnis 2008] or the cavity mounting must be rigid to some extent. However to achieve minimum vibration sensitivity, the forces acting on the cavity must be symmetric. This can be achieved to a good degree with elastic supports [Webster 2007] but presents a particular challenge for rigid supports. A variety of routes to portable reference cavities are currently being explored at NPL.

Figure 14. Two different designs of vibration-insensitive cavity. Left: vertical cavity design developed at JILA (reproduced from [Ludlow 2007]. Right: cut-out cavity design developed at NPL [Webster 2007].
### 3.1.3.3 Thermal noise

With good temperature control at the zero thermal expansion point and with the effects of vibrations suppressed, the frequency stability of an optical local oscillator will become limited by dimensional changes due to thermal fluctuation of the mirror substrates and their coatings [Numata 2004].

The fluctuation dissipation theorem [Callen 1952] relates the spectrum of random motion to the mechanical loss in a system, and has been used to develop a theoretical model of the effects of thermal noise in a rigid optical reference cavity [Numata 2004]. This model was based on measured mechanical quality factors $Q$ of typical cavity materials and a numerical solution of the mechanical equation of motion with the direct application of the fluctuation dissipation theorem to obtain the thermal noise spectrum. The power spectral density $G(f)$ of mirror thermal noise displacement, which typically dominates over the spacer contribution (Table 6), is given by the following function of frequency $f$:

$$G(f) = \frac{4kT}{2\pi f} \left( 1 - \sigma^2 \right) \phi_{\text{sub}} \left[ 1 + \frac{2}{\sqrt{\pi}} \frac{(1 - 2\sigma) \phi_{\text{coat}}}{\phi_{\text{sub}} w_0} \right],$$

...(8)

where $k$ is the Boltzmann constant, $T$ is the temperature, $\sigma$ is Poisson’s ratio, $E$ is Young’s modulus, $w_0$ is the beam radius, $\phi$ ($=1/Q$) is the loss of the substrate or coating (as indicated by the subscript), and $d$ is the thickness of the mirror coating. The term in front of the brackets is the contribution of the substrate, whilst the factor inside the brackets gives the modification due to the coating. This $1/f$ noise in power spectral density gives rise to a fractional frequency stability (Allan deviation) that is independent of averaging time. Using this model, Numata et al. have shown that thermal noise sets a fundamental limit for the frequency stability achieved with a rigid reference cavity that is very similar to the state-of-the-art performance achieved. A systematic and detailed experimental confirmation of this thermal noise model has been carried out at JILA, where a comparison has been made of the frequency noise of lasers locked to a variety of rigid reference cavities of differing lengths and mirror substrate materials [Notcutt 2006].

<table>
<thead>
<tr>
<th>Cavity component</th>
<th>Root spectral power density of thermal noise $S_{\nu f}^{1/2}$ mHz</th>
<th>% contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacer</td>
<td>10.7</td>
<td>5.5</td>
</tr>
<tr>
<td>Mirrors (substrate only)</td>
<td>157</td>
<td>81.3</td>
</tr>
<tr>
<td>Mirrors (substrates + coatings)</td>
<td>192</td>
<td>99.5</td>
</tr>
<tr>
<td>Total</td>
<td>193</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 6. Breakdown of contributions to thermal noise in the cavity used by Webster et al., in which both the spacer and the mirrors are made from ULE [Webster 2008]. The thermal noise is observed to be dominated by the mirror substrates, with the contribution from the coatings being approximately 20%.
Figure 15. Allan deviation of the beat note between two Nd:YAG lasers that are stabilized to two independent vibration-insensitive ULE cavities [Webster 2008]. A calculation of the thermal noise limit is also shown.

Currently ULE is used for optical reference cavities because it has zero expansivity close to room temperature. A significant reduction in the thermal noise could be obtained by changing to silica, which has a much lower mechanical loss. The thermal noise would then be dominated by the coating, which is at a lower level. However, silica has a relatively large thermal expansivity and so very good control of the temperature would be required. An alternative would be to use a ULE spacer, which contributes very little to the thermal noise, and a silica substrate [Notcutt 2006].

The mirror coatings are alternating layers of silica (SiO$_2$) and tantala (Ta$_2$O$_5$), with the tantala layers contributing most to the thermal noise [Penn 2003]. Significant work is underway in the gravitational-wave-detection community to develop lower loss coatings, and a factor of two reduction has been achieved by doping the tantala layers with titania (TiO$_2$) [Harry 2007].

Cooling the cavity itself would also reduce the thermal noise, since it scales as the square root of the temperature. However in the case of ULE this would mean operating far from the zero thermal expansion point and so may lead to unacceptably large drift rates. Silicon, which has a zero in expansivity at 114 K [Okada 1984], has been proposed as an alternative material. The problem of the noise in the coating remains, however, and unfortunately this gets worse at low temperature since the mechanical loss coefficient of tantala has a resonance at 70 K [Rowan 2008]. In addition, silicon is only transparent for wavelengths above 1.2 μm, and so a frequency comb would have to be used to transfer the frequency stability of the local oscillator to the frequency required for the clock transition (section 3.4.2.1).

Further reductions in thermal noise are possible by altering the cavity geometry so as to increase the spot size on the mirrors, which could be achieved using mirrors with larger radius of curvature. However there may be some trade-off between beam diameter and pointing stability. The fractional effect of the thermal noise can also be reduced by increasing the cavity length (which can be achieved without adversely affecting the vibration insensitivity [Webster 2007]). This is the reason why the fractional frequency instability of the NIST dye laser, which is stabilized to a longer than usual (24 cm) cavity, is lower than the other systems listed in Table 5.

By a judicious choice of cavity geometry and materials we can thus expect that the thermal noise in optical local oscillators may in future be reduced to the $10^{-16}$ level.
3.2 Atomic reference: trapped ion optical clocks

The ability to confine a single isolated ion in a small region of space under ultra-high-vacuum conditions with minimal electromagnetic and collisional perturbation makes forbidden optical transitions in such systems very promising as highly accurate frequency standards.

3.2.1 Principles of operation

3.2.1.1 Trapping and cooling a single ion

Trapped ion optical frequency standards use radio-frequency or Paul-type ion traps [Paul 1990], in which a time-varying quadrupolar potential of the form

$$\phi(r, z, t) = A(t)\left(r^2 - 2z^2\right)$$

is used to create a dynamic pseudopotential well between the trap electrodes. For a given ion mass and charge, and for given trap dimensions, the ion motion in such a field will only be stable for certain ranges of the applied voltage amplitude and frequency. Within these regions of stability, the ion motion can be separated into two parts: a driven oscillatory motion at the trap drive frequency (micromotion) and a slower motion associated with the time-averaged confining potential (secular motion).

Conventional Paul traps employ hyperbolic electrodes, which give the best approximation to a harmonic potential over a large volume, but optical access to such a structure is relatively poor. Fortunately, when carrying out experiments on a single laser-cooled ion, the potential need only be harmonic close to the centre of the trap, and so simpler electrode structures can be used. Variants of the Paul trap commonly used for trapped ion optical frequency standards include Paul-Straubel traps [Schrama 1993, Yu 1995], endcap traps [Schrama 1993, Sinclair 2001] and linear Paul traps [Rosenband 2007]. Examples of these types of trap are shown in Figure 16.

Figure 16. Examples of ion traps used for optical frequency standards. Left: Paul-Straubel trap used at PTB for a $^{171}\text{Yb}^+$ optical frequency standard. Centre: Endcap trap used for a $^{88}\text{Sr}^+$ optical frequency standard at NPL. Right: Linear ion trap developed in Innsbruck for trapping $^{40}\text{Ca}^+$ ions.
Using laser cooling techniques, it is possible to confine the ion to a region of space with dimensions less than the wavelength of the light used to probe the reference transition for the optical clock. Under these circumstances, known as the Lamb-Dicke regime, the absorption spectrum of the clock transition will be free of the first-order Doppler effect, exhibiting an unshifted narrow carrier with weak sidebands at the secular motion frequency away from the carrier [Dicke 1953]. Micromotion of the ion leads to amplitude modulation of the cooling laser fluorescence, via the Doppler effect, at the trap drive frequency. The micromotion can thus be monitored using rf photon correlation techniques [Berkeland 1998], and minimized by applying small dc voltages to additional compensation electrodes. In this way the second-order Doppler shift can be greatly reduced and the ion confined to the trap centre where electric field perturbations are minimized. Operating the trap under ultrahigh vacuum conditions ensures that collisional effects are negligible.

Ions are often loaded into the trap by electron bombardment of neutral atoms. However the atomic flux required causes patch potentials to form on the electrode surfaces and static charge also tends to accumulate on insulating surfaces near to the trapping region, modifying the trap potential and necessitating frequent adjustments of the voltages applied to the compensation electrodes. An alternative approach is to use photoionization techniques [Kjaergaard 2000, Gulde 2001, Brownnutt 2007], which lead to significant reductions in both the atomic flux required to load the trap and the fluctuations of the applied voltages used to minimize the ion micromotion.

3.2.1.2 Probing the reference transition

Although a single trapped ion possesses the intrinsic advantages of a nearly isolated quantum system, detecting the absorption of individual photons at the frequency of the narrow reference transition poses a challenge. Direct detection of fluorescence or absorption by the reference transition is not feasible because the transition is intrinsically weak and the typical fluorescence detection efficiency is low. The solution is to use the electron shelving technique [Dehmelt 1975, Nagourney 1986], which involves observing quantum jumps in the strong cooling laser fluorescence from the single ion when the ion is excited into the upper metastable level of the clock transition (Figure 17). This enables the narrow reference transition to be detected with nearly 100% efficiency.

![Figure 17](image.png)

**Figure 17.** In a trapped ion optical frequency standard the ion is laser-cooled using a strong transition to a short-lived excited state, and the ion is detected by monitoring the fluorescence from this transition. When the probe laser induces the weak reference transition the ion is shelved in the long-lived state and the fluorescence disappears until the ion decays back to the ground state.
The line profile of the clock transition can be built up from the statistics of the number of quantum jumps observed for a fixed number of probe laser interrogation pulses as the frequency of the laser is scanned across the transition. The probe laser can be stabilised to the clock transition by repeatedly stepping back and forth between the estimated half-intensity points and monitoring the quantum jump rate imbalance between these two points. This imbalance serves as the frequency discriminant from which the correcting steer to the probe laser is derived. The optimization of such interrogation and stabilisation schemes is discussed in references [Riis 2004] and [Peik 2006].

3.2.2 Candidate systems and current performance

Research is underway at a number of laboratories worldwide to investigate the stability and accuracy of different trapped ion optical frequency standards, including those based on single cold ions of Al$^+$ [Rosenband 2008], Ca$^+$ [Champenois 2004, Matsubara 2008, Chwalla 2008], Sr$^+$ [Margolis 2004, Dubé 2005], In$^+$ [von Zanthier 2000], Yb$^+$ [Tamm 2007, Hosaka 2005] and Hg$^+$ [Oskay 2006]. Some of the key features of each of these standards, namely the wavelength and theoretical linewidth of the clock transition, are compared in Table 7, which also lists the laboratories in which each system is being studied.

The optical frequency standards listed in Table 7 fall into two categories. In the first category are those that are based on ions with alkali-like or quasi-alkali-like atomic structure, including Ca$^+$, Sr$^+$, Yb$^+$ and Hg$^+$. In these ions, the lowest excited $^2D$ states lie below the lowest $^2P$ states in energy and so are metastable, decaying to the $^2S_{1/2}$ ground state via an electric quadrupole transition. The natural linewidths of these quadrupole transitions lie in the range 0.2–3 Hz, making it relatively straightforward to drive the weak clock transition. The highest $Q$ factor achieved experimentally to date is for the $^{199}$Hg$^+$ ion, where the Fourier-transform-limited cold ion linewidth of 6.7 Hz corresponds to a $Q$-factor of $1.6 \times 10^{14}$ [Rafac 2000], although $Q$-factors approaching this value have been observed for several other systems [Peik 2004, Barwood 2007]. The $^{171}$Yb$^+$ ion also has a low-lying $^2F_{7/2}$ state that has an exceptionally long lifetime of around 6 years [Roberts 2000], and which can only decay by an electric octupole transition to the ground state. Although this transition is significantly harder to drive than the electric quadrupole transition, it offers the potential advantage of longer interrogation times limited by the probe laser coherence time rather than the natural decay of the upper level of the clock transition. It therefore offers potentially higher frequency stability, albeit at the expense of somewhat increased experimental complexity.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>$\lambda$ / nm</th>
<th>$\Delta \nu_{\text{nat}}$ / Hz</th>
<th>Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al$^+$</td>
<td>$^3S_0 - ^3P_0$</td>
<td>267</td>
<td>$8 \times 10^{-3}$</td>
<td>NIST</td>
</tr>
<tr>
<td>$^{40,43}$Ca$^+$</td>
<td>$^3S_{1/2} - ^3D_{5/2}$</td>
<td>729</td>
<td>0.14</td>
<td>Innsbruck, NICT, Marseilles</td>
</tr>
<tr>
<td>$^{88}$Sr$^+$</td>
<td>$^3S_{1/2} - ^3D_{5/2}$</td>
<td>674</td>
<td>0.4</td>
<td>NPL, NRC</td>
</tr>
<tr>
<td>$^{111}$In$^+$</td>
<td>$^3S_0 - ^3P_0$</td>
<td>237</td>
<td>0.8</td>
<td>MPQ, Erlangen</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^3S_{1/2} - ^3D_{5/2}$</td>
<td>436</td>
<td>3.1</td>
<td>PTB, NPL</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^3S_{1/2} - ^3F_{7/2}$</td>
<td>467</td>
<td>$\sim 10^{-9}$</td>
<td>NPL, PTB</td>
</tr>
<tr>
<td>$^{199}$Hg$^+$</td>
<td>$^3S_{1/2} - ^3D_{5/2}$</td>
<td>282</td>
<td>1.8</td>
<td>NIST</td>
</tr>
</tbody>
</table>

Table 7. Properties of trapped ion optical frequency standards currently under development, including the spectroscopic designation of the clock transitions, their wavelengths $\lambda$ and natural linewidths $\Delta \nu_{\text{nat}}$.  

35
The second category of standards includes those ions that have an atomic structure similar to that of the alkaline earth elements, with two valence electrons. The reference transition in these systems is the strongly spin-forbidden $^1S_0 - ^3P_0$ transition, which has a natural linewidth of 0.8 Hz for $^{115}$In$^+$ and approximately 8 mHz for $^{27}$Al$^+$. These systems have advantages with respect to certain systematic frequency shifts; in particular there is no electric quadrupole shift of the clock transition frequency and the UV clock transition frequencies result in a small blackbody Stark shift (section 3.2.3). However the construction of laser sources at the deep UV wavelengths required for cooling and probing the ion presents a significant experimental challenge. The $^{27}$Al$^+$ ion has a particularly inaccessible cooling wavelength of around 167 nm. To overcome this difficulty, the $^{27}$Al$^+$ ion is trapped together with an auxiliary $^9$Be$^+$ ion that can be cooled at the more convenient wavelength of 313 nm. The Coulomb interaction couples the two ions, leading to sympathetic cooling of the $^{27}$Al$^+$ ion. The normal electron shelving scheme cannot be used to probe the weak $^{27}$Al$^+$ clock transition because there is no cooling fluorescence from this ion. Instead, techniques originally developed for quantum information processing experiments are used to map the clock transition information back to the $^9$Be$^+$ ion for readout [Schmidt 2005].

The fractional uncertainty with which the various trapped ion optical clock transitions have been measured relative to the SI second is shown in Table 8. The uncertainty of the best absolute frequency measurement, that for $^{199}$Hg$^+$ [Oskay 2006, Stalnaker 2007], is dominated by the uncertainty of the caesium microwave primary frequency standard and the associated measurement statistics, and the measurements for $^{88}$Sr$^+$ [Margolis 2004, Margolis 2006] and the $^{171}$Yb$^+$ quadrupole transition [Tamm 2007] are also approaching this limit. This points to the need for direct comparison of optical frequency standards without an intermediate microwave frequency reference. This task may be accomplished using a femtosecond optical frequency comb that is phase-locked to the frequency of one standard to determine the frequency ratio of the two standards. A comparison of the $^{199}$Hg$^+$ and $^{27}$Al$^+$ standards carried out in this way has led to the frequency ratio of their clock transitions being determined with a relative uncertainty of $5.2 \times 10^{-17}$ [Rosenband 2008], which means that the absolute frequency of the $^{27}$Al$^+$ clock transition is known with essentially the same relative frequency uncertainty as that of the $^{199}$Hg$^+$ clock transition.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>$\lambda$/nm</th>
<th>$(\delta \nu/\nu) / 10^{-15}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>267</td>
<td>0.65</td>
<td>[Rosenband 2008]</td>
</tr>
<tr>
<td>$^{40}$Ca$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>729</td>
<td>2.4</td>
<td>[Chwalla 2008]</td>
</tr>
<tr>
<td>$^{88}$Sr$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>674</td>
<td>3.8</td>
<td>[Margolis 2004, Margolis 2006]</td>
</tr>
<tr>
<td>$^{115}$In$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>237</td>
<td>180*</td>
<td>[von Zanthier 2000]</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>436</td>
<td>3.2</td>
<td>[Tamm 2007]</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2F_{7/2}$</td>
<td>467</td>
<td>11</td>
<td>Not yet published</td>
</tr>
<tr>
<td>$^{199}$Hg$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>282</td>
<td>0.65</td>
<td>[Oskay 2006, Stalnaker 2007]</td>
</tr>
</tbody>
</table>

Table 8. The fractional uncertainty $\delta \nu/\nu$ with which the clock transitions in trapped ion optical frequency standards have been measured. (*There is a discrepancy of 1.3 kHz between these two independent measurements of the $^1S_0 - ^3P_0$ transition in $^{115}$In$^+$, the reason for which has not yet been established.)
Figure 18. Theoretical quantum-limited instability for trapped ion optical frequency standards. These calculations assume an interrogation time equal to the natural linewidth of the upper state of the clock transition, except for the $^{171}\text{Yb}^+$ electric octupole transition and the $^{27}\text{Al}^+$ clock transition, for which a 5 s interrogation time has been used.

In terms of frequency stability, the best results reported to date are for the comparison of the $^{199}\text{Hg}^+$ and $^{27}\text{Al}^+$ standards, where a combined fractional frequency instability of $4 \times 10^{-15} \tau^{-1/2}$ has been demonstrated for averaging times up to 2000 s, representing an upper limit to the instability of each standard [Rosenband 2008]. For the $^{171}\text{Yb}^+$ quadrupole standard a frequency stability of approximately $9 \times 10^{-15} \tau^{-1/2}$ has been observed in the comparison of two similar systems [Peik 2006]. In both cases this is greater than the theoretical quantum-limited instability of the standards (Figure 18), because the clock transitions have not yet been interrogated at their natural linewidth.

It is clear from the results above that the performance of the best trapped ion optical frequency standards now exceeds that of the best caesium microwave primary frequency standards. These developments have led to the introduction of so-called secondary representations of the second by the international time and frequency metrology community, which is seen as a way to assist in the characterization of these highly reproducible standards [Gill 2006]. Three trapped ion standards, based on the electric quadrupole transitions in $^{199}\text{Hg}^+$, $^{88}\text{Sr}^+$ and $^{171}\text{Yb}^+$ were ratified as approved secondary representations of the second at the 2006 meeting of the International Committee for Weights and Measures (CIPM).

### 3.2.3 Systematic frequency shifts

Systematic frequency shifts that must be considered to determine the overall uncertainty budget for a trapped ion optical frequency standard include Zeeman shifts due to external magnetic fields, the electric quadrupole shift, second-order Doppler shifts, Stark shifts due to thermal motion and micromotion of the ion or due to applied light fields, and shifts due to the blackbody radiation field. As for any frequency standard, there will also be a gravitational redshift, which means that to achieve $10^{-18}$ fractional accuracy the height of the standard above the geoid needs to be known to 1 cm.
There are several methods by which systematic shifts may be studied experimentally. Absolute frequency measurements may be carried out relative to microwave standards [Barwood 2004, Margolis 2004], or frequency shifts may be measured relative to a high stability optical cavity [Oskay 2005]. However a more detailed characterisation of systematic frequency shifts and evaluation of system reproducibility requires direct comparison between two or more nominally similar systems [Barwood 2001, Schneider 2005]. The best results reported to date are for the comparison of the frequencies of the 2S_{1/2} → 2D_{3/2} reference transition in 171Yb⁺ for two single ions stored in two independent traps [Schneider 2005]. A mean fractional frequency difference of 3.8(6.1) × 10^{-16} was observed, comparable to the agreement found in the most accurate comparisons between caesium primary frequency standards.

3.2.3.1 Zeeman shifts

In general, the interaction between the magnetic moment of atomic states and an external magnetic field leads to a linear Zeeman shift of a transition frequency. However for clock transitions in the odd isotopes of alkali-like ions, which have half-integral nuclear spin, m_F = 0 → m_F = 0 clock transitions exist that are field-independent to first order. This is the case for the ¹⁹⁹Hg⁺ and ¹⁷¹Yb⁺ ions, which have nuclear spin I = 1/2 and hence relatively simple hyperfine structure. The second-order Zeeman shift remains (Table 9), but with operation in low magnetic fields of around 1 µT it should be possible to reduce the fractional uncertainty in this shift to the 10^{-17} to 10^{-18} level or lower, depending on the particular ion species.

Although transitions that are independent of magnetic field to first order can be identified in ⁸⁷Sr⁺ [Barwood 2003] and ⁴³Ca⁺ [Kajita 2005], these systems have rather complicated hyperfine structure due to their large nuclear spin (I = 9/2 for ⁸⁷Sr⁺ and I = 7/2 for ⁴³Ca⁺). It is therefore preferable to use the even isotope ions ⁸⁸Sr⁺ and ⁴⁰Ca⁺, and to eliminate the first-order Zeeman shift by repeatedly and sequentially probing two Zeeman components that are symmetrically placed about the centroid of the Zeeman multiplet [Bernard 1998]. The ¹⁶S₀ → ¹⁶P₀ transitions in ¹¹⁵In⁺ and ²⁷Al⁺ also have a linear dependence on magnetic field, which is eliminated in a similar way [Rosenband 2007]. The second-order Zeeman shift remains, but is negligible compared to other sources of uncertainty for the magnetic field strengths typically employed.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>λ / nm</th>
<th>Quadratic Zeeman shift / mHz µT⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>¹⁷¹Yb⁺</td>
<td>2S_{1/2} → 2D_{3/2}</td>
<td>436</td>
<td>60</td>
</tr>
<tr>
<td>¹⁷¹Yb⁺</td>
<td>2S_{1/2} → 2F_{7/2}</td>
<td>467</td>
<td>−1.72</td>
</tr>
<tr>
<td>¹⁹⁹Hg⁺</td>
<td>2S_{1/2} → 2D_{3/2}</td>
<td>282</td>
<td>−19</td>
</tr>
<tr>
<td>⁸⁸Sr⁺</td>
<td>¹⁶S₀ → ¹⁶P₀</td>
<td>674</td>
<td>0.0056</td>
</tr>
<tr>
<td>⁴⁰Ca⁺</td>
<td>²S_{1/2} → ²D_{3/2}</td>
<td>729</td>
<td>0.026, 0.017, 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>⁰m_j = 1/2, ³m_j = 3/2, ⁵m_j = 5/2</td>
</tr>
<tr>
<td>¹⁹⁹Hg⁺</td>
<td>²S_{1/2} → ²D_{3/2}</td>
<td>282</td>
<td>−19</td>
</tr>
</tbody>
</table>

Table 9. Quadratic Zeeman shift coefficients for the different trapped ion optical frequency standards.
3.2.3.2 Electric quadrupole shift

The electric quadrupole shift is due to the interaction between the electric quadrupole moments of the atomic states with any residual electric field gradient present at the position of the trapped ion. The magnitude of this effect depends on the ion species. For the \(^1\text{S}_0 - ^3\text{P}_0\) transitions in \(^{115}\text{In}^+\) and \(^{27}\text{Al}^+\) there is no quadrupole shift because both the upper and the lower levels of the clock transition have zero angular momentum and hence no quadrupole moment. For the other systems being studied as optical frequency standards, the electric quadrupole shift is due entirely to the shift of the upper level of the clock transition because the spherically symmetric \(^2\text{S}_{1/2}\) state has no quadrupole moment. Experimental measurements of quadrupole moments have been carried out for the \(^2\text{D}_{5/2}\) states in \(^{88}\text{Sr}^+\) [Barwood 2004], \(^{199}\text{Hg}^+\) [Oskay 2005] and \(^{40}\text{Ca}^+\) [Roos 2006] and for the \(^2\text{D}_{3/2}\) state in \(^{171}\text{Yb}^+\) [Tamm 2007], and are in reasonable agreement with theoretical calculations [Itano 2006, Sur 2006]. The results are summarized in Table 10.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>(\lambda) / nm</th>
<th>Electric quadrupole moment / ea_0^2</th>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{27}\text{Al}^+)</td>
<td>(^1\text{S}_0 - ^3\text{P}_0)</td>
<td>267</td>
<td>—</td>
<td>0</td>
<td>[Roos 2006]</td>
</tr>
<tr>
<td>(^{40}\text{Ca}^+)</td>
<td>(^2\text{S}<em>{1/2} - ^2\text{D}</em>{5/2})</td>
<td>729</td>
<td>1.83(1)</td>
<td>1.917</td>
<td>[Itano 2006]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.916</td>
<td>[Sur 2006]</td>
</tr>
<tr>
<td>(^{88}\text{Sr}^+)</td>
<td>(^2\text{S}<em>{1/2} - ^2\text{D}</em>{5/2})</td>
<td>674</td>
<td>2.6(3)</td>
<td>3.048</td>
<td>[Itano 2006]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.94</td>
<td>[Sur 2006]</td>
</tr>
<tr>
<td>(^{115}\text{In}^+)</td>
<td>(^1\text{S}_0 - ^3\text{P}_0)</td>
<td>237</td>
<td>—</td>
<td>0</td>
<td>[Itano 2006]</td>
</tr>
<tr>
<td>(^{171}\text{Yb}^+)</td>
<td>(^2\text{S}<em>{1/2} - ^2\text{D}</em>{5/2})</td>
<td>436</td>
<td>2.08(11)</td>
<td>2.174</td>
<td>[Itano 2006]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[Tamm 2007]</td>
</tr>
<tr>
<td>(^{171}\text{Yb}^+)</td>
<td>(^2\text{S}<em>{1/2} - ^2\text{F}</em>{7/2})</td>
<td>467</td>
<td>—</td>
<td>-0.22</td>
<td>[Webster 2004b]</td>
</tr>
<tr>
<td>(^{199}\text{Hg}^+)</td>
<td>(^2\text{S}<em>{1/2} - ^2\text{D}</em>{5/2})</td>
<td>282</td>
<td>-0.510(18)</td>
<td>-0.56374</td>
<td>[Itano 2006]</td>
</tr>
</tbody>
</table>

Table 10. Comparison of electric quadrupole moments of the upper state of the clock transition for trapped ion optical frequency standards.

Even with low flux ion loading techniques and good 3D micromotion compensation, residual field gradients within the trap can easily lead to quadrupole shifts of several hertz or more. Fortunately there are several ways of nulling this shift. Firstly, if measurements are carried out for three mutually perpendicular orientations of the applied magnetic field, the quadrupole shift averages to zero [Itano 2000]. The level of cancellation achievable using this technique depends on the accuracy in setting the three magnetic field directions, and has already been demonstrated to approximately 5 parts in \(10^{17}\) for \(^{199}\text{Hg}^+\) [Oskay 2006]. An alternative scheme, which does not rely on an accurate knowledge of the magnetic field direction, involves taking the average of measurements for several different Zeeman components [Dubé 2005]. Taking the example of the \(^{88}\text{Sr}^+\) ion, if measurements are carried out for three different pairs of Zeeman components that correspond to transitions for which the magnitude of the magnetic quantum number \(m_j\) of the upper level is 1/2, 3/2 and 5/2 respectively, the average quadrupole shift is zero, independent of the magnetic field direction. This nulling technique has been demonstrated to give good agreement with the first scheme [Margolis 2004], but has the advantage that the direction of the magnetic field does not need to be determined (it need only remain stable over the period of the measurement). A closely related scheme that has been proposed for nulling the quadrupole shift involves averaging the result over levels with different magnetic quantum numbers [Roos 2006].
3.2.3.3 Second-order Doppler shifts

Second-order Doppler shifts arise from residual thermal motion and micromotion of the ion within the trap.

The second-order Doppler shift due to the thermal (secular) motion can be estimated from the equation [Madej 2004]

$$\frac{\Delta \nu_{\text{Doppler}}}{\nu_{\text{thermal}}} = -\frac{3kT}{2Mc^2},$$  \hspace{1cm} (10)$$

where $k$ is the Boltzmann constant, $T$ is the ion temperature, $M$ is the mass of the ion and $c$ is the speed of light in vacuum. As long as the ion is cooled close to the Doppler cooling limit [Wineland 1979]

$$T_{\text{min}} = \frac{\hbar \gamma}{2k},$$  \hspace{1cm} (11)$$

where $\gamma$ is the natural linewidth of the cooling transition, then the fractional frequency shift due to the secular motion will approach the $10^{-18}$ level (Table 11). The effect can be further reduced by cooling the ion to the ground state of its motion within the trap, e.g. by resolved sideband cooling [Diedrich 1989]. As can be seen from equation (10), for similar ion temperatures the second-order Doppler shift due to the secular motion is smallest for the heaviest clock ions.

The second-order Doppler shift due to the micromotion can be much more significant, with careful minimisation of the micromotion in three dimensions being necessary to reduce this to better than the part in $10^{17}$ level. The size of the micromotion can be determined by measuring the effects of the first-order Doppler shift on an optical transition with natural linewidth $\gamma$ [Berkeland 1998], and minimised by adjusting the dc voltages applied to compensation electrodes within the trap.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>$\lambda$ / nm</th>
<th>$(\gamma/2\pi)$ / MHz</th>
<th>$T_{\text{min}}$ / mK</th>
<th>Fractional 2nd-order Doppler shift (secular motion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}\text{Ca}^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>729</td>
<td>22</td>
<td>[Jin 1993]</td>
<td>0.54</td>
</tr>
<tr>
<td>$^{88}\text{Sr}^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>674</td>
<td>22</td>
<td>[Gallagher 1967]</td>
<td>0.52</td>
</tr>
<tr>
<td>$^{115}\text{In}^+$</td>
<td>$^1S_{0} - ^1P_{0}$</td>
<td>237</td>
<td>0.36</td>
<td>[Peik 1994]</td>
<td>0.009</td>
</tr>
<tr>
<td>$^{171}\text{Yb}^+$</td>
<td>$^2S_{1/2} - ^2D_{3/2}$</td>
<td>436</td>
<td>20</td>
<td>[Berends 1993]</td>
<td>0.47</td>
</tr>
<tr>
<td>$^{171}\text{Yb}^+$*</td>
<td>$^2S_{1/2} - ^2F_{7/2}$</td>
<td>467</td>
<td>20</td>
<td>[Berends 1993]</td>
<td>0.47</td>
</tr>
<tr>
<td>$^{199}\text{Hg}^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>282</td>
<td>69</td>
<td>[Bergquist 1986]</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 11. Fractional second-order Doppler shift due to the secular motion, assuming that the ion is cooled to the Doppler cooling limit. The natural linewidths $\gamma$ of the cooling transitions and the Doppler cooling limit $T_{\text{min}}$ are also shown. The $^{27}\text{Al}^+$ standard is not included since in this case the ion is sympathetically cooled by a laser-cooled beryllium ion and secular mode heating causes the ion to be heated above the Doppler cooling limit. In the current state of development of the $^{27}\text{Al}^+$ standard, the second-order Doppler shift due to the secular motion is $(-1.6 \pm 0.8) \times 10^{-17}$ [Rosenband 2008].
In the case where the angular frequency of the trapping field $\Omega \gg \gamma$, the micromotion can be monitored by measuring the ratio between the scattering rate $R_1$ when the laser is tuned to the first micromotion sideband and the scattering rate $R_0$ when it is tuned to the carrier. In the low intensity limit the micromotion contribution to the second-order Doppler shift is then given by

$$\left( \frac{\Delta V_{\text{Doppler}}}{V} \right)_{\text{micromotion}} \approx -\left( \frac{\Omega}{\omega \cos \phi} \right)^2 \frac{R_1}{R_0},$$

where $\omega$ is the angular frequency of the transition and $\phi$ is the angle between the probe laser beam and the direction of the micromotion. In this case the optical transition used would typically be the clock transition.

In the case where $\Omega \ll \gamma$, the rf photon correlation technique can be used to monitor the modulation of the cooling laser fluorescence from the ion due to the first-order Doppler shift. In this case

$$\left( \frac{\Delta V_{\text{Doppler}}}{V} \right)_{\text{micromotion}} \approx -\frac{1}{4} \left( \frac{\gamma}{\omega \cos \phi} \frac{\Delta R_d}{R_{\text{max}}} \right)^2,$$

where $\Delta R_d$ is the amplitude of the detected fluorescence signal synchronous with the trap drive frequency and $R_{\text{max}}$ is the signal when the laser is tuned to line centre.

It can be seen from equations (12) and (13) that the micromotion contribution to the second-order Doppler shift scales quadratically with the wavelength of the clock transition.

### 3.2.3.4 Stark shifts

Stark shifts of the clock transition frequency can arise in a number of ways, including motionally-induced exposure of the ion to electric fields, interactions with the blackbody radiation field, and exposure to the various light fields required to cool and probe the trapped ion [Itano 2000, Madej 2004].

Considering first the motionally-induced Stark shift, the micromotion and the thermal motion of the ion cause it to experience a non-zero rms value of the electric field $E$, the magnitude and constancy of which can be affected by stray charge within the trap. With careful micromotion compensation, it should be possible to reduce the micromotion-induced Stark shift [Berkeland 1998] to a few parts in $10^{18}$. As for the micromotion-induced second-order Doppler shift, the micromotion-induced Stark shift scales quadratically with the wavelength of the clock transition. The thermal motion contribution to the Stark shift is given by

$$\left( \Delta V_{\text{Stark}} \right)_{\text{thermal}} \approx \sigma_S \frac{3M\Omega^2kT}{q^2},$$

where $\sigma_S$ is the Stark shift coefficient ($\partial \nu / \partial E$) of the clock transition, $M$ is the mass of the ion and $q$ is its charge. For an ion cooled to the Doppler cooling limit, this secular motion contribution to the Stark shift is likely to be substantially lower than the contribution from micromotion.

There will also be a blackbody Stark shift because the ion experiences a spectrum of blackbody radiation due to the temperature $T$ of the surrounding apparatus. For a trap that is operated at room temperature, the associated blackbody Stark shift (referenced to absolute
zero) is typically in the region of 100 – 500 mHz, depending on the scalar polarisabilities for the levels concerned (Table 12). The shift is relatively large but reasonably constant, with a temperature variation of 1 K leading to a change of typically 10 mHz in the blackbody Stark shift. The uncertainty in the absolute value of the correction, however, is determined by the typical 30% uncertainties in the Stark shift coefficients. Amongst the room temperature systems being investigated as optical frequency standards, the $^1S_0 - ^3P_0$ transition in $^{27}$Al$^+$ has the lowest fractional blackbody Stark shift at $-8(3)$ parts in $10^{18}$ for a temperature of 300 K [Rosenband 2006]. The blackbody Stark shift scales as $T^4$. This means that for the $^{199}$Hg$^+$ standard, which is operated in a liquid helium cryostat, the blackbody Stark shift is seven orders of magnitude lower than at room temperature, and hence negligible.

### Table 12. Fractional blackbody Stark shifts at 300 K for systems being studies as trapped ion optical frequency standards. Where no uncertainty is given, it is unknown. The $^{199}$Hg$^+$ optical clock transition is not included in the table because this standard is operated at 4.2 K and so the shift is negligible.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Clock transition</th>
<th>$\lambda$ / nm</th>
<th>Fractional blackbody Stark shift at 300 K ($\times 10^{16}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>267</td>
<td>$-0.08(3)$ [Rosenband 2006]</td>
</tr>
<tr>
<td>$^{40}$Ca$^+$</td>
<td>$^4S_{1/2} - ^2D_{5/2}$</td>
<td>729</td>
<td>9.3(2) [Arora 2007]</td>
</tr>
<tr>
<td>$^{90}$Sr$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>674</td>
<td>7.4(2) [Lea 2006]</td>
</tr>
<tr>
<td>$^{115}$In$^+$</td>
<td>$^1S_0 - ^1P_0$</td>
<td>237</td>
<td>0.38 [Wang 2006]</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2D_{3/2}$</td>
<td>436</td>
<td>$-5.4(7)$ [Tamm 2007]</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2F_{7/2}$</td>
<td>467</td>
<td>$-2.4(1.1)$ [Lea 2006]</td>
</tr>
</tbody>
</table>

Ac Stark shifts can arise due to the interaction of the electric field of the probe laser with the light-field-induced electric dipole moment in the ion. The overall shift is primarily due to the off-resonant interaction of the light with transitions out of the two levels of the clock transition, and therefore depends on the light intensity. For typical probe laser intensities used to drive the clock transition, the associated shifts are very small. The exception is the 467 nm electric octupole transition in $^{171}$Yb$^+$. At current probe laser linewidths, significant light intensity is required to drive this very weak transition at a reasonable rate, and so the transition frequency is measured as a function of laser power and extrapolated to zero power to determine the unshifted value [Hosaka 2005]. However, as the probe laser linewidth is improved towards the 0.1 Hz level, the power needed to maintain the spectral intensity necessary to drive the transition at the same rate will be reduced in proportion, and the ac Stark shift will cease to dominate the uncertainty budget.

Ac Stark shifts can also arise from the cooling or repumper radiation, and so it is important that these beams are properly extinguished during the probe laser interrogation periods.

### 3.2.3.5 Other effects

The potential existence of other, more technical, frequency shifts must also be considered. To take just one example, recent studies of the $^{199}$Hg$^+$ and $^{27}$Al$^+$ standards at NIST investigated potential residual first-order Doppler shifts, which may occur if the ion trap moves in a way that is correlated with the clock laser pulses [Rosenband 2008]. The list of systematic frequency shifts considered here is therefore not exhaustive and in general shifts that were not previously accounted for can become significant as the accuracy of optical frequency standards continues to improve.
3.2.4 Technology considerations

The laser wavelengths required for cooling and probing the various trapped ion species used as optical clocks are shown in Table 13, along with other technology factors that affect the feasibility of constructing an optical clock for space applications. Technology aspects of the different systems are discussed in more detail in the following subsections.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Cooling laser wavelength</th>
<th>Clock laser wavelength</th>
<th>Other technology considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al$^+$</td>
<td>Sympathetically cooled using $^9$Be$^+$ ion that is laser cooled at 313 nm</td>
<td>267 nm</td>
<td>Quantum logic techniques required; state readout requires second laser at 267 nm</td>
</tr>
<tr>
<td>$^{40}$Ca$^+$</td>
<td>397 nm (+ 866 nm repumper)</td>
<td>729 nm</td>
<td>Clearout laser @ 854 nm desirable</td>
</tr>
<tr>
<td>$^{88}$Sr$^+$</td>
<td>422 nm (+ 1092 nm repumper)</td>
<td>674 nm</td>
<td>Clearout laser @ 1033 nm desirable</td>
</tr>
<tr>
<td>$^{115}$In$^+$</td>
<td>231 nm</td>
<td>237 nm</td>
<td></td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>369 nm (+ 935 nm repumper)</td>
<td>436 nm</td>
<td>Clearout laser @ 638 nm essential</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>369 nm (+ 935 nm repumper)</td>
<td>467 nm</td>
<td>Clearout laser @ 638 nm essential</td>
</tr>
<tr>
<td>$^{199}$Hg$^+$</td>
<td>194 nm</td>
<td>282 nm</td>
<td>Cryogenic (liquid helium) operation</td>
</tr>
</tbody>
</table>

Table 13. Laser wavelengths required to cool and probe different trapped ion species.

3.2.4.1 $^{27}$Al$^+$

Although impressive performance has been demonstrated for the $^{27}$Al$^+$ optical clock by the NIST group [Rosenband 2008], it is technically more complex than the other systems being studied. As discussed in section 3.2.2, the lack of a suitable laser system to access the deep UV cooling transition at 167 nm necessitates the use of a second ion species such as $^9$Be$^+$ to sympathetically cool the ion and quantum logic spectroscopy [Schmidt 2005] to map the information about the clock state of the $^{27}$Al$^+$ ion to the $^9$Be$^+$ ion for readout. Although 99.94% detection fidelity of the clock state has been demonstrated for this system at NIST [Hume 2007], the quantum logic techniques lead to additional complexity in the algorithm for observation of the clock transition.

The laser wavelengths required are also challenging (Figure 19). In the NIST arrangement, the 313 nm beams required to drive the Doppler-cooling, stimulated Raman and repumping transitions in $^9$Be$^+$ are derived from three frequency-doubled dye lasers [Monroe 1995], a fourth frequency-doubled dye laser at 267.0 nm is used to drive the $^1S_0$–$^3P_1$ transition in $^{27}$Al$^+$ (used for optical pumping and quantum state transfer) [Schmidt 2005] and the clock laser is a frequency-quadrupled fibre laser at 267.4 nm [Rosenband 2007]. One possibility being investigated at NIST is to change the sympathetic cooling ion from Be$^+$ to Mg$^+$, which uses a simpler (fibre-based) cooling laser at 280 nm.
Figure 19. Partial term diagrams for the $^{27}$Al$^+$ and $^9$Be$^+$ ions (not to scale), showing the 313 nm transitions used for Doppler and Raman cooling of the $^9$Be$^+$ ion, the inaccessible cooling transition at 167 nm in $^{27}$Al$^+$ and the clock transition at 267.4 nm. The additional transition at 267.0 nm is required for the process of quantum state transfer between the $^{27}$Al$^+$ and $^9$Be$^+$ ions [Rosenband 2007].

Since there is a linear Zeeman shift in $^{27}$Al$^+$, it is necessary to probe two Zeeman components symmetrically placed around line centre in order to cancel the shift to first order [Rosenband 2007]. Magnetic shielding is therefore essential to prevent fluctuations of the linear Zeeman shift from limiting the stability of the standard. One further issue for a space-borne clock may be the ion storage time; in the present arrangement the ion pair is typically destroyed after several hours by a chemical reaction with background gas atoms [Rosenband 2007].

3.2.4.2 $^{40}$Ca$^+$

The $^{40}$Ca$^+$ optical clock has the advantage that all wavelengths necessary for laser cooling and probing the trapped ion (Figure 20), including those required for photoionization, can be generated by commercially available laser diodes.

Figure 20. Partial term diagram for $^{40}$Ca$^+$ (not to scale), showing the laser wavelengths required for operation of an optical clock based on the $^3S_{1/2} - ^3D_{5/2}$ transition at 729 nm.
The cooling wavelength for the $^{40}$Ca$^+$ standard is 397 nm. One possibility is to use a UV diode laser to obtain this wavelength directly [Matsubara 2008]. However UV diodes have significantly reduced tuning range compared to those in the visible and near-infrared, and obtaining diodes close to the 397 nm transition is becoming increasingly difficult as the industry standard moves closer to 405 nm. An alternative is to generate the cooling wavelength by frequency doubling of 794 nm radiation. Since tapered amplifiers exist at this wavelength relatively simple single pass doubling in a periodically poled crystal should generate sufficient power at 397 nm for the three cooling beams required to provide micromotion control in three dimensions. Diode lasers are also available at the clock transition wavelength of 729 nm, the repumper wavelength of 866 nm and the clearout wavelength of 854 nm [Matsubara 2008]. As for $^{27}$Al$^+$, there is a linear Zeeman shift of the clock transition frequency and so magnetic shielding is necessary.

### 3.2.4.3 $^{88}$Sr$^+$

The $^{88}$Sr$^+$ optical clock, like $^{40}$Ca$^+$, has the technical advantage that all wavelengths necessary for laser cooling and probing the trapped ion (Figure 21), including those required for photoionization [Brownnutt 2007], can be generated by commercially available laser diodes.

The laser cooling wavelength is 422 nm, and single pass doubling of an 844 nm diode laser in periodically poled KTP can readily provide enough power at 422 nm for the three cooling beams required to provide micromotion control in three dimensions. For the clock transition, extended cavity diode lasers at 674 nm are readily available, and DFB lasers are available at the repumper wavelength of 1092 nm and the clearout wavelength of 1033 nm. As for $^{27}$Al$^+$ and $^{40}$Ca$^+$, there is a linear Zeeman shift of the clock transition frequency in $^{88}$Sr$^+$, and so magnetic shielding is essential to avoid degrading the stability of the standard.

![Figure 21. Partial term diagram for $^{88}$Sr$^+$ (not to scale), showing the laser wavelengths needed to cool and probe the trapped ion.](image)

### 3.2.4.4 $^{115}$In$^+$

The $^{115}$In$^+$ optical clock is technically challenging because it requires UV radiation both to cool the ion and to probe the clock transition (Figure 22). Although the clock transition has a useful coincidence with the fourth harmonic of a Nd:YAG laser at 946 nm, the two doubling stages required mean that efficient power build-up must be maintained in two enhancement cavities [Hollemann 1995]. The $^{115}$In$^-$ ion is cooled on the $5s^2 3^2S_0 - 5s5p 3^2P_1$ intercombination line at 230.6 nm, which is significantly weaker than the cooling transitions in other systems, meaning that higher power levels and bichromatic sideband cooling [Peik 1999] are required.
In early experiments a frequency-doubled dye laser was used to generate the cooling laser radiation, but in more recent work this has been replaced by a diode-based laser system in which an amplified laser at 922 nm is frequency quadrupled using two sequential enhancement cavities [Eichenseer 2003].

![Figure 22. Partial term diagram for $^{115}\text{In}^+$ (not to scale), showing the laser wavelengths required for operation of an optical clock based on the $^1S_0 \rightarrow ^3P_0$ transition at 236.5 nm.](image)

### 3.2.4.5 $^{171}\text{Yb}^+$

The $^{171}\text{Yb}^+$ quadrupole standard requires frequency-doubling stages to produce both the cooling and the clock transition wavelengths (369 nm and 436 nm respectively). Diode lasers can be used in both cases [Tamm 2000], although the fundamental wavelength of 738 nm for the cooling transition represents a slightly difficult region from the point of view of diode laser availability. Since the extremely long-lived $^2F_{7/2}$ state is populated at a rate of up to $1 \text{ hour}^{-1}$ [Tamm 2000], a clearout laser at 638 nm is necessary in addition to the repumper laser at 935 nm. Diode lasers are readily available at both these wavelengths. Since the $^{171}\text{Yb}^+$ standard is based on an odd isotope, the energy level structure is complicated by hyperfine structure (Figure 23), meaning that modulated or stepped wavelengths (or even two separate wavelengths) are required at the cooling, repumper and clearout transitions.

![Figure 23. Low-lying levels of $^{171}\text{Yb}^+$ (not to scale), showing the laser wavelengths necessary for operation of an optical clock based on the $^2S_{1/2} \rightarrow ^2D_{3/2}$ transition at 436 nm or the $^2S_{1/2} \rightarrow ^2F_{7/2}$ transition at 467 nm.](image)
The $^{171}\text{Yb}^+$ octupole transition is a more challenging option for a space-borne optical clock. Although the 467 nm radiation needed to probe this clock transition can again be produced using a frequency-doubled diode laser, the probe laser beam has to be focussed very tightly to ensure that a spectral intensity sufficient to drive the extremely weak clock transition at a reasonable rate is achieved. Although the narrow natural linewidth of the clock transition means that longer interrogation times can in principle be achieved compared to other systems, this results in longer servo correction times and hence places greater constraints on the thermal noise of the ULE cavity used to stabilize the probe laser. These factors combine to make stand-along operation for long periods very challenging.

3.2.4.6 $^{199}\text{Hg}^+$

The $^{199}\text{Hg}^+$ optical clock is a relatively complex system compared to many of the other ions being studied. One technical challenge is the need to generate deep UV cooling laser radiation at 194 nm (Figure 24). In the NIST system this is achieved by sum-frequency generation of 792 nm and 257 nm radiation, with the 257 nm light being generated by frequency doubling the light from a single-frequency argon-ion laser and the 792 nm light being produced by either an amplified diode laser [Berkeland 1997] or an injection-locked Ti:sapphire laser [Tanaka 2003]. As for $^{171}\text{Yb}^+$, the energy level structure is complicated by hyperfine structure, and so an additional offset-locked 194 nm beam is required to avoid weak off-resonant pumping into the $F = 0$ ground state. The clock laser radiation at 282 nm is produced by a frequency-doubled diode laser [Young 1999a], but an alternative would be to use a frequency-quadrupled fibre laser.

Another technical challenge is the need for cryogenic cooling of the ion trap apparatus to prevent loss of the Hg$^+$ ion, which at room temperature can result from collisions with background neutral mercury and hydrogen in which Hg$_2^+$ or HgH$^+$ ions are formed [Tanaka 2003]. The existing clock apparatus at NIST uses liquid helium cooling, but a closed-cycle refrigerator could in principle be used instead, and cryogenic cooling with sorption coolers is an established space technology.

![Figure 24. Partial term diagram for $^{199}\text{Hg}^+$ (not to scale), showing the laser wavelengths needed to cool and probe the trapped ion.](image_url)
3.3 Atomic reference: neutral atom optical lattice clocks

The alternative technology for developing optical frequency standards is based on ensembles of cold atoms. The potential advantage of these standards is the $\sqrt{N}$ improvement in stability offered by ensembles of $N$ atoms within a cloud, an optical lattice, or a continuous beam.

3.3.1 Principles of operation

Although neutral atom optical frequency standards have better potential stability than optical clocks based on single ions due to the increased signal-to-noise ratio available from a large number of atoms, until recently they suffered from larger systematic errors. This was primarily due to the fact that the atoms could not be held trapped during the clock measurement because of induced ac-Stark or magnetic-field-related shifts of the atomic energy levels. Instead they were studied in atomic beams, such as in high-resolution spectroscopy of the two-photon $1S - 2S$ transition in atomic hydrogen [Niering 2000, H"ansch 2005], or were released from a magneto-optical trap (MOT) to expand ballistically during the measurement of the clock transition frequency, as in the extensively studied calcium optical frequency standard based on the $1S_0 - 3P_1$ intercombination line at 657 nm in atomic calcium [Degenhardt 2005, Wilpers 2006, Wilpers 2007]. The ballistic expansion of the atomic cloud under gravity limits the maximum interaction time that can be achieved, restricting clock operation to the relatively broad $1S_0 - 3P_1$ transition (natural linewidth 400 Hz in Ca). The ballistic expansion also gives rise to residual first-order Doppler shifts.

Single ions, in contrast, are held in traps that do not strongly perturb the energy levels of the ion, reducing the effects of systematic frequency shifts in the clock measurement. A paradigm shift occurred when it was postulated that neutral atoms could be held in a far-detuned lattice dipole trap (Figure 25) during the clock interrogation cycle, provided that the frequency of the lattice laser was chosen such that it shifted both the ground and excited states of the clock transition by precisely the same amount [Katori 2002, Katori 2003]. The phrase “optical lattice clock” was coined to describe such a clock system, and clock spectroscopy of $^{87}\text{Sr}$ atoms held in an optical lattice at the “magic” zero-shift wavelength was demonstrated for the first time in 2003 [Takamoto 2003].

![Figure 25. A 1D optical lattice trap formed by the interference pattern generated when light is retroreflected back on itself.](image)

48
3.3.2 Candidate systems and current performance

At present, there are a number of groups around the world working with neutral atom optical frequency standards, and although some of the ballistic neutral atom optical standards are still in use, the majority of laboratories actively developing frequency standards have begun programmes in optical lattice clocks. Due to the long coherence times inherent in lattice trapping clocks as well as advances in the stabilization of extremely narrow linewidth lasers, lattice clocks are able to exploit the much narrower hyperfine-induced $^1S_0 - ^3P_0$ transitions found in the fermionic (odd) isotopes of alkaline-earth elements such as Be, Mg, Ca and Sr. Similar transitions are also found in the group IIb transition metal elements such as Zn, Cd and Hg, in rare-gas atoms and in Yb in the lanthanide series. The linewidths of these clock transitions range from around 100 mHz to less than 1 mHz (Table 14), and it was the $^1S_0 - ^3P_0$ transition in $^{87}$Sr that was described as the first potential candidate for an optical lattice clock [Katori 2003].

<table>
<thead>
<tr>
<th>Atom</th>
<th>$I$</th>
<th>$\lambda$ / nm</th>
<th>$\Delta\nu_{\text{nat}}$ / mHz</th>
<th>[Source]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{25}$Mg</td>
<td>5/2</td>
<td>457</td>
<td>0.44</td>
<td>[Porsev 2004a]</td>
</tr>
<tr>
<td>$^{43}$Ca</td>
<td>7/2</td>
<td>659</td>
<td>2.2</td>
<td>[Porsev 2004a]</td>
</tr>
<tr>
<td>$^{87}$Sr</td>
<td>9/2</td>
<td>698</td>
<td>7.6</td>
<td>[Porsev 2004a]</td>
</tr>
<tr>
<td>$^{171}$Yb</td>
<td>1/2</td>
<td>578</td>
<td>43.5</td>
<td>[Porsev 2004a]</td>
</tr>
<tr>
<td>$^{173}$Yb</td>
<td>5/2</td>
<td>578</td>
<td>38.6</td>
<td>[Porsev 2004a]</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>1/2</td>
<td>266</td>
<td>$\sim$100</td>
<td>[Bigeon 1967]</td>
</tr>
<tr>
<td>$^{201}$Hg</td>
<td>3/2</td>
<td>266</td>
<td>$\sim$100</td>
<td>[Bigeon 1967]</td>
</tr>
</tbody>
</table>

Table 14. Properties of fermionic neutral atom optical frequency standards currently under development, including the nuclear spin $I$ and the wavelength $\lambda$ and natural linewidth $\Delta\nu_{\text{nat}}$ of the clock transition.

However there have also been a number of ideas raised for using the highly forbidden $^1S_0 - ^3P_0$ transition in the bosonic (even) isotopes of these atoms, which are spectroscopically simpler because there is no hyperfine structure. Schemes that have been proposed include methods based on multi-photon excitation [Hong 2005], electromagnetically-induced transparency (EIT) [Santra 2005], and state mixing induced by externally applied magnetic [Taichenachev 2006] or electric (circularly-polarized lattice) fields [Ovsiannikov 2007]. The only technique that has been explored experimentally to date is the application of a small magnetic field to mix the $^3P_1$ and the $^3P_0$ states so that the $^1S_0 - ^3P_0$ transition becomes allowed. The first observation of the $^1S_0 - ^3P_0$ clock transition in $^{174}$Yb [Barber 2006] followed quickly from the original proposal, and the first accuracy evaluation of an optical lattice clock based on the $^1S_0 - ^3P_0$ transition in $^{88}$Sr has recently been reported [Baillard 2007]. One unique feature of this method is that the linewidth of the clock transition is tunable, so that it may be chosen to yield optimum clock performance.

Although a significant level of research is still required to yield an improved knowledge of the magnitude of various frequency shifts (section 3.3.3) under actual operating conditions, the development of lattice clocks has proceeded rapidly. Probe-limited resonance linewidths of 1.8 Hz have recently been achieved for the $^1S_0 - ^3P_0$ clock transition in $^{87}$Sr, corresponding to a $Q$-factor of $2.4 \times 10^{14}$, the highest obtained for any coherent spectral feature [Boyd 2006]. Furthermore, high accuracy frequency measurements of this transition have been reported by three separate groups (JILA, LNE-SYRTE and Tokyo), with agreement at the 1 part in $10^{14}$ level [Takamoto 2006, Baillard 2008, Campbell 2008], and this system was accepted as a secondary representation of the second [Gill 2006] at the 2006 meeting of the International Committee for Weights and Measures (CIPM). The best absolute frequency measurement
reported to date has an uncertainty of 0.37 Hz, corresponding to a fractional uncertainty of $8.6 \times 10^{-16}$ [Campbell 2008], and is limited mainly by the performance of the hydrogen maser used to compare the $^{87}$Sr standard to the NIST-F1 caesium fountain primary frequency standard. By comparison with a calcium optical atomic clock with high short-term stability, the systematic fractional frequency uncertainty has been estimated to be $1.5 \times 10^{-15}$, with the largest contribution arising from the blackbody Stark shift (section 3.3.3.2). The stability of the strontium-calcium comparison was approximately $4 \times 10^{-15} \tau^{-1/2}$. To date there is only a single high accuracy measurement of the frequency of the $^{1}S_{0} - ^{3}P_{0}$ clock transition in $^{88}$Sr, with an fractional uncertainty of $7 \times 10^{-14}$ [Baillard 2007]. In addition to the experiments discussed above, neutral strontium lattice clocks are also under development at a number of other laboratories worldwide, including NPL, PTB, LENS and NRC.

Lattice clocks based on the $^{1}S_{0} - ^{3}P_{0}$ transitions in $^{171}$Yb, $^{173}$Yb or $^{174}$Yb are being actively studied at NIST [Barber 2006] and KRISS [Park 2003], with additional experiments being set up at the University of Düsseldorf, INRIM and NMIJ/AIST. At NIST, work has concentrated on the bosonic isotope $^{174}$Yb, in which magnetically-induced spectroscopy has been used to observe the $^{1}S_{0} - ^{3}P_{0}$ transition with a width of 5 Hz, limited by the spectroscopic probe time [Barber 2007]. The absolute frequency of the clock transition has been measured with a fractional uncertainty of $1.7 \times 10^{-15}$, and a stability of better than $5.5 \times 10^{-15} \tau^{-1/2}$ demonstrated by comparison with the NIST $^{199}$Hg$^{+}$ optical clock [Poli 2008].

Alternatives to the strontium and ytterbium lattice clocks are clocks based on the $^{1}S_{0} - ^{3}P_{0}$ transitions in mercury, which is being studied at LNE-SYRTE and the University of Tokyo, and in magnesium, which is being investigated at the University of Hannover. These are of interest because of their relatively low sensitivity to blackbody radiation (section 3.3.3.2). Due to the technological challenges associated with the deep UV laser sources required (section 3.3.4), magneto-optical trapping of mercury isotopes has only recently been demonstrated [Hachisu 2008], but was shortly followed by the first direct observation of the $^{1}S_{0} - ^{3}P_{0}$ clock transition in the $^{199}$Hg and $^{201}$Hg isotopes [Petersen 2008]. The clock transition frequencies were determined with a fractional uncertainty of approximately $5 \times 10^{-12}$. Laser-cooling of magnesium also presents technical challenges, and it is only recently that temperatures below the Doppler cooling limit of 1.9 K have been reported [Mehlstäubler 2008]. However only the $^{1}S_{0} - ^{3}P_{1}$ clock transition has been studied to date [Friebe 2007].

### 3.3.3 Systematic frequency shifts

Since the atoms are held in the optical lattice during the spectroscopy of the clock transition, many of the systematic shifts that arise from the motion of the atoms through the spectroscopy laser in the beam or ballistic standards are eliminated. In particular, for atoms that have been cooled to the Lamb-Dicke regime [Dicke 1953] in the lattice, Doppler and recoil effects become insignificant.

The systematics relevant to an optical lattice clock depend on whether a fermionic (odd) or bosonic (even) isotope is used, and according to the spectroscopic method. In the following sections we consider Zeeman shifts arising from external magnetic fields, Stark shifts (including the effects of blackbody radiation) and collisional shifts.

#### 3.3.3.1 Zeeman shifts

Although the $^{1}S_{0}$ and $^{3}P_{0}$ states in odd isotopes of alkaline-earth atoms are magnetically insensitive to lowest order, the hyperfine-induced state mixing that renders the clock transition observable also leads to differential $g$-factors between the clock states. This effect results in a linear Zeeman shift of the $^{1}S_{0} - ^{3}P_{0}$ transition (Table 15). For $^{87}$Sr, the differential
g-factor has been measured accurately [Boyd 2007a] and is in good agreement with theory. The linear Zeeman shift can cause undesirable effects in terms of broadening of the transition, as well as line centre shifts due to asymmetric distribution of atomic population between the magnetic sublevels. However in practice, the linear Zeeman shift can be eliminated by carrying out experiments on resolved sublevels and averaging out the effect by alternating between measurements of levels with equal magnitude but opposite signs of the $m_F$ quantum number, for example using atoms that are spin polarized to the two stretched states with $m_F = \pm \frac{9}{2}$ in $^{87}$Sr [Takamoto 2006, Boyd 2007a]. This greatly reduces the otherwise stringent requirements on the control and size of the magnetic fields that can be present during the clock measurement.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Linear Zeeman shift (for $\pi$-polarized transitions) / Hz µT$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{87}$Sr</td>
<td>$-1.088(4)m_F$ [Boyd 2007a]</td>
</tr>
<tr>
<td>$^{171}$Yb</td>
<td>$-4.1 m_F$ [Porsev 2004b]</td>
</tr>
<tr>
<td>$^{171}$Yb</td>
<td>$+1.1 m_F$ [Porsev 2004b]</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>$+6.6 m_F$ [Hachisu 2008]</td>
</tr>
<tr>
<td>$^{201}$Hg</td>
<td>$-2.5 m_F$ [Hachisu 2008]</td>
</tr>
</tbody>
</table>

Table 15. Linear Zeeman shift of the $^1S_0 - ^3P_0$ clock transition ($\pi$-polarized transitions) in neutral atom optical frequency standards based on odd isotopes. The shifts for the odd isotopes of Mg and Ca have not yet been determined.

In the case of the even isotopes, the linear Zeeman shift of the clock transition is zero because the $g$-factors of the ground and excited states are essentially identical. However there remains a second-order Zeeman shift (Table 16).

<table>
<thead>
<tr>
<th>Atom</th>
<th>Quadratic Zeeman shift / Hz mT$^{-2}$</th>
<th>Fractional shift for a field of 1 mT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>$-217$ [Taichenachev 2006]</td>
<td>$-3.3 \times 10^{-13}$</td>
</tr>
<tr>
<td>Ca</td>
<td>$-83.5$ [Taichenachev 2006]</td>
<td>$-1.8 \times 10^{-13}$</td>
</tr>
<tr>
<td>Sr</td>
<td>$-24.9(1.7)$ [Baillard 2008]</td>
<td>$-5.8 \times 10^{-14}$</td>
</tr>
<tr>
<td>Yb</td>
<td>$-6.12(10)$ [Poli 2008]</td>
<td>$-1.2 \times 10^{-14}$</td>
</tr>
<tr>
<td>Hg</td>
<td>$-24.4$ [Hachisu 2008]</td>
<td>$-2.2 \times 10^{-14}$</td>
</tr>
</tbody>
</table>

Table 16. Quadratic Zeeman shift of the $^1S_0 - ^3P_0$ clock transition in neutral atom optical frequency standards.

### 3.3.3.2 Stark shifts

Stark shifts of the clock transition frequency can arise from exposure to the light fields required to trap and probe the atoms, and from interactions with the blackbody radiation field. Ac Stark shifts scale linearly with laser intensity (quadratically with electric field). The ac Stark shift due to the laser used to probe the clock transition is due mainly to the off-resonant coupling of the $^3P_0$ state with the $^3S_1$ state. In principle this shift is zero in even isotopes, but is rendered non-zero by the magnetic field-induced state mixing that is used to make the transition observable. For either the even or the odd isotopes, the shift must be explored experimentally because the shift depends strongly on the atom temperature and trap parameters, which are hard to measure precisely in real time. However the shift is straightforward to investigate experimentally by interrogating the clock transition with varying probe laser intensities. For normal experimental parameters in $^{87}$Sr, the shift is of
order 10 – 100 mHz [Boyd 2007b, Baillard 2008], but the lowest shift so far demonstrated for a bosonic system is around 7 Hz for $^{174}$Yb, with 0.2 Hz uncertainty [Poli 2008]. This can, however, be expected to reduce as the linewidth of the transition is reduced in future experiments. Shifts due to the lasers used for cooling and detection can be eliminated by switching them off during the clock transition interrogation using a combination of acoustooptic modulators and mechanical shutters.

Lattice clock operation with low systematic frequency uncertainty of course relies very strongly on the ability to operate at the “magic” wavelength where the ac Stark shifts of the lower and upper levels of the clock transition due to the lattice laser are equal to first order (Table 17). This means that it is important to consider the possibility of second- or higher-order effects. The clock transition frequency in a lattice of well depth $U_0$ can be written in terms of the unperturbed frequency $\nu_0$ plus first and higher-order shift terms:

$$\nu = \nu_0 + v_1 \frac{U_0}{E_r} + \nu_0 \left( \frac{U_0}{E_r} \right)^2 + \cdots,$$

(15)

where $E_r$ is the absorption recoil energy and $v_1$ is zero at the magic wavelength. However the higher order terms do not vanish and may be enhanced by nearby two-photon resonances. For example, in $^{87}$Sr, the 5s5p $^1P_0$ – 5s7p $^1P_1$ and the 5s5p $^1P_0$ – 5s4f $^3F_2$ two-photon transitions at 813.36 nm and 818.57 nm lie very close to the 813.428 nm magic wavelength. The former transition is only 30 GHz away from the magic wavelength, but is forbidden to leading order, whilst the latter is further away but fully allowed. Theoretical calculations of the contributions from these two-photon transitions indicate that they should be in the microhertz range under typical operating conditions [Katori 2003], in line with experimental evidence [Brusch 2006]. The effect of two-photon resonances on the magic wavelength has also been investigated for $^{174}$Yb [Barber 2008], and is not expected to limit the accuracy of a lattice clock at the $10^{-17}$ level. The magnitude of higher-order light shifts in a mercury lattice clock remains to be explored experimentally.

Experimentally, the overall ac Stark shift of the clock transition frequency in $^{87}$Sr as a function of lattice detuning from the magic wavelength has been found to scale as 2 mHz / MHz [Ludlow 2006]. In the most precise studies of the $^{87}$Sr lattice clock reported to date, typical operating parameters result in an ac Stark shift due to the lattice laser of $6.5(5) \times 10^{-16}$ compared to zero intensity [Ludlow 2008a]. In bosonic isotopes, lattice polarization effects occur in the presence of the magnetic fields that are needed to observe the clock transition [Taichenachev 2007]; however experimental geometries have been identified that can be used to reduce or potentially eliminate these effects completely.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Magic wavelength (nm)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>432</td>
<td>[Ovsiannikov 2007]</td>
</tr>
<tr>
<td>Ca</td>
<td>680</td>
<td>[Ovsiannikov 2007]</td>
</tr>
<tr>
<td>$^{87}$Sr</td>
<td>813.428(1)</td>
<td>[Brusch 2006]</td>
</tr>
<tr>
<td>$^{174}$Yb</td>
<td>759.35374(7)</td>
<td>[Barber 2008]</td>
</tr>
<tr>
<td>Hg</td>
<td>360</td>
<td>[Hachisu 2008]</td>
</tr>
</tbody>
</table>

Table 17. “Magic” wavelengths at which the ac Stark shifts of the $1S_0$ and $3P_0$ levels of the clock transition due to the lattice laser are exactly equal. The values for $^{87}$Sr and $^{174}$Yb are experimental; the remainder are calculated values. (All wavelengths are vacuum values.)
There will also be a blackbody Stark shift because the atoms are bathed in a spectrum of blackbody radiation due to the temperature \( T \) of their surroundings. Calculated values of the fractional blackbody Stark shift at 300 K are shown in Table 18, and the shift scales as \( T^4 \). The shift is smallest for Hg, and relatively large for the two most commonly studied systems, Sr and Yb. A temperature change of 1 K leads to a change of 30 mHz in the blackbody Stark shift for Sr, corresponding to a fractional frequency shift of the clock transition of \( 7 \times 10^{-17} \). Experimentally, significant work therefore still needs to be done to fully quantify the blackbody Stark shift for this and other systems at the level necessary to reach uncertainties at the \( 10^{-17} \) level or below. In particular uncertainties in the blackbody Stark shift coefficients need to be reduced and the distribution of ambient temperature (e.g. due to the operation of magnetic field coils and any line of sight to the hot oven) needs careful consideration. The next generation of laboratory experiments is being built with this in mind.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Fractional blackbody Stark shift at 300 K (( \times 10^{16} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>-3.94(11) [Porsev 2006]</td>
</tr>
<tr>
<td>Ca</td>
<td>-25.8(4) [Porsev 2006]</td>
</tr>
<tr>
<td>Sr</td>
<td>-54.9(7) [Porsev 2006]</td>
</tr>
<tr>
<td>Yb</td>
<td>-26(3) [Porsev 2006]</td>
</tr>
<tr>
<td>Hg</td>
<td>-1.6 [Hachisu 2008]</td>
</tr>
</tbody>
</table>

Table 18. Blackbody Stark shift at 300 K for the \( ^1S_0 - ^3P_0 \) clock transitions in neutral atom optical frequency standards. Where known, the uncertainty is also given.

### 3.3.3.3 Collisional shift

Another potentially important frequency shift that must be considered in optical lattice clocks is the cold collisional shift resulting from the strength of the resonant dipole-dipole interactions between the atoms trapped in the lattice sites. This shift is proportional to the atomic density and to the ground-state scattering length. The scattering cross section is difficult to calculate and so it is important to measure the collisional shift experimentally. For most experiments performed to date, in which the density of the atomic sample is typically of order \( 10^{11} \) cm\(^{-3} \), the measured collisional shift has been consistent with zero to within the experimental uncertainty, regardless of whether the atomic sample is spin-polarized [Takamoto 2005, Boyd 2007b, Baillard 2008, Poli 2008]. However, the recent improvement in sensitivity obtained from optical-to-optical comparisons has led to the first observation of a non-zero collisional shift for a spin-polarized sample of \(^{87}\)Sr atoms [Ludlow 2008a]. The shift depends on a number of experimental parameters, such as the spectroscopic excitation fraction and the nuclear spin state. For a sample at a density of \( 10^{11} \) cm\(^{-3} \), spin-polarized to the \( m_F = \pm 9/2 \) states and with an excitation fraction of approximately 16%, the fractional frequency shift was measured to be \(-8.9(8) \times 10^{-16} \).

At sufficiently low temperatures, the collision of two particles can be described by their dominant S-wave interaction because higher-order partial wave collisions are suppressed by their centrifugal barrier. If spin-polarized ensembles of atoms are used, then this can greatly reduce the scattering rate due to Pauli-blocking [Gupta 2003]. Nonetheless, in the most precise \(^{87}\)Sr experiments carried out to date, a density-dependent frequency shift was observed even when atoms are polarized to a single spin state. This was attributed to possible P-wave interactions or loss of indistinguishability due to inhomogeneous excitation [Ludlow 2008a]. However, by choosing the spectroscopic excitation fraction appropriately, it was possible to reduce the collisional shift to a value consistent with zero [Ludlow 2008b]. Alternatively, there are various lattice configurations, especially in three dimensions, that can be used to greatly reduce the collisional frequency shift [Chang 2004]. Ultimately the intention is to
move to 3D lattice configurations such that the lattice sites could be loaded with an efficiency of less than one atom per site, eliminating the collisional frequency shift altogether [Katori 2002]. Methods envisaged to achieve single atom occupancy at a lattice site include driving a photoassociative loss process until one atom remains in each site, or making use of a Mott insulator transition [Greiner 2002], whereby a Bose-Einstein condensate (BEC) is gradually lowered into the lattice to achieve the single occupancy.

In addition to the effects discussed above, collisions can also cause line-pulling effects in optical lattice standards that use Ramsay spectroscopy. A detailed theoretical analysis can be found in [Band 2006].

3.3.3.4 Other effects

As for trapped ion optical frequency standards, the potential existence of other, more technical frequency shifts must be considered in addition to the effects discussed above, and may become significant as the accuracy of the optical lattice clocks continues to improve. Finally, as for any frequency standard, there will also be the gravitational redshift of the clock transition frequency, which means that to achieve $10^{-18}$ fractional accuracy the height of the standard above the geoid needs to be known to 1 cm.

3.3.4 Technology considerations

In order to efficiently load an optical lattice trap whilst maintaining parameters conducive to making a good clock, the temperature of the atoms must be less than 50 µK. An ideal way to achieve this is to first cool and trap clock atoms using a magneto-optical trap (MOT). For experiments on alkali atoms, MOTs are usually loaded from the slow tail of an atomic vapour released in the vacuum chamber. Unfortunately for the alkaline-earth and alkaline-earth-like atoms of interest for optical clocks, the vapour pressure needed for efficient loading necessitates heating the entire chamber to high temperatures [Vogel 1998]. This greatly increases the blackbody frequency shift (section 3.3.3.2), which is unacceptable for an optical clock. The alternative is to load the MOT from an atomic beam produced from an oven, which is usually fitted with some type of collimation nozzle. To increase the loading efficiency from the beam, Ca and Yb experiments have used a far-detuned laser beam propagating against the flow of atoms to increase the number of atoms whose velocity is within the trapping range of the MOT [Wilpers 2006, Hoyt 2005]. Adding in a Zeeman slower [Phillips 1982], which creates a specific magnetic field profile that keeps more of the atoms in resonance with the far-detuned beam as they are slowed, increases experimental complexity, but also increases the number of trapped atoms by at least a factor of ten, which can be expected to improve the potential stability of the clock. Historically Zeeman slowers were built using a wire-wrapped conical former, needing large power supplies to provide a sufficiently strong magnetic field and water cooling to take away the heat produced. More recently a novel Zeeman slower based on permanent magnets has been developed at NPL, which eliminates the need for (and the noise of) the power supplies and water cooling [Ovchinnikov 2007]. This type of Zeeman slower has not yet been tested experimentally, but could be used for all atomic species.

For most of the atoms of interest for optical clocks, cooling on the $^1S_0 - ^1P_1$ transition yields Doppler-limited temperatures of a few millikelvin. For Mg, Ca, Sr and Yb, a second stage of cooling on the narrow $^1S_0 - ^3P_1$ transition is therefore necessary to be able to cool the atoms down to the microkelvin temperatures needed for efficient loading of the lattice trap. Ca and Mg require an additional quenching (or repumping) laser to facilitate the cooling, as the transition is otherwise too long lived to cool effectively [Curtis 2003, Rehbein 2007]. For cooling using broader $^1S_0 - ^3P_1$ transitions, the usual practice is to initially further broaden the line by frequency modulating the laser beam in order to cool a larger velocity class of atoms.
in a short time, and then to follow this with a single frequency cooling stage to produce the lowest temperatures. This second-stage cooling can either be done within the MOT, which is then used to load ultracold atoms into the lattice, or while the atoms are being loaded into the lattice trap.

Mercury differs from the other atomic systems being studied as optical clocks in that atoms can be loaded into a MOT from a vapour cell operated slightly below room temperature [Petersen 2008]. There is therefore no need for an oven, an atomic beam, or a Zeeman slower of any type. Cooling is achieved using the $^1S_0 - ^3P_1$ transition alone, which is sufficiently broad (1.3 MHz linewidth) to obtain fast and efficient cooling, but narrow enough to achieve a Doppler-limited temperature of approximately 30 µK, which is suitable for direct loading of the atoms into the optical lattice for clock spectroscopy.

The laser wavelengths required for cooling, trapping and probing the atoms that are actively being pursued as optical lattice clocks are shown in Table 19. The technological challenges associated with producing these wavelengths are discussed in the following sections.

<table>
<thead>
<tr>
<th>Atom</th>
<th>First-stage cooling $\lambda$ / nm</th>
<th>Second-stage cooling $\lambda$ / nm</th>
<th>Lattice laser $\lambda$ / nm</th>
<th>Clock transition $\lambda$ / nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr</td>
<td>461</td>
<td>689</td>
<td>813</td>
<td>698</td>
</tr>
<tr>
<td>Yb</td>
<td>399</td>
<td>556</td>
<td>759</td>
<td>578</td>
</tr>
<tr>
<td>Hg</td>
<td>254</td>
<td>Not required</td>
<td>360</td>
<td>266</td>
</tr>
</tbody>
</table>

Table 19. Laser wavelengths required to cool, trap and probe neutral atoms that are being studied as optical lattice clocks.

3.3.4.1 Strontium

The first-stage cooling for the strontium atom requires radiation at 461 nm (Figure 26), which may be produced by frequency doubling the output from a high power master oscillator power amplifier (MOPA) diode system. The second-stage cooling wavelength is 689 nm, which is also accessible using diode lasers. The magic wavelength for the optical lattice is 813 nm, which may be produced using a MOPA system. The clock transition wavelength is 698 nm and can also be produced using a diode laser. Thus the laser systems for the strontium lattice clock are relatively straightforward, with only one frequency doubling stage required.

Figure 26. Partial term diagram for neutral strontium (not to scale), showing the laser wavelengths required to cool the atom and to probe the $^1S_0 - ^3P_1$ optical clock transition. The hyperfine structure for the $^{87}\text{Sr}$ isotope (nuclear spin $I = 9/2$) is not shown.
3.3.4.2 Ytterbium

The primary cooling wavelength required for neutral ytterbium is 399 nm (Figure 27), reachable with InGaN diode lasers [Park 2003, Hoyt 2005], whilst the secondary cooling wavelength is 556 nm, which may be obtained using a frequency doubled fibre laser at 1112 nm [Hoyt 2005]. The magic wavelength for the optical lattice is 759 nm, which in current experiments is produced using a Ti:sapphire laser [Barber 2006, Poli 2008], but which could also be generated using a MOPA system. For the clock wavelength of 578 nm, experiments have used either dye lasers [Hoyt 2005, Barber 2006] or, more recently, sum frequency generation of a Nd:YAG laser at 1319 nm and a Yb-fibre laser at 1030 nm in a MgO-doped periodically-poled lithium niobate waveguide [Barber 2007, Poli 2008]. However there are now both diode lasers and fibre lasers commercially available at 1156 nm. So far these exhibit low power, and so the power available after second harmonic generation will be limited, but is estimated to be sufficient to induce the clock transition. Thus, with the present state of technology, the ytterbium lattice clock is slightly more complex than the strontium lattice clock in that it requires one additional frequency doubling stage.

![Figure 27. Energy levels of atomic ytterbium (not to scale), showing the wavelengths of the cooling and clock transitions. The hyperfine structure of the $^{171}$Yb (nuclear spin $I = 1/2$) and $^{173}$Yb (nuclear spin $I = 5/2$) isotopes is not shown.](image)

3.3.4.3 Mercury

Optical lattice clocks based on Hg offer the prospect of very low systematic uncertainties, but are considered challenging for space operation because of the deep UV cooling and clock transitions (Figure 28). The $^1S_0 - ^3P_1$ transition, used for first stage cooling in the other atomic systems, is at the inaccessible wavelength of 185 nm, but as discussed above, fast and efficient cooling is possible using the $^1S_0 - ^3P_0$ transition alone. This has the advantage of reducing the number of lasers required. The cooling transition is at 254 nm, and can be accessed using either a frequency-quadrupled Yb:YAG thin disk laser [Petersen 2008] or a frequency-quadrupled amplified diode laser [Hachisu 2008]. The clock transition at 266 nm has been probed using a frequency-quadrupled DFB laser that is injection locked to an ultra-stable laser source consisting of a Yb-doped DFB fibre laser stabilized to a ULE cavity [Petersen 2008]. The wavelength required for the optical lattice is approximately 360 nm, and could be produced using a frequency-tripled fibre laser.
3.4 Optical frequency combs

Since the development of octave-spanning femtosecond optical frequency combs in the late 1990s, they have become indispensable tools for optical frequency metrology, making it possible to compare optical frequency standards in a fast and efficient way as well as to link the optical and microwave regions of the spectrum in a single step. The development and use of these combs has been the subject of a number of comprehensive review articles [Cundiff 2003, Hollberg 2005, Hall 2006, Hänsch 2006].

3.4.1 Principles of operation

A femtosecond optical frequency comb is based on a mode-locked femtosecond laser that emits a periodic train of ultrashort pulses. The spectrum of this emission corresponds to a comb of phase-coherent frequency modes whose spacing is equal to the repetition rate \( f_{\text{rep}} \) at which pulses are emitted from the mode-locked laser. The pulse-to-pulse phase shift \( \Delta \phi \) of the optical carrier wave with respect to the peak of the pulse envelope, which arises due to the difference between the group and phase velocities within the laser cavity, causes the comb modes to be offset from integer harmonics of \( f_{\text{rep}} \) by an amount \( f_0 = f_{\text{rep}} (\Delta \phi / 2\pi) \). The frequency of the \( n \)th mode of the comb is thus given by

\[
f_n = n f_{\text{rep}} + f_0 .
\]

(16)

The frequency \( f_0 \) is generally referred to as the carrier-envelope offset frequency.

To determine the absolute frequency of each mode in the comb therefore requires an accurate knowledge of both the repetition rate \( f_{\text{rep}} \) and the offset frequency \( f_0 \). It is relatively straightforward to determine the pulse repetition rate \( f_{\text{rep}} \) from the beat signal between adjacent comb modes or between modes that are separated by a harmonic of the repetition rate, although there are stringent demands on the noise properties of this measurement. Measurement of \( f_0 \) is a more involved task, and a number of schemes of varying degrees of complexity have been proposed [Telle 1999].

Figure 28. Energy levels of atomic mercury (not to scale), showing the laser wavelengths required for the cooling and clock transitions. The hyperfine structure of the \(^{199}\text{Hg}\) (nuclear spin \( I = 1/2 \)) and \(^{201}\text{Hg}\) (nuclear spin \( I = 3/2 \)) isotopes is not shown.
The simplest and most commonly employed method to determine \( f_0 \) requires a comb that spans a complete optical octave, i.e. a factor of two in frequency (Figure 29). In this case the measurement of \( f_0 \) can be accomplished by frequency doubling a portion of the low frequency end of the comb and then comparing this with the high frequency end of the comb [Jones 2000, Holzwarth 2000]. The beats between the comb modes \( f_m = m f_{\text{rep}} + f_0 \) and the frequency doubled modes \( 2f_n = 2n f_{\text{rep}} + 2f_0 \) with \( m = 2n \) yield a signal at the frequency \( f_0 \). This is known as the \( f - 2f \) self-referencing scheme.

A femtosecond optical frequency comb can be stabilized in two different ways. One mode of operation is to stabilize the comb to a microwave reference and hence generate precise and stable optical frequencies [Holzwarth 2000]. An optical clock uses the second mode of operation, in which the comb is stabilized to an optical frequency standard (atomic reference) and used to transfer the stability of this standard to other optical frequencies or into the microwave region of the spectrum, as shown in Figure 30 [Diddams 2001].
3.4.2 Accuracy and stability tests

3.4.2.1 Tests in the optical domain

A variety of experiments have been carried out to test the validity of equation (16). The first tests compared the frequency comb from a mode-locked Ti:sapphire laser (both before and after broadening in microstructure fibre) with an optical frequency interval divider, confirming that the modes of the frequency comb were equally spaced to within the experimental uncertainty of $3 \times 10^{-18}$ [Udem 1999, Holzwarth 2001b]. However to test that the absolute frequencies of the comb modes can be controlled at this level, a better test is to compare several independent systems.

In one category of tests, the femtosecond comb is referenced to a microwave standard. In an early test a comparison was made between absolute frequency measurements made using a femtosecond optical frequency comb at JILA and a traditional harmonic frequency synthesis chain at the National Research Council (NRC) of Canada [Ye 2000]. In this case the measurement accuracy of $1.6 \times 10^{-12}$ was limited by the stability of the transportable iodine-stabilized HeNe laser used as a flywheel reference. To eliminate this uncertainty, simultaneous frequency measurements must be carried out using the two independent devices. For example in a comparison between a self-referenced frequency comb and a $3.5f - 4f$ frequency chain, an uncertainty limit of $5 \times 10^{-16}$ was derived [Holzwarth 2000], whilst agreement at the $1.2 \times 10^{-15}$ level was demonstrated in a comparison between three maser-referenced frequency combs [Ma 2004a]. All these tests were performed using Ti:sapphire-based combs; however absolute frequency measurements carried out using two fibre-based combs were also shown to agree at the $6 \times 10^{-16}$ level [Kubina 2005].

In the above tests the measurement uncertainty was limited by the short-term instability imposed by the microwave reference, which means that long averaging times are required. Significantly improved short-term stability can be obtained when the femtosecond comb is referenced to an optical standard (“optical clock” mode of operation). In an early test of this sort, two octave-spanning Ti:sapphire-based combs were locked to a common optical reference in such a way that the two combs had the same value of $f_{rep}$ but different values of $f_0$. By measuring and analysing the heterodyne beat between the two combs in different spectral regions (Figure 31), the intrinsic fractional frequency noise of the comb modes was shown to be less than $6.3 \times 10^{-16}$ for an averaging time of 1 s and the absolute frequencies of the modes of the two combs were found to agree to $4 \times 10^{-17}$ across the entire octave [Diddams 2002]. Similar experiments demonstrated that the individual components of a femtosecond comb can exhibit instrument-limited, sub-hertz linewidths relative to an ultrastable reference laser [Bartels 2004, Swann 2006].

![Figure 31. Experimental arrangement for optical heterodyne comparison of two optically referenced femtosecond optical frequency combs.](image-url)
Later this type of experiment was extended to comparisons between four different Ti:sapphire-based frequency comb systems, two of which were octave-spanning combs generated by spectral broadening in microstructure fibre and two of which directly emitted a broadband spectrum from the laser that was sufficient for self-referencing by $2f - 3f$ comparison. The reproducibility demonstrated in these experiments established that the relative frequency uncertainty introduced by the femtosecond comb was less than $8 \times 10^{-20}$ [Ma 2004b, Ma 2007]. High accuracy results have also been obtained using the “transfer oscillator” scheme [Telle 2002] in which a deliberate choice of frequency mixing processes is employed to eliminate the noise of the femtosecond laser when used to compare two different frequencies, so that it is not necessary for the femtosecond comb to be tightly stabilized. Applying this method with a Ti:sapphire-based comb, the frequency ratio between the fundamental and second harmonic frequencies of a Nd:YAG laser has been measured with an accuracy of $7 \times 10^{-19}$ [Stenger 2002]. More recent experiments in which two fibre-based femtosecond optical frequency combs were compared showed that the transfer oscillator scheme may be used to synthesize arbitrary optical frequencies (i.e. transfer stability around the optical region of the spectrum) with an accuracy of $8 \times 10^{-18}$ [Grosche 2008]. Similar experiments performed at NIST with tightly phase locked combs and with the addition of short (a few hundred metres) optical fibre links between the combs have demonstrated the transfer of frequency stability around the optical region of the spectrum with a fractional accuracy of $5 \times 10^{-19}$ and a fractional instability of approximately $6 \times 10^{-17} \tau^{-1/2}$ [Coddington 2007].

This level of performance in the optical domain is more than sufficient to support the high accuracy local comparison of optical frequency standards, with the most accurate comparison reported to date being a measurement of the frequency ratio of the $^{27}$Al$^+$ and $^{199}$Hg$^+$ trapped ion standards at NIST [Rosenband 2008]. The total fractional uncertainty in this frequency ratio was $5.2 \times 10^{-17}$, of which $3.0 \times 10^{-17}$ was systematic uncertainty in the two standards and $4.3 \times 10^{-17}$ was statistical measurement uncertainty, dominated by the instability of the trapped ion standards.

### 3.4.2.2 Microwave synthesis from an optically referenced femtosecond comb

To determine how well the femtosecond comb can transfer the stability of an optical frequency standard into the optical domain, two independent femtosecond frequency combs can be locked to a common optical reference and their emerging pulse trains compared. This comparison can be carried out in two different ways [Bartels 2003].

The first method is to use optical nonlinear cross-correlation to compare the optical pulse train outputs (Figure 32 (a)). Here the optical pulse trains from the two separate combs are crossed in a nonlinear crystal, and when pulses from each laser arrive at the crystal simultaneously, the sum frequency signal is generated in a direction that bisects the angle between the two crossed pulse trains. The repetition rate of the sum-frequency pulse train will be equal to the difference in the repetition rates between the two combs, and can be detected using a photomultiplier tube and measured using a frequency counter. This method essentially measures relative fluctuations in the arrival times of the two optically referenced pulse trains.

The second method is to use high speed photodetection of the individual pulse trains to produce two signals, which are compared using an electronic mixer (Figure 32 (b)). In addition to providing a signal at the fundamental repetition rate $f_{\text{rep}}$, the signal from the photodiodes also provides a comb of frequencies at harmonics of $f_{\text{rep}}$, extending out to the bandwidth of the photodiodes used, and so the comparison may be carried out at any of these harmonics.
Although optical heterodyne comparisons between two optically referenced femtosecond combs can exhibit a fractional instability of $10^{-17}$ at 1 s, averaging down to the $10^{-19}$ range in a few thousand seconds (Figure 33 (d)), this performance has not yet been matched in the microwave domain [Ma 2007]. The Allan deviation for nonlinear cross-correlation measurements exhibits a similar $\tau^{-1}$ behaviour, but with instability approximately two orders of magnitude higher (Figure 33 (c)). In this measurement approach, the intensity of the sum-frequency signal, and hence the counting of this signal, depends on the power in the two frequency combs. This means that amplitude fluctuations can be translated into apparent phase or timing noise, and so the results could potentially be improved by stabilization of the optical power in the two combs. Nonetheless, this comparison demonstrates that a femtosecond comb can transfer the accuracy of an optical standard to the femtosecond optical pulse train with a relative frequency uncertainty of $1.7 \times 10^{-18}$.

However, microwave signals extracted by photodetection of the optical pulse trains exhibit significantly greater instability (Figure 33 (a) and (b)), showing that excess noise is introduced in the photodetection process. There are a number of technical and fundamental noise sources that can affect the stability of the microwave signals synthesized from the comb, including the performance of the optical phase locked loops, laser short noise, phase noise introduced by microwave amplifiers, amplitude-to-phase noise conversion in the photodetectors used to extract the microwave signal, and laser beam pointing fluctuations [Ivanov 2003]. Efforts to reduce a number of these noise sources have led to improvements in performance, with the result that 10 GHz signals have been synthesized with an instability of $6.5 \times 10^{-16}$ at 1 s introduced by the femtosecond comb [Bartels 2005], although for averaging times longer than 1 s the instability does not improve as $\tau^{-1}$. The best reported phase noise from such a system is $-100$ dBC/Hz at 1 Hz offset from the 10 GHz carrier [McFerran 2005, Ivanov 2007].
Figure 33. Instability observed in different types of comparisons between optically referenced femtosecond combs. (a) Comparison of the repetition rates by free space photodetection at $f_{\text{rep}} = 1$ GHz. (b) Comparison of the repetition rates by fibre-coupled photodetection at $10 f_{\text{rep}} = 10$ GHz. (c) Comparison of the pulse trains by optical nonlinear cross-correlation. (d) Optical heterodyne comparison. (Figure reproduced from [Ma 2007].)

The above studies of the excess noise associated with the photodetection process were all carried out using Ti:sapphire-based femtosecond combs. However there are also a few reports of the use of fibre-based combs for low noise microwave synthesis. In the only experiment to report direct measurements of frequency stability [Hartl 2006], the stability of a 10 GHz signal derived from an integrated fibre comb was compared with an established high stability frequency comb based on a mode-locked Ti:sapphire laser. Both combs were locked to a common optical reference with an established stability of around $3 \times 10^{-15}$ at 1 s. Excellent short-term stability of $2 \times 10^{-14}$ at 0.1 s was achieved, a result comparable to that achieved in comparisons between Ti:sapphire-based combs. However the stability curve remained fairly flat at approximately the $10^{-14}$ level out to timescales of 10 s. The cause of the observed instability at longer averaging times remains to be identified; however this work is at a rather preliminary stage and it is likely that significant improvements in performance will be achieved in future.

One promising way to circumvent amplitude-to-phase noise conversion in the photodetectors and the microwave mixers is to carry out the phase detection directly in the optical domain using a balanced optical-microwave phase detector as demonstrated by Kim et al. [Kim 2006a]. In this scheme phase detection is based on electro-optic sampling with a differentially biased Sagnac loop, and is potentially less sensitive to power and thermal drifts. Using this technique, 12.8 fs relative timing jitter integrated from 10 Hz to 10 MHz and a timing drift below 48 fs in 1 hour has been demonstrated between two 10 GHz signals locked to the 44 MHz pulse train from a passively mode-locked fibre laser [Kim 2007a]. In this experiment the long term drift was limited mainly by the drift of the characterization system itself, whilst the short term jitter was limited by excess technical noise. By identifying and removing some of these noise sources, and by using a balanced optical-microwave phase detector for monitoring the out-of-loop timing jitter as well as for synchronization of the optical and microwave signals, performance has been improved to demonstrate 6.8 fs rms integrated timing jitter in 1 MHz bandwidth integrated over 10 hours [Kim 2008]. From this measurement a relative stability of $1.9 \times 10^{-19}$ for an averaging time of 10 hours can be inferred, although no direct measurements of frequency stability have yet been made.
3.4.3 Technology considerations

A femtosecond optical frequency comb as implemented in an optical clock consists of a femtosecond laser system, usually with some form of additional spectral broadening, together with a nonlinear interferometer for self-referencing, optics for beat detection, and electronics to stabilize the comb and to derive a microwave output signal. In this section we focus on the first two elements of the frequency comb, since the space qualification of these is considered to present the most challenging task.

3.4.3.1 Laser technology

A variety of different laser sources have been used for femtosecond optical frequency combs, and so it is important to consider the factors that will influence the selection of the optimum femtosecond laser technology for a future space-borne optical clock.

Obvious requirements are that the comb should be compact, light, robust against environmental disturbances and capable of long-term reliable operation. The electrical to optical efficiency is also extremely important for space applications, where power consumption and heat dissipation are vital considerations. In this respect a relevant consideration is the difference between the pump wavelength and the laser centre wavelength, which indicates how much energy per photon is dissipated in order to generate a photon at the laser wavelength (quantum defect).

Another factor to consider is the repetition rate of the laser source. Higher repetition rates correspond to smaller cavity lengths and hence offer intrinsically smaller system footprints. For a given output power from the laser, a high repetition rate system also yields higher power per comb mode, which is advantageous because it leads to a higher signal to noise ratio in the heterodyne beat between a cw laser and an individual mode of the comb. However for higher repetition rate lasers the power per pulse decreases (assuming the same average output power) and so the efficiency of spectral broadening decreases. Although the spectral broadening can be enhanced by using as short a pulse as possible, the maximum useful repetition rate is probably no higher than a few GHz. Reduced peak power can also be an issue for maintaining Kerr-lens mode-locking within the laser cavity.

The wavelength coverage of the laser source and resulting frequency comb are also important considerations because they determine the range of overlap with the local oscillators used to probe the clock transitions used in optical frequency standards, most of which lie in the 500 – 1100 nm range. Combs that emit further into the infrared can also be used in conjunction with the standards but this necessitates an extra nonlinear frequency conversion stage, which increases system complexity and reduces the power efficiency. A femtosecond source that emits an octave-spanning spectrum directly is also advantageous in that it eliminates the need for alignment-sensitive microstructured fibre.

Finally, the residual noise of the optical frequency comb is clearly vital to the performance of the optical clock. In this context it is important to recognize that, due to its highly nonlinear nature, microstructured fibre can significantly amplify noise on the input light [Newbury 2003, Corwin 2003], and so careful attention has to be paid to minimizing this effect in the laser design.

In the following subsections of this report we discuss the extent to which different laser sources used for femtosecond combs meet these requirements.
**Ti:sapphire**

Kerr-lens mode-locked femtosecond Ti:sapphire lasers [Spence 1991] were the first to be used for optical frequency metrology and the reliability and low noise of this technology means that it continues to be used in many laboratories. As a gain medium, Ti:sapphire has a number of advantages, since it has a large cross section, good thermal properties and a broad gain bandwidth that supports ultrashort pulses of 5 fs duration [Morgner 2001]. In some cavity designs [Asaki 1993] the dispersion compensation necessary to produce ultrashort pulses is achieved through the use of a pair of intracavity prisms [Fork 1984], whilst other lasers [Stingl 1994, Bartels 1999] employ dispersion-compensating mirror coatings [Szipoecs 1994, Kärtner 1997]. The octave-spanning spectrum required for \( f - 2f \) self-referencing is usually generated by using a short length of microstructured fibre [Ranka 2000] or photonic crystal fibre [Knight 1996] outside the laser cavity, in which additional comb modes are generated by self-phase modulation or four-wave mixing. However there are now Ti:sapphire lasers based on chirped mirror technology that can provide octave-spanning combs directly without additional spectral broadening in nonlinear fibres [Fortier 2003, Matos 2004, Mücke 2005, Fortier 2006].

Octave spanning Ti:sapphire femtosecond combs have been reported with repetition rates up to 5 GHz [Bartels 2007], resulting in a small system footprint. As discussed in the preceding section, this type of comb has also demonstrated impressive levels of stability and accuracy. However when considering space applications Ti:sapphire lasers suffer from the serious drawback that they are typically pumped by high power (5–8 W) frequency-doubled Nd:YVO\(_4\) lasers and generally require regular optical adjustments. Recently, a low-threshold self-referenced Ti:sapphire optical frequency comb has been demonstrated that requires only 1 W of pump power [Kirchner 2006], which represents a significant step forward for applications that require portability and efficiency. The low pump power used in this system also reduces undesirable thermal effects with the result that the free-running offset frequency beat is significantly more stable than in higher power Ti:sapphire systems.

**Erbium-doped fibre**

Another option is to use a frequency comb based on a mode-locked erbium-doped fibre laser (Figure 34). Compared to Ti:sapphire lasers, erbium-doped fibre lasers have a number of advantages in that they can be efficiently pumped by laser diodes, while the light guiding in the fibre means that they are insensitive to thermal effects or external disturbances, require less alignment and can be very compact. Centred around 1550 nm, fibre lasers generate octave-spanning combs covering the 1000–2000 nm range [Tauser 2003, Washburn 2004, Kubina 2005], and the spectral coverage can be extended by nonlinear frequency conversion [Hong 2003]. These systems are very flexible in that parallel amplifier branches can be independently configured for self-referencing and frequency measurements in a particular spectral region [Adler 2004]. The intrinsic stability of the fibre laser designs allows for continuous operation, and all-fibre collinear self-referencing architectures can be implemented [Schibli 2004, Hartl 2005]. Furthermore mode-locked erbium-doped fibre lasers are largely based on optical telecommunications components, which have been developed for reliable long-term operation and are relatively cheap and readily available.

However, although femtosecond erbium-doped fibre lasers can be much more compact, robust, lighter and power-efficient than a bulk optic solid state laser system, the highest repetition rates of fundamentally mode-locked systems are around 250 MHz. Higher repetition rates are hard to achieve because the overall fibre cavity length has to accommodate both the active and the passive functional fibre-optic elements, and erbium-doped fibre is a relatively low gain material.
Figure 34. Commercial femtosecond optical frequency comb based on a mode-locked erbium-doped fibre laser.

Optical frequency combs generated by erbium-doped femtosecond fibre lasers also typically exhibit significantly more high frequency noise than Ti:sapphire based systems. This causes broad optical linewidths, particularly in the wings of the comb and on the carrier-envelope offset frequency beat signal [Hundertmark 2004, Washburn 2004, McFerran 2006]. This noise, which has been the subject of detailed investigations [Washburn 2005, Newbury 2007a], is primarily a result of white amplitude noise on the pump diodes which, when coupled with the sensitivity of the laser to pump fluctuations, causes a breathing motion of the comb about a central fixed frequency. Due to the long fluorescence lifetime of erbium, the pump power provides only a slow mechanism for stabilization of the offset frequency, and so phase advance circuitry is required to increase the bandwidth and hence to reduce the linewidth to below 1 Hz [McFerran 2007].

**Ytterbium-doped fibre**

Femtosecond fibre lasers based on ytterbium-doped fibres are interesting because they can be diode-pumped with high efficiency at several wavelengths and the broad gain bandwidth has been shown to support pulses as short as 36 fs [Ilday 2003]. The output spectrum of these lasers is centred in a region between Ti:sapphire and erbium-doped fibre lasers. The high possible doping concentration of the Yb$^{3+}$ ions in silica fibres leads to high gain and enables shorter cavities, and hence higher repetition rates, to be realized compared to erbium-doped fibre lasers. However, all conventional silica fibres have normal dispersion at the emission wavelength of ytterbium-doped fibres and so dispersion compensation requires the incorporation of additional free space elements or photonic crystal fibres into the cavity.

The technologies of cladding pumping and chirped pulse amplification give these systems excellent power scalability [Hartl 2007] and recently an amplified ytterbium-doped femtosecond fibre laser has been used to generate an optical frequency comb with more than 10 W average output power that is passively more stable than any previously reported comb systems [Schibli 2008]. When stabilized to an ultrastable cw laser system and compared with a Ti:sapphire-based comb, this system demonstrated submillihertz optical linewidths.
**Yb:KYW**

Ytterbium-doped potassium yttrium tungstate (Yb:KY(WO₄)₂ or more commonly Yb:KYW) is a promising material for the development of highly efficient and compact frequency comb sources. This material has a strong absorption band that can be optically pumped by common high-brightness InGaAs laser diodes at 980 nm. It emits in the 1020–1080 nm spectral range, meaning that the quantum defect is low. This leads to high efficiencies and low thermal loads on the crystal. Repetition rates up to 300 MHz have been demonstrated for Kerr-lens mode-locked Yb:KYW lasers with an optical – optical conversion efficiency of 53\% [Lagatsky 2004], and higher repetition rates should be achievable. Its emission bandwidth is significantly lower than that of Ti:sapphire, but it has been shown to be capable of supporting sub-100 fs pulses [Liu 2001].

An octave-spanning frequency comb based on a Yb:KYW femtosecond laser has recently been reported [Meyer 2008]. The femtosecond oscillator, which is pumped by two fibre-coupled 600 mW laser diodes, is mode-locked using a semiconductor saturable absorber (SESAM) and produces 290 fs pulses at a repetition rate of 160 MHz with 240 mW average output power. The output from the oscillator is amplified using a single-mode Yb-doped fibre amplifier, which leads to some spectral broadening, and then temporally compressed to 80 fs before being launched into a one metre length of microstructured fibre with a zero dispersion wavelength of 945 nm and a core diameter of 3.2 µm. This produced a spectrum spanning from 650–1450 nm with strong peaks at around 680 nm and 1360 nm that are used in an \( f - 2f \) nonlinear interferometer to detect and stabilize the carrier envelope offset frequency via feedback to the pump laser current. It was also possible to obtain an octave-spanning spectrum without any amplification of the oscillator output when the cavity length was extended to give a repetition rate of 91 MHz, which leads to significantly shorter (160 fs) pulses. Although in these initial experiments the phase-locked offset frequency was less stable than in the more mature Ti:sapphire-based combs, further improvements are anticipated in future. In particular the unlocked offset frequency was observed to have a relatively clean spectrum without the high frequency noise commonly observed in fibre-based femtosecond combs.

**Cr:LiSAF**

Chromium-doped LiSrAlF₆ (Cr:LiSAF) has broadly similar laser characteristics to Ti:sapphire, but with a strong absorption band between 550 and 750 nm [Payne 1989], which means that it can be pumped using commercially available GaInP or AlGaInP laser diodes around 670 nm, and so has the potential for developing compact laser sources. Cr:LiSAF has a broad emission spectrum of almost 400 nm, meaning that it is capable of supporting very short pulses, and it has a better emission-cross section – upper-state lifetime product than Ti:sapphire, ensuring low operational thresholds [Agate 2002]. However it has certain disadvantages in that it is mechanically soft, its thermal conductivity is about an order of magnitude less than that of Ti:sapphire and at a crystal temperature above 50 – 60 °C thermal quenching substantially lowers the gain. The nonlinearity of the crystal is also 5 – 10 times lower than for Ti:sapphire, meaning that to achieve Kerr-lens mode-locked operation it is necessary to focus very tightly into the laser cavity.

The poor beam quality of the high-power red laser diodes that are currently available reduces the soft-aperturing effect on which many self-mode-locked lasers rely, as well as reducing the achievable power density in the gain medium. However despite these difficulties Kerr-lens mode-locking of a diode-pumped Cr:LiSAF laser was first reported in 1993 [French 1993] and other reasonably compact systems have since been demonstrated [Dymott 1994, Uemura 1999]. Diode-pumped self-mode-locked Cr:LiSAF lasers have demonstrated output powers up to 1.1 W [Kopf 1994], repetition rates up to 1 GHz [Kemp 2001] and pulse durations as
short as 10 fs [Uemura 2000]. Of particular relevance to space applications is the development of highly compact and efficient resonator designs by the group at the University of St Andrews [Hopkins 2002, Agate 2002]. For example by using narrow-stripe, single spatial mode laser diodes to achieve compact pump states and improved use of available pump power, and methods of prismless dispersion and SESAM-based mode-locking to reduce the cavity component count, the group have demonstrated a portable, battery-powered Cr:LiSAF laser with a footprint of just 22 cm × 28 cm, an optical to optical conversion efficiency of over 20% and an electrical to optical efficiency of approximately 4%.

The only report of frequency comb generation from a mode-locked Cr:LiSAF laser comes from the group at the Max-Planck Institute for Quantum Optics in Garching [Holzwarth 2001a]. This Kerr-lens mode-locked laser uses two battery-powered 350 mW pump diodes and has a linear cavity design with several bounces off a pair of chirped mirrors in each cavity arm, leading to a compact laser but with a relatively low repetition rate of 93 MHz. In the reported experiments the laser was operated with 115 mW output power at a central wavelength of 894 nm and with a spectral width of 24 nm FWHM. To achieve an octave-spanning frequency comb the uncompressed pulses (duration 57 fs) in a fraction of the output beam (42 mW) were launched into a 20 cm length of photonic crystal fibre that had a core diameter of 1 µm and zero group velocity dispersion near 580 nm. The output spectrum from the fibre exhibited peaks near 530 nm and 1060 nm, enabling an offset beat frequency signal $f_0$ with a signal-to-noise ratio exceeding 40 dB in 100 kHz resolution bandwidth to be obtained. This is sufficient to enable $f_0$ to be phase-locked and therefore such a system offers the prospect of a compact and efficient system for optical frequency metrology. Higher repetition rates would, however, be desirable.

**Cr:forsterite**

Operating at 1.3 µm, Cr$^{4+}$:forsterite (Mg$_2$SiO$_4$) has attracted interest as a femtosecond source in the near infrared. Cr:forsterite has a broad absorption band around 1 µm, which is a good match to a number of commercially available pump sources, including compact diode-pumped Yb fibre laser sources and InGaAs diode lasers operating around 970 nm.

Kerr-lens mode-locking of Cr:forsterite was first demonstrated in 1992 [Seas 1992], and since then substantial progress has been made in generating shorter pulses. By using specially designed and fabricated double-chirped mirrors in combination with high-index PBH71 prisms to compensate for higher-order dispersion in the Cr:forsterite laser cavity, Chudoba et al. have produced 14 fs pulses with 80 mW average power at a repetition rate of 100 MHz [Chudoba 2001].

A femtosecond comb based on a Cr:forsterite femtosecond laser has been developed at NIST [Thomann 2003, Corwin 2004]. This laser, which is pumped by a 10 W Yb fibre laser, uses dispersion-compensating mirrors in a ring cavity geometry to generate 35 fs pulses at a repetition rate of 433 MHz with approximately 500 mW of average power. The output spectrum is centred at around 1.26 µm and has a FWHM of approximately 50 nm. Various fibres have been used for spectral broadening. In early experiments the broadening achieved fell short of the complete optical octave required for $f – 2f$ self referencing and so an alternative stabilization scheme was employed whereby one of the comb modes was locked to an optical frequency reference whilst the repetition rate was locked to a microwave reference [Corwin 2004]. However in later experiments the use of an improved dispersion-flattened highly nonlinear fibre (HNLF) for spectral broadening produced an octave-spanning comb from 1.0 µm to 2.2 µm, which enabled the carrier-envelope offset frequency to be detected and stabilized using a nonlinear $f – 2f$ interferometer [Kim 2005]. Significant enhancement of the beat between the frequency-doubled comb and a cw laser at 657 nm was reported by using a 2 m long piece of dispersion-flattened HNLF containing a fibre Bragg grating to enhance
the continuum around 1314 nm [Kim 2006b]. Comparisons between this Cr:forsterite based comb and a well characterized Ti:sapphire comb with both combs locked to the optical reference at 657 nm demonstrated agreement at the $7.1 \times 10^{-17}$ level [Kim 2006b]. The noise on the Cr:forsterite comb has been studied by looking at heterodyne optical beat notes with cw lasers at different wavelengths [Kim 2007b]. Strong correlations of the frequency noise were observed between different spectral components of the comb, relative to a fixed point close to the 1.3 µm carrier of the Cr:forsterite laser, and were found to be due to amplitude noise on the pump laser. The amplitude of this frequency noise was observed to be amplified in the process of spectral broadening, with the amplification factor depending on the type of nonlinear fibre used.

**Cr:YAG**

Another option for a femtosecond source in the near infrared is Cr$^{4+}$:YAG, which operates around 1.5 µm. Like Cr:forsterite, Cr:YAG has a broad absorption band around 1 µm, so it can be pumped using a Yb fibre laser or an InGaAs diode laser at 970 nm.

The emission bandwidth of Cr:YAG extends over the spectral region 1200 – 1600 nm and is capable of supporting the generation of ultrashort optical pulses with broad spectra and high peak intensities. Pulses as short as 20 fs with 400 mW average power at a repetition rate of 110 MHz have been reported from a Cr:YAG laser that uses double-chirped mirrors for dispersion compensation [Ripin 2002]. The corresponding pulse spectrum is peaked at 1450 nm and has a FWHM that extends from 1310 – 1550 nm. Compact, high-repetition rate Cr:YAG lasers have also been reported with pulse repetition rates up to 4 GHz [Leburn 2004], although in this case the transform-limited pulse duration is longer at 80 fs.

A 30 nm wide frequency comb direct from a mode-locked Cr:YAG laser with a repetition rate of 130 MHz has been used to measure frequency intervals between saturated absorption transitions in acetylene [Alcock 2005, Madej 2006]. However to date there have been no reports of self-referenced optical frequency combs based on Cr:YAG lasers.

**Comparison between different laser sources and adaptation to space**

A trade-off analysis of the different laser sources described above has, in their present state of development, led to the conclusion that fibre-based femtosecond combs appear to be the most promising implementation for space, and these are now being investigated under an ESA-funded GSTP project [ESA 2005]. One particularly critical issue is the sensitivity of the active fibres to radiation damage. Although tests have been carried out on a variety of rare-earth doped fibres, for example to establish radiation-induced attenuation losses [Henschel 1998, Van Uffelen 2004, Girard 2007, Fox 2008], this does not immediately yield information about the suitability of these fibres for a space-borne femtosecond laser. In the ESA-funded project, the effects of radiation on the mode-locking of ytterbium and erbium mode-locked femtosecond fibres lasers is being studied directly, along with mechanisms for radiation-hardening the fibres.

### 3.4.3.2 Nonlinear interferometer

The nonlinear interferometer used for self-referencing the comb can be set up in either a two-arm or a one-arm (collinear) configuration. In the two arm configuration the octave-spanning spectrum from the frequency comb is divided up into a long wavelength part that is frequency doubled and a short wavelength part that is subsequently overlapped with the frequency doubled radiation. A delay line in one arm is necessary in order to ensure that the pulses overlap temporally as well as spatially [Jones 2000, Holzwarth 2000]. The one arm interferometer is a simpler configuration in which the long and short wavelength parts of the
frequency comb propagate collinearly in space. A nonlinear crystal is used for second harmonic generation of the long wavelength part, whilst the short frequency part is simply transmitted by the crystal. With appropriate dispersion management of the incident octave-spanning spectrum, temporal overlap of the fundamental and frequency doubled pulses can be achieved in a compact and robust setup [Schibli 2004, Jiang 2005].

An alternative monolithic scheme for stabilizing the carrier envelope offset frequency in the case where the laser emits very short (i.e. spectrally broad) pulses has been demonstrated by Fuji et al. [Fuji 2005]. This scheme is based on a combination of self phase modulation and second harmonic generation or difference frequency generation in a single periodically poled crystal, and also leads to a compact and robust arrangement.

If the spectrum does not span a full octave, then more complicated arrangements involving several nonlinear steps are required for self-referencing [Morgner 2001, Ramond 2002], a clear disadvantage when considering the development of a space-qualified optical frequency comb.

3.5 Optical frequency comparison

For most applications of space-borne optical clocks, a high accuracy method for ground to satellite clock comparison will be essential, with others potentially requiring intersatellite links in addition. The current state of the art is considered in section 3.5.1, along with the prospects for future improvements. During the development phase of a space-borne optical clock, methods for high accuracy comparison of remotely located ground-based optical clocks will also play an important role. In section 3.5.2 methods for optical frequency comparison via optical fibre networks are therefore discussed.

In each case, the frequency transfer technique should ideally have a performance that matches or exceeds the performance of the highly stable and accurate optical atomic clocks.

3.5.1 Satellite frequency transfer techniques

In this section techniques for frequency transfer between a ground station and a satellite, and between satellites, are discussed. The state of the art is considered for both microwave and optical methods, as well as possible future developments. Because the techniques are closely related to the methods used for frequency and time transfer between ground timing centres and significant research is being carried out in these areas, these ground-based methods will also be discussed.

3.5.1.1 Microwave frequency transfer between ground timing centres

Precise frequency and time transfer between timing centres operating atomic clocks today relies predominantly on satellite-based methods. Recent studies [Bauch 2006] have demonstrated that the lowest instabilities for frequency intercomparisons can be obtained by either GPS-based carrier phase (GPS CP) observations or two-way satellite time and frequency transfer (TWSTFT). Both of these methods will be discussed, in particular TWSTFT, which is closely related to space microwave links.

GPS carrier phase frequency transfer between ground institutes is of interest as it provides a close parallel with microwave code-based frequency transfer between satellites. For ground-based frequency transfer, measurements of code and carrier-phase signals from all tracked GPS satellites with respect to a local 1 pulse per second (PPS) reference are recorded every 30 s using a dual-frequency carrier-phase GPS receiver. The data are subsequently processed
to determine the receiver and satellite clock parameters, making use of additional
determinations of satellite orbits, troposphere delays and Earth orientation available from the
International GNSS Service (IGS). Frequency comparisons by this method have been
demonstrated to have an instability of $1 \times 10^{-15}$ over a measurement time of 1 day, and
$2 \times 10^{-14}$ over 1 hour [Bauch 2006]. However, it is not at present routinely used by the
metrology community, largely because of the complexity of the data analysis.

GPS-based time and frequency transfer is feasible not only between two ground stations, but
also between a ground station and a low Earth orbit (LEO) satellite, or between two LEO
satellites. The latter has been demonstrated by the GRACE dual-satellite experiment, in which
both satellites carry an ultra-stable oscillator and the two oscillators are synchronised to about
150 ps using carrier-only dual-frequency GPS receivers [Bertiger 2003].

The preferred technique used by national timing institutes for intercomparisons of primary
frequency standards, as well to contribute atomic clock data for the realisation of Coordinated
Universal Time (UTC), is TWSTFT [Piester 2008]. This method is based on the simultaneous
exchange of microwave signals (in the K$_u$-band) between two timing centres via a
geostationary satellite. At present, twelve institutes in Europe, two in the USA and seven in
the Asia-Pacific region operate TWSTFT stations, although line of sight limitations restrict
the number of feasible links. Measurements are carried out between pairs of stations
according to an agreed schedule. The transmitted signal is modulated by a pseudo-random
noise (PRN) code sequence, using binary phase-shift keying, that includes a timing marker
related to the 1 PPS output of the reference clock. The receiving station employs a correlator
to reconstitute the 1 PPS marker from the received signal, and measures its time offset from
the local clock over a specified period, typically at 1 second intervals over 2 minutes. The
measurement data are subsequently exchanged and differenced, cancelling the signal
propagation times to first order, and additional corrections applied for the Sagnac effect and
known equipment delays.

For time transfer, the accuracy of TWSTFT is limited to around 1 ns by uncertainties in the
calibration of the absolute delays of the earth station components. For frequency transfer,
however, only the instabilities of the delays are significant. A number of factors contribute to
these instabilities.

1. Environmental factors, in particular laboratory and outdoor temperatures and
relative humidities, have been shown to affect the stability of TWSTFT
measurements. Control of these parameters should limit the resultant instability to a
time deviation (TDEV) of around 10 ps.

2. The ionospheric delay is generally not the same for the transmitted and received
signals due to path asymmetry and geographical variations in the electron content of
the ionosphere. This difference can give rise to an instability at the level of
1 – 10 ps TDEV.

3. The diurnal motion of the geostationary satellite gives rise to a difference in the
path delays of the two signals of up to a few tens of ps.

4. For longer link baselines, the two signals will pass through different transponders
on the satellite. A significant change in the loading of one of the transponders can
give rise to an uncompensated change in its delay of up to several ns. Even when
both signals use the same transponder, an increase in the number of stations
transmitting simultaneously increases the measurement noise. For example, a
change from one pair of stations transmitting to six pairs causes the frequency
transfer instability (Allan deviation, or ADEV) to increase by a factor of 1.4.
Although these satellite-based frequency transfer techniques are reasonably well matched to the performance of current caesium microwave primary frequency standards, they are not adequate for comparison of higher performance optical atomic clocks with stabilities and accuracies at the $10^{-17} - 10^{-18}$ level.

Currently, operational TWSTFT stations in Europe and the USA employ a modulation rate of 2.5 Mchip/s, limited by the allocated satellite transponder bandwidth. This chip rate results in a relative frequency instability (ADEV) of approximately $2 \times 10^{-15}$ at 1 day, varying with the measurement time $\tau$ according to $\tau^{-1}$. Increasing the chip rate, to 20 Mchip/s or even 100 Mchip/s, would significantly reduce the measurement noise and hence the Allan deviation.

Other means of improving the performance of TWSTFT frequency transfer are being researched. One particularly promising method is the measurement of the $K_u$-band carrier phase in addition to the code. Studies and preliminary experiments [Fonville 2005] have indicated that a frequency transfer uncertainty of $2 \times 10^{-15}$ at 1 s should be feasible, although further work is required. Current work in Japan aims to achieve uncertainties of around $10^{-12}$ at 1 s and below $10^{-16}$ at 1 day.

### 3.5.1.2 Microwave frequency transfer between ground stations and satellites

Two-way microwave links between a satellite and ground stations have been in operation for many years, an example being the PRARE ranging system on board the ERS-2 Earth sensing satellite launched by ESA in 1995. Microwave links are often installed even when alternatives, such as optical links, are also available because they are unaffected by weather conditions.

The current state of the art for microwave frequency transfer between a satellite and a ground station is represented by the Microwave Link (MWL) that forms part of the ACES payload that is intended to be installed onboard the International Space Station (ISS). The requirements of the MWL are optimised for the very low orbit of the ISS, with the link stability specified for an averaging time of 350 s that corresponds to the effective time in view during one pass over a ground station [Uhrich 2000].

The design of the ACES MWL is illustrated in Figure 35. The system is based on a bi-directional $K_u$-band link with an additional S-band down-link [Seidel 2007]. Each of the links is modulated with a PRN sequence, using a high modulation rate of 100 MChip/s in the case of the $K_u$-band signals. The very fast code modulation provides precise absolute signal delay measurements that allow the cycle ambiguity of the $K_u$-band carrier to be resolved. The additional S-band frequency enables the ionospheric delay to be determined and cancelled, including the correction of short-term variations in the ionosphere. The system is designed to operate in close to real time, processing the carrier ambiguities and ionosphere delay in less than 20 s.
The performance of the MWL will provide a time instability at 300 s averaging time of $TDEV \leq 0.23 \text{ ps}$, corresponding to a relative frequency stability of $ADEV \leq 1.9 \times 10^{-15}$, comparable to the expected instability of the space hydrogen maser (SHM), the clock in the ACES payload with the highest stability over this period. At an averaging time of 1 day, the specified performance is $TDEV \leq 5.5 \text{ ps}$, or $ADEV \leq 1.6 \times 10^{-16}$, which is again comparable to or better than the most stable clock in the ACES package over this averaging time, the PHARAO cold-atom caesium standard. This assumes that the MWL exhibits a white frequency noise behaviour for averaging times longer than 1000 s. If we assume no cycle slips, i.e. white phase noise behaviour, then intercontinental frequency comparison accuracies at the $10^{-17}$ level should be achievable after one week of averaging [Cacciapuoti 2007].

An enhanced version of the ACES microwave link has been suggested for use in the context of the EGE mission proposal [Schiller 2007]. This is a two-way three-frequency microwave link, with two bi-directional links operating in $K_a$-band and $K_u$-band, and a third down-link operating in S-band (Figure 36). The microwave carrier frequencies are generated directly from the microwave output of the femtosecond optical frequency comb of the optical clock, yielding signals in the 10 GHz frequency range with extremely low close-to-carrier phase noise (section 3.4.2.2) without the need for intermediate frequency dividers or synthesizer stages. The use of a $K_u$-band carrier reduces the noise by a factor of two compared to the Ku-band, while the use of three bands permits higher-order ionospheric corrections to be determined. Together with several other enhancements, these changes are expected to improve the performance of the MWL, making it compatible with the expected performance of optical clocks and allowing frequency comparisons between remote clocks at the $10^{-17}$ level after only one day of averaging [Schiller 2007].
3.5.1.3 Optical frequency transfer between ground stations and satellites

Links at optical frequencies promise even lower instability for frequency transfer and greater accuracy for time transfer than microwave links, and provide a complementary method that is subject to different performance limitations and causes of instability. The use of laser links for frequency and time transfer to satellites is well-established, an early example being the LASSO (laser synchronisation from stationary orbit) time transfer experiment that was launched on the geostationary Meteosat-P2 satellite in 1988, which demonstrated time transfer with an uncertainty of around 100 ps and a frequency stability of around $10^{-13}$ over 1000 s [Fridelance 1995]. More recent projects incorporating laser time and frequency transfer include NASA’s Mars Polar Lander, lost during landing in 1999, and the Chinese laser time transfer (LTT) payload that was launched with the Compass-M1 navigation satellite in April 2007.

The basic principle of the method is the emission of timed pulses of laser light from a ground station towards the satellite. At the satellite the pulses are retro-reflected back to the ground station and their arrival times measured against the on-board clock. The time of arrival of the reflected pulses back at the ground station is also measured, enabling both the offset between the ground and satellite clocks and the range to be determined. The technique requires the use of a suitable high-powered laser, typically one of the 40 or more satellite laser ranging (SLR) stations that routinely perform ranging measurements on satellites or the Moon as part of the International Laser Ranging Service (ILRS). The lasers employed are typically frequency-doubled Nd:YAG lasers that emit 532 nm pulses of 10 ps to 500 ps duration.

Figure 36. EGE-MWL concept and characteristics, reproduced from [Schiller 2007].
The present state of the art in frequency and time transfer using laser pulses is demonstrated by the T2L2 (time transfer by laser link) experiment. This project initially aimed to place a payload on the Russian space station Mir in 1999 and was subsequently considered for ACES, but has now been integrated onto the Jason-2 ocean observation satellite that was launched in June 2008 [Guillemot 2006]. The system is expected to provide ground to space time transfer with an uncertainty of less than 100 ps, with an ultimate stability better than 10 ps over 1 day \((1.2 \times 10^{-16})\) [Weick 2007].

A deep space optical laser link has been proposed in the context of the SAGAS mission [Wolf 2007]. This is based on the exchange of continuous two-way laser signals between the ground station and the spacecraft, with heterodyne detection systems at either end to detect the frequency difference between the local oscillator and the incoming signal. The scheme allows for asynchronous operation, which means that measurements taken at different times can be combined in an optical way to minimise the uncertainties.

### 3.5.1.4 Inter-satellite frequency transfer

Requirements for frequency transfer between satellites are found in two types of scenario: multi-satellite deep-space missions, and within satellite constellations in earth orbit, whether between LEO, MEO or GEO satellites, or any combination of these. In most systems, time synchronisation is required rather than frequency transfer, and the system optimised accordingly. A range of two-way or reflected microwave and optical techniques can be employed, as for ground-to-satellite links, and similar levels of performance can in general be achieved.

Data transfer and communication links between satellites often provide some capability for time and frequency transfer, an example being NASA’s Tracking and Data Relay Satellite System (TDRSS), in operation for more than 20 years, which can perform time synchronisation although with a limited accuracy of around 100 ns [Detoma 1990]. The optical inter-satellite links (OISLs) that are increasingly being carried to provide data transfer at rates of up to 10 Gb/s can also be configured to transfer frequency via the optical carrier.

GNSS satellite signals provide a readily available mechanism for frequency or time transfer between LEO satellites equipped with suitable receivers. Most previous work in this area has addressed time rather than frequency transfer, demonstrating for example synchronisation of the clocks onboard the two GRACE satellites to within 150 ps. These satellites also have crosslinks at 24 GHz and 32 GHz for precise ranging by means of phase measurements, and the two independent measurements of clock frequency offset have been shown to agree within 0.06 ps/s \((6\times10^{-15})\) [Bertiger 2003]. Future GNSS satellites are expected to have enhanced cross-links for data exchange. For example, the ESA GNSS+ project is aimed at assessing the benefits of inter-satellite ranging and communications between GNSS satellites, including the use of precise satellite clock offset determinations in generating the system time scale.

### 3.5.2 Frequency comparison via optical fibres

Transmission via optical fibre networks has been identified as the most viable alternative to more traditional satellite-based techniques for the transfer of high stability frequency references between remote locations. Although some demonstration experiments have been performed at other wavelengths, there is a general consensus that long-distance remote frequency comparison will use lasers at around 1.5 μm (telecommunications C-band), where the attenuation of single mode fibre (SMF) is at its minimum and off-the-shelf components are readily available at moderate cost. The potential advantages of working at the zero-dispersion wavelength of SMF, around 1.3 μm, are offset by the greater attenuation at this wavelength.
Three methods of transferring frequency references via optical fibre for timing and metrological applications have been explored to date:

1. Microwave or radio-frequency transfer by amplitude modulation of a continuous wave (cw) optical carrier;

2. Direct transfer of the optical carrier;

3. Simultaneous transfer of optical and microwave references by transmission of an optical frequency comb.

As described in section 3.4.2, a femtosecond comb can be used to transfer the stability of an optical frequency standard either to a 1.5 µm transfer laser or down to the microwave or radio frequency region of the spectrum, depending on the transfer technique selected.

In any of these techniques, noise added in propagation through the fibre must be corrected for. The correction to be applied is determined by comparing the phase of the received signal (i.e., the signal after propagation) with that of the transmitted signal (the signal before propagation). This comparison can only be done locally, whereas the correction needs to be applied to compensate for the phase error at the remote location. This is achieved by returning the signal from the remote location. The out-and-back signal is compared with the transmitted signal and the phase error assumed to be twice the phase error accumulated in one-way transmission, i.e., the correction applied to the transmitted signal is half the detected phase error. This assumption of reciprocity is inherently flawed due to the non-zero propagation time of the signal: if the fibre experiences a time-varying perturbation, the accumulated phase error on the return trip will be different from that accumulated on the outward trip. The round-trip propagation time \( \tau_{rt} \) limits the servo bandwidth (i.e., the maximum perturbation frequency for which a correction can be applied) to \( 1/(2\pi\tau_{rt}) \). For a 100 km length of standard silica fibre, the round trip delay is about 1 ms, limiting the bandwidth to approximately 160 Hz.

The three techniques for frequency transfer have been compared over a 6.9 km (round trip) optical fibre link between JILA and NIST (Boulder) [Foreman 2007a]. Whilst this work is of great importance in linking the ensemble of optical frequency standards at these laboratories with the NIST caesium fountain primary frequency standard, the fibre length is too short to provide a realistic test of the feasibility of fibre frequency transfer at the scale required for frequency comparison between European laboratories. Longer-distance tests over installed fibre have been performed using an 86 km (round trip) installed fibre link between LNE-SYRTE and LPL in Paris [Lopez 2008, Jiang 2008], on spooled fibre in the laboratory [Marra 2008], or on combinations of installed and spooled fibre [Newbury 2007b]. A more detailed comparison of the three different techniques is presented in the following sections.

### 3.5.2.1 Microwave / rf transfer by amplitude modulation of an optical carrier

Transmission of an rf or microwave signal over fibre by amplitude modulation (AM) of the optical carrier is essentially the standard telecommunications technique for data transmission, and has been used on the JILA–NIST and LNE-SYRTE–LPL links as well as being routinely used on NASA’s Deep Space Network (DSN) at Goldstone in the Mojave desert.

The DSN has distributed maser reference frequencies at 100 MHz over unstabilised fibre links of up to 30 km since at least the late 1980s, with fibre stabilisation being employed from the early 1990s. The stability requirement for radio science experiments with the Cassini spacecraft, \( 1.5 \times 10^{-16} \) at 100 MHz for averaging times greater than 10³ s, has required improved fibre link stabilization with distribution at 1 GHz [Calhoun 2002]. Stabilisation is achieved by means of a temperature-controlled fibre spool inserted into the link.
In the JILA–NIST work, a 1.3 μm single-mode DFB laser is the carrier for a maser-referenced 1 GHz signal. A round-trip instability of $2.5 \times 10^{-14}$ at 10 s averaging time was obtained, without phase compensation [Ye 2003]. With fibre length stabilisation using a fibre stretcher, the instability has been reduced below $10^{-16}$ for averaging times longer than $10^4$ s [Foreman 2007b].

The LNE-SYRTE–LPL link consists of a parallel pair of fibres following a 43 km route. Two continuous lengths of fibre were constructed by fusion splicing a few tens of sections of unused SMF 28 single mode fibres from France Telecom’s Paris area metropolitan fibre network. The carrier wavelength is 1.55 μm, generated by a DFB laser. Initially, AM at 100 MHz was used without any fibre stabilisation, yielding a round-trip (86 km) instability of $7 \times 10^{-14}$ at 1 s and $7 \times 10^{-16}$ at $10^4$ s [Amy-Klein 2004]. Two stabilisation schemes were subsequently demonstrated in parallel, with a 100 MHz signal transmitted over one fibre and a 1 GHz signal transmitted over the other. Electronic phase compensation was used on the 100 MHz link and optical path length correction on the 1 GHz link, using a fibre stretcher constructed by wrapping fibre around a piezo-electric actuator for compensation of fast (acoustic) fluctuations and a temperature-controlled fibre spool for slow (thermal) fluctuations in the fibre length [Daussy 2005]. In both schemes, the return signal was generated by modulating a DFB laser at the remote end of the fibre at 100 MHz derived from the transmitted frequency. Although the outward and return signals are transmitted over the same fibre, their polarizations differ stochastically, leading to uncorrelated phase fluctuations as a result of polarization mode dispersion (PMD) [Narbonneau 2006]. This effect can be overcome by rapid scrambling of the polarisation. Using this technique and configuring the two fibres as a single 86 km link with modulation at 1 GHz, a link instability of $5 \times 10^{-15}$ at 1 s has been demonstrated, reaching $2 \times 10^{-18}$ at an averaging time of 1 day (Figure 38 (c)) [Lopez 2008]. The advantages of working at higher frequency (relatively smaller phase fluctuations with fibre length) have recently been demonstrated in the transmission of a 9.2 GHz signal over the same link, with a 5 dB improvement in stability (Figure 38 (d)) [Jiang 2008]. At this frequency, the AM has to be applied using an external modulator rather than directly to the DFB laser current.

### 3.5.2.2 Direct optical frequency transfer

Transmission of an optical frequency standard by optical fibre between remote laboratories was first demonstrated at 778 nm using a specially installed 3 km fibre link between LNE-SYRTE and the Laboratoire Kastler-Brossel (LKB) in Paris, in support of spectroscopy of atomic hydrogen for measurement of the Rydberg constant [de Beauvoir 1997]. A fractional frequency uncertainty of $8 \times 10^{-15}$ was obtained on this unstabilised link.

The basic principles of phase compensation of fibre-induced noise in transmission of stable optical frequencies over optical fibre were demonstrated more than a decade ago by Hall and co-workers at JILA [Ma 1994]. In this early work a short fibre length (25 m) was used and hence a large servo bandwidth was achievable, with acousto-optic modulators (AOMs) being used for phase correction. This phase-noise cancellation technique is widely used for short, intra-laboratory fibre links, for example between optical frequency standards and femtosecond combs (Figure 37). In one such demonstration at NIST, an overall fractional frequency transfer instability of $6 \times 10^{-17} \tau^{-1/2}$ was demonstrated over a coherent network of 750 m optical fibre with several intermediate frequency conversions using femtosecond optical frequency combs [Coddington 2007]. In another example, the JILA group have applied this technique to optical frequency transfer at 1064 nm over the 7 km JILA–NIST fibre link, obtaining a round-trip instability of $6 \times 10^{-18} \tau^{-1/2}$ for averaging times up to 1000 s [Foreman 2007a].
The LNE-SYRTE group have extended this technique to longer distances. In initial experiments, carried out in collaboration with PTB, light from a 1.5 μm fibre laser stabilised via a fibre-based femtosecond comb to an ultrastable laser at 698 nm was transmitted over the 86 km round-trip link, with the return signal used to stabilise the optical phase via an AOM at the fibre input. The link was extended to 211 km by the addition of fibre spools in the laboratory. In this configuration an erbium-doped fibre amplifier (EDFA) was included to compensate for the 100 dB round-trip attenuation in optical power. Instabilities of $6 \times 10^{-18}$ and $2 \times 10^{-17}$ respectively were obtained after 8000 s averaging time [Grosche 2007]. In this configuration, with the fibre pair connected at the LPL end of the route, the “remote” end of the fibre is located at LNE-SYRTE and is thus accessible for direct comparison of the received and transmitted signals. Out-of-loop comparison of these with the out-and-back signal used for compensation will provide a test of the reciprocity of the fibre. Further experiments with this configuration have used an ultrastable 1.5 μm fibre laser. An AOM is used to offset the return frequency from the transmitted frequency by a few tens of megahertz. By the addition of further frequency offsets together with EDFAs at the “remote” end of the link, it is possible to recirculate the light twice more around the fibre, creating a 172 km link. In this configuration, the link instability is $4 \times 10^{-16}$ at 1 s, reaching $4 \times 10^{-19}$ at an averaging time of $3 \times 10^4$ s (Figure 38 (e)) [Jiang 2008]. This is the longest distance demonstration of optical frequency transfer entirely over installed fibre. A similar experiment has been performed over 110 km of an urban installed fibre network between NMIJ/AIST and Kashiwa, Japan, with the reported link instability being $1.5 \times 10^{-15}$ at 1 s [Musha 2008].
3.5.2.3 Microwave / rf transfer using an optical frequency comb

Transmission of the pulse train from a mode-locked laser offers the possibility of simultaneous transmission of a microwave frequency (the pulse repetition rate \( f_{\text{rep}} \)) and a comb of ultra-stable optical frequencies. Work reported to date has focused mainly on the stability with which the repetition rate can be transmitted. With this technique, temporal dispersion of the pulses due to the chromatic dispersion of the fibre results in a loss of coherence in the repetition rate signal over long fibre length. This can be compensated for by the introduction of a matched length of dispersion compensation fibre (DCF) or eliminated by the use of dispersion shifted fibre (DSF). A link using the former approach is known as a dispersion-managed link and is widely used for ultra-high data-rate transmission. The latter approach requires a link entirely laid with non-standard fibre, and so is likely to be less generally applicable. A further drawback of this method of frequency transmission is that, since the repetition rate signal is a beat note between comb modes, its attenuation is the square of the attenuation for an optical signal.

Using DCF, the JILA group have demonstrated a fractional instability of \( 9 \times 10^{-15} \tau^{-1/2} \) over the 6.9 km JILA-NIST network (Figure 38 (a)) [Holman 2005] and have demonstrated that this technique can be applied to remote synchronisation of mode-locked femtosecond lasers [Hudson 2006]. At NPL a similar link instability has been demonstrated over 100 km of spooled fibre in the laboratory (Figure 38 (b)) [Marra 2008].

3.5.2.4 Comparison of transmission techniques

The three techniques for frequency transfer over optical fibre are compared in Figure 38. This clearly illustrates that the best results in terms of fractional frequency stability have been obtained using the optical carrier as the transmitted frequency standard. This can be expected given that direct transfer of the optical carrier has the advantage of high spectral resolution. Furthermore, the optical carrier technique uses the femtosecond comb to transfer the stability of the optical frequency standard into the 1.5 µm band, which (as discussed in section 3.4.2.1), can be achieved at a level that does not degrade the stability of state-of-the-art optical frequency standards. The rf / microwave techniques, on the other hand, require downconversion of the optical frequency standard to the rf / microwave domain via the femtosecond comb, which (as described in section 3.4.2.2) at present introduces excess noise. Direct transfer of an optical carrier frequency hence offers the best prospects for high accuracy comparison of remotely located optical frequency standards, and is being pursued at a number of laboratories around the world.
Figure 38. Fractional frequency stability of transfer over fibre spools in the laboratory and installed optical fibre links: (a) microwave transfer at 774 MHz over a 6.9 km installed fibre link using a mode-locked fibre laser [Holman 2005]; (b) microwave transfer at 1.5 GHz over 100 km of spooled fibre in the laboratory [Marra 2008]; (c) microwave transfer at 1 GHz over an 86 km installed fibre link using an AM cw laser [Lopez 2008]; (d) microwave transfer at 9.2 GHz over an 86 km installed fibre link using an AM cw laser [Jiang 2008]; (e) optical frequency transfer at 1.5 µm over a 172 km installed fibre link [Jiang 2008]; (f) optical frequency transfer at 1.5 µm over a 251 km link comprising 76 km of installed fibre and 175 km of spooled fibre [Newbury 2007b].

3.5.2.5 Further experimental investigations

The experiments reported above have demonstrated that cw optical frequency transfer over installed fibre links can be performed with a level of instability that will not degrade the stability of state-of-the-art ultrastable lasers and optical frequency standards, at least at distances up to around 200 km.

The shortest distance between two European NMIs active in the development of optical frequency standards is 470 km between NPL and LNE-SYRTE, whilst the distance from either of these to PTB is closer to 900 km. Optical frequency transfer over such a long installed fibre link has yet to be demonstrated, although work is currently in progress on a 900 km link between PTB and MPQ in Germany (section 6.1.1.2). Erbium-doped fibre amplifiers will be required at intermediate stages of the link to compensate for the unavoidable fibre losses. It may also be necessary to implement intermediate lasers as active noise filters to remove accumulated phase noise due to transmission through the fibre. This is a technique that has yet to be demonstrated.

The question as to whether it is technically feasible to transmit ultrastable optical frequencies over fibre links carrying other traffic has not yet been explored experimentally.
4 Selection of optical atomic clock hardware

4.1 Technology selection

Recent progress in the development of optical frequency standards has been impressive, with orders of magnitude improvement in performance levels having been achieved since the introduction of femtosecond optical frequency combs (Figure 39). As discussed in sections 3.2 and 3.3, there are a number of candidate cold ion and neutral atom species with intrinsically narrow optical transitions, each of which has its own advantages and disadvantages with respect to the laser technology required for cooling and probing the atomic reference, and its intrinsic sensitivity to systematic frequency shifts. The performance of the best optical frequency standards already exceeds that of microwave standards, but the various clock systems being studied are currently at different stages of their development, new results are being reported frequently and significant further improvements are expected for all systems. For this reason, parallel development of a number of different systems is considered desirable for optical atomic clocks to reach a high level of technical maturity and to reach the state where the best option for any particular mission scenario can be selected with confidence.

Figure 39. The accuracy of caesium microwave atomic clocks has increased by more than five orders of magnitude over the past 50 years (blue circles), and the best caesium fountain clocks now have systematic uncertainties better than 1 part in 10¹⁵. However recent progress in the development of optical frequency standards has been even more rapid, with the uncertainties of the best absolute frequency measurements (green circles) now limited by the uncertainty of the caesium primary standards, and direct comparisons between optical frequency standards (red circles) showing an even higher level of reproducibility.
Neutral atom optical clocks offer the prospect of higher stability than trapped ion optical clocks at short timescales, whilst the trapped ions may provide higher accuracy since they generally have lower blackbody Stark shifts than neutral atom species. The single trapped ion architecture is also simpler, and hence offers the prospect of a more compact clock for space operation. Since different parameters of the clock (stability, accuracy, size, etc.) are likely to be of critical importance for different applications, we therefore recommend that advanced portable prototypes of both types of clock should be developed.

Of the trapped ion clocks discussed in section 3.2, the technologically simplest contenders for a space-borne optical clock are the $^{171}\text{Yb}^+$, $^{88}\text{Sr}^+$ and $^{40}\text{Ca}^+$ electric quadrupole transitions. There is little to choose between these systems in terms of projected frequency instability (Figure 18), the magnitude of the electric quadrupole moment of the upper state (Table 10) or the fractional blackbody Stark shift (Table 12). Our final recommendation for development of an advanced portable prototype is therefore determined by technology considerations.

Overall, $^{171}\text{Yb}^+$ is more complicated than the even-isotope options of $^{88}\text{Sr}^+$ and $^{40}\text{Ca}^+$ because its hyperfine structure means that two different frequencies are required for the cooling, repumper and clearout transitions (section 3.2.4.5). In addition it requires frequency-doubling stages for both the clock transition and the cooling transition. The $^{88}\text{Sr}^+$ and $^{40}\text{Ca}^+$ systems both exhibit a linear Zeeman shift of the clock transition frequency (section 3.2.3.1), meaning that magnetic shielding is required, but the absence of hyperfine structure eliminates the need for multiple laser frequencies for each transition. Taking into account the ease of obtaining diode lasers at the relevant wavelengths (sections 3.2.4.2 and 3.2.4.3), and the available power levels for both clock and cooling transitions, our recommendation is to develop an advanced portable prototype of an optical clock based on the $^2\!S_{1/2} - ^2\!D_{5/2}$ electric quadrupole transition at 674 nm in $^{88}\text{Sr}^+$. We consider this to represent an excellent prospect for a light and compact optical clock with low power consumption, which can be expected to reach a level of performance suitable for a wide range of mission opportunities.

However, in the longer term, some of the technologically more challenging options for trapped ion optical clocks offer the prospect of higher stability and accuracy. The most attractive option from this point of view is the $^1\!S_0 - ^3\!P_0$ intercombination transition in $^{27}\text{Al}^+$, which combines excellent potential stability (Figure 18) with the lowest room-temperature blackbody Stark shift of all systems being studied (Table 12), a very low quadratic Zeeman shift (Table 9) and zero electric quadrupole shift (Table 10). However quantum-logic techniques are required to access the clock transition, increasing the complexity of the trap technology, and this is compounded by the need for deep UV laser technology (section 3.2.4.1), which is likely to increase the complexity, size and power consumption of the clock as well as presenting additional challenges for space qualification. Nonetheless, due to its expected superior performance, we recommend that an advanced portable prototype of a $^{27}\text{Al}^+$ optical clock should be developed in parallel with the $^{88}\text{Sr}^+$ optical clock discussed above.

Although optical clocks based on cold neutral atoms are currently at a lower technological readiness level than trapped ion optical clocks, they offer a better prospect of reaching high stability at short timescales. However the accuracy they are able to offer will depend on the extent to which fluctuations in systematic frequency shifts can be controlled. Of particular concern is the blackbody Stark shift, which is generally higher than for trapped ions.

Of the three atom species that are actively being pursued as optical lattice clocks at present, strontium and ytterbium are significantly simpler than mercury in terms of the laser technology required. There is little to choose between these systems in terms of systematic frequency shifts, since the blackbody Stark shift differs only by a factor of two (Table 18). As a result, our final recommendation for development of an advanced portable prototype is determined by technology considerations.
In this respect, all the laser wavelengths required to cool, trap and probe the strontium atom can be produced using diode laser technology, with only one frequency doubling stage required (section 3.3.4.1). The ytterbium lattice clock uses a combination of diode lasers and fibre lasers, but in this case two frequency doubling stages are required (section 3.3.4.2). In view of the slightly greater complexity of the ytterbium clock, our recommendation is therefore to develop an advanced portable prototype of a lattice clock based on the $^1S_0 \rightarrow ^3P_0$ transition in atomic strontium. Present the fermionic isotope $^{87}$Sr is preferred; however if future developments turn out to favour the bosonic isotope $^{88}$Sr then this could be studied using essentially the same apparatus.

In the longer term, however, an optical lattice clock based on neutral mercury offers the prospect of significantly higher accuracy, because it has a much lower blackbody Stark shift (Table 18). This system has the added advantages that it can be operated without a Zeeman slower and fewer lasers are required since efficient laser cooling is possible using the $^1S_0 \rightarrow ^3P_1$ transition alone. However deep UV lasers are needed for both the cooling and the clock transitions (section 3.3.4.3). Nevertheless, in spite of the challenging nature of the laser technology developments required for space operation, we recommend that in light of the improved performance expected from this system, an advanced portable prototype of a mercury optical lattice clock should be developed in parallel with the strontium lattice clock.

In summary, we consider that to reach the stage where the best option for any particular mission scenario can be selected with a high degree of confidence and minimum technological risk, advanced portable prototypes of four different optical atomic clocks should be developed in parallel. The four systems recommended for development are as follows:

1. A trapped ion optical clock based on $^{88}$Sr$^+$;
2. A strontium atom optical lattice clock;
3. A quantum-logic-based trapped ion optical clock using $^{27}$Al$^+$;
4. A mercury atom optical lattice clock.

This multiple-clock development approach will allow for the highest performance specifications to be achieved during the prototyping phase whilst, at the same time, undertaking the engineering developments necessary to prepare the clocks for space integration. It will lead to a number of different clock options with individual advantages of compactness, stability or accuracy, from which the best option for any particular mission scenario can be selected with a high degree of confidence and minimum technological risk. Some elements of the clock are common to all four systems, in particular the optical frequency comb and many aspects of the optical local oscillator. The rationale for the particular choices and the synergies between them is discussed below.

The clock options can be divided into two broad approaches. These target a pair of options that are technologically more advanced at this time, and another pair that are currently more technologically challenging, but which point to clocks of higher performance (by an order of magnitude) in the longer term. Whilst the latter options are likely to reach a lower technology readiness level (TRL) by the end of the initial prototyping period, the strategy aims to "future-proof" the optical atomic clock development as far as possible, by facilitating the opportunities for future missions with on-board optical atomic clocks providing state-of-the-art performance at the point of engineering model build.
The methodology adopted in the selection of the different atomic reference choices identified **key synergies** between the different possibilities, as a fundamental prerequisite that allows for time- and cost-effective solutions, avoiding multiple and un-correlated technology development being undertaken by the collaborating institutes during the prototyping phase.

![Diagram of optical clock development plan](image)

**Figure 40.** Optical clock development plan showing four options for the atomic reference and the synergies between the particular choices. The femtosecond comb development package, common to all options, is not shown. (LO: local oscillator; SHG: second harmonic generation; SFG: sum frequency generation).

Looking at figure 2, the Sr⁺ ion and Sr atom clocks represent the simpler, more technically advanced systems at present, and share a good deal of common laser technology. Laser cooling is achieved with infra-red and red laser diodes, nearly all of which are already commercially available in extended cavity or DFB format. Only one stage of frequency doubling (SHG) to the blue is required for the primary cooling radiation; auxiliary lasers in the red (Sr) and infra-red (Sr⁺) are available as DFBs. On the local oscillator front, both clock lasers are based on red diodes at similar wavelengths and have already demonstrated linewidths at the hertz level using vibration-insensitive cavities at two of the major collaborator institutes. Improved cavity designs for space application are expected to lead to rapid testing and incorporation of the same cavity and laser technology into these two clock options.

The more technically demanding options (the quantum logic Al⁺ ion clock and the Hg neutral atom lattice clock) represent optical clock systems that have intrinsic advantages in respect of their low sensitivities to particular frequency shifts, offering better accuracy in the long term. This pairing is qualified by the similar requirements for UV cooling and clock laser technology. The generic laser technology allowing access to the UV is centred on high power narrow-linewidth fibre lasers in the 1.0 – 1.1 µm region, with two stages of frequency doubling / sum frequency generation to reach the required UV wavelengths. The
commonality between the 1 µm laser systems and the harmonic generation stages again allows for the same technology development to satisfy both opportunities. In particular, the UV local oscillator wavelengths are almost identical, being within ~1 nm of each other, and will also benefit from the common cavity technology developed and tested on the red local oscillator systems.

It is important to view the relativity of this clock development plan with other ESA activities in this area, such as the ELIPS II “Space optical clock” (SOC) project, which is managed by the ESA HME directorate and targeted on a mission on board the International Space Station. The SOC project concentrates on neutral atom lattice clocks, and has activity at European laboratories on two options, Sr and Yb. In drawing up this TEC-MME plan, we considered all the various atom options including Sr, Yb, Mg and Hg and concluded that Sr and Yb were very similar with respect to limiting frequency shifts. The future-proofing considerations offered by the longer-term perspective of Hg, and the fit with the quantum logic ion technology, led us to the conclusion that the Sr and Hg neutral options were a better combination. Of course, this does not negate the significance of Yb neutral clock research and development carried out within the SOC project. Rather, we view it as additional value to a combined optical atomic clock development, where funding issues do not allow total coverage of all opportunities. In fact, we recognise that there is an implicit synergy within the development with respect to the neutral Yb lattice activity within SOC. This arises from the requirement for two stages of frequency doubling from IR fibre laser fundamental wavelengths to reach the UV local oscillator wavelengths. The technology for first stage of doubling to the green region is entirely consistent with the single stage doubling needed to access the Yb atom LO wavelength at 578 nm. As a result, these 1st stage doubling outputs will have good impact both for Yb and the UV clock options. With respect to the Sr lattice clock activity within SOC, we view the development plan as building on the progress already achieved by members of the SOC project (who are also part of this TEC-MME plan), but focusing on the need to push development via advanced prototypes that target mass, volume and power budgets in sufficient detail that will allow direct and efficient conversion to EM and FM build in the second-phase post-prototyping period. In order to achieve this, we believe it is imperative to increasingly incorporate the design experience of space integrators during the initial prototyping phase.

One of the generic objectives of this four-strand development approach (Figure 41) is to take account of the different rates of progress being demonstrated for the different clock choices and allow for their respective capabilities and specifications to further evolve during the prototyping period, whilst at the same time undertaking the technology adaptation necessary to prepare them for space integration. This approach will thus provide the competitive drive necessary to achieve the highest possible performance levels, and lead to a number of different clock options with individual advantages of compactness, stability or accuracy. One further important point to note in the context of fundamental physics missions designed to search for a dependence of the fundamental constants on gravitational potential (section 2.1.1.1) is that these four clocks have different sensitivities to variation in the fine structure constant $\alpha$ (Table 1). Some elements of the clock will of course be common to all four systems, in particular the optical frequency comb and many aspects of the optical local oscillator. In parallel it will be necessary to develop improved techniques for frequency comparison of the optical clocks in order to allow their true performance to be assessed (section 6).
4.2 Technology developments necessary for space operation

The advantages of the multi-track development approach proposed in the previous section becomes clear once one considers the relatively short timescale available to bring optical clock capability to the space-qualified level by 2020. In order to cope with the still evolving state-of-the-art within the development of a space clock payload on this timescale, the development strategy has been broken down into two main periods (Figure 41). The first period covers the development of optical clock sub-system assemblies (both parallel-track atomic reference sub-units and the common local oscillator and frequency comb sub-units) to the advanced prototype level by five years. Towards the end of this period it is expected that each sub-system will be integrated into an optical clock package for system testing, and so provision must be made for fully compatible interfacing between the respective sub-units. This will include electrical and optical fibre interfacing, provision for standardised central processor control and a common structural support approach that can allow the separate sub-units to be structurally integrated within a complete unit. The sub-units will conform to indicative size and mass and power budgets, which will be critically assessed against space payload requirements. These indicative budgets are given below for each sub-unit and for the total optical atomic clock systems.

With the completion of the five-year advanced prototyping period, the technology development would move to a second phase, covering the development of full engineering models (EMs) and subsequent space-qualified flight models (FMs). At this stage, and taking account of mission proposals including the outputs from the Cosmic Vision 2011 call, the preferred atomic reference sub-unit(s) will be selected to go forward to the EM development phase.
Whilst our recommendation is that the advanced prototyping activities should primarily involve the European space and clock metrology research community together with the specialist space-oriented time and frequency companies, the need for early inputs on the space integration side (i.e. common interfacing and control, and thermal and structural systems management) will require consultancy and outline platform design from one or two leading space integration companies as a precursor to the EM development phase. It would then be expected that one of these space integrators would take the prime contractual role as we move into this second phase.

Within these terms of reference, the necessary technology development for achieving complete optical clock sub-unit technologies at the advanced prototype level (as defined above) can be broken down into sub-unit development packages. These packages can be pursued by different metrology and space institutes and companies, with the proviso that proper co-ordination in the developmental activities are maintained throughout the breadth of the sub-unit structure. This will require strong co-ordination between the prime contractor for the advanced prototype period and the primary sub-unit co-ordinators.

The technology selection process described above takes account of projected clock performance and achievable specification within the first five-year period, but also the technical issues that will likely determine the ease of technical development and integration throughout this period and beyond into the EM and FM phases. In addition it also considered commonalities and synergies relevant to the different atomic reference choices, such as commonality of local oscillator technology between the Sr$^{+}$ ion and Sr neutral lattice clocks. This allows for the maintenance of the wide parallel approach, but with improved efficiency due to the reduction in intrinsically different wavelength-dependent local oscillator technology components. Thus the technology development plan constructed addresses generic sub-unit processes that are applicable to more than one atomic reference opportunity.

4.2.1 Sub-unit technology developments

Technology considerations for the range of possible atom and ion optical clock species have already been discussed in section 3. Here we concentrate on those options selected for development. Each atomic reference is considered in turn, before addressing the common sub-unit systems. A number of issues are common to all sub-systems, and are initially described in detail within the ion clock development plan, and simply referred to in subsequent plans.

4.2.1.1 $^{88}$Sr$^{+}$ ion clock

The $^{88}$Sr$^{+}$ ion clock physics package can be broken down into the ultra-high vacuum (UHV) trap sub-component, the cooling and auxiliary laser block, and the local oscillator unit. In common with all the other sub-units, there are electrical and fibre-optic interface requirements, together with central processor control, servo control and data acquisition. Also common is the requirement for high stability and high stiffness support structures and adequate thermal management of each sub-unit. The trap package comprises a miniature (mm-dimension) rf end-cap trap or ring trap, capable of ionizing, trapping and storing a single ion for long periods of time within a UHV chamber with a volume of approximately one litre and evacuated to a pressure of $10^{-10}$ mbar using miniature sputter ion or evaporable getter pumps. External to the chamber is a high numerical aperture lens system and photomultiplier combination with a magnification of a few times, capable of sampling single ion count rates of a few thousand s$^{-1}$ and three-axis magnetic field coils with two layers of mu-metal shielding, contributing, in total, a volume of several litres.
The cooling laser platform uses all-diode-laser technology. The primary cooling is now achieved with a 150 mW high power extended cavity laser diode at 844 nm, single-pass frequency-doubled to 422 nm in periodically-poled KTP. For redundancy purposes, a minimum of two cooling laser units should be considered. Possibilities also exist for 844 nm high power DFB systems. Currently, the auxiliary repumper and clear-out lasers at 1092 nm and 1033 nm are DFB and extended cavity lasers respectively, but simple temperature and current controlled DFBs, locked to simple low-drift cavities will suffice for both these lasers. These DFBs are small and a redundancy package of minimum three DFBs per wavelength should be built in, although our experience with the 1092 nm DFBs is that the original diode is still in use after ten years. A total volume footprint of approximately 10 litres looks viable.

The local oscillator (LO) for the $^{88}\text{Sr}^+$ quadrupole transition is a red diode laser at 674 nm within an extended cavity, frequency stabilised to an evacuated, temperature-controlled high finesse ULE reference cavity, providing state-of-the-art linewidths at the 1 Hz level. The dominant volumetric package is the vacuum housing for the ULE cavity. With cylindrical cavity dimensions of 5 cm length and 3 cm diameter, a vacuum cavity package of less than one litre looks feasible, leading to a total local oscillator volume including conditioning optics of approximately 8 litres.

Optical integration of the sub-units will be achieved using single mode fibre optic launchers and beam combiners. Already, we are supplying cooling and probe laser light to our ion traps using multi-way fibre couplers designed to split and deliver the required intensities to the trap. Additionally, we also use fibre pick-off from each wavelength to a diagnostic wavemeter for high resolution frequency monitoring and control. The current volume for this arrangement is approximately 7 litres. Separate from the optical integration and control is the ion quantum state preparation, control and clock probe sequencing algorithm development. This is primarily a software issue written into the central processor control software with appropriate analogue-digital (A-D) inputs and digital-analogue (D-A) outputs to and from the relevant opto-electronic transducers situated within the various sub-units. The sequencing algorithms are largely routine for basic clock operation.

The final sub-component is the support structure. Here, considerations of support stability and stiffness, and insensitivity to environmental conditions encountered on spacecraft, are paramount. Whilst optical breadboards constructed from Zerodur blocks, machined out to provide a low-mass ribbed structure underneath the optical surface, have been designed into the LISA Pathfinder, other lightweight but stiff platforms with nominal zero expansion in the $x$-$y$ plane, produced from carbon fibre matrices, are a fairly common support method for space platforms. Here the experience of the large space integrators will be very helpful in choosing the most appropriate material.

### 4.2.1.2 $^{27}\text{Al}^+$ quantum logic ion clock

The quantum logic ion clock comprises a combination of logic ion and clock ion of different species. For a number of potential clock species such as $\text{Al}^+$, $\text{B}^+$ or $\text{Tl}^+$, the clock transition is well into the UV region of the spectrum, but the cooling transition is located even further into the deep UV (well below 200 nm) where no appropriate cooling laser sources currently exist. As a result, not only is cooling problematic, but furthermore there is no cooling fluorescence available with which to observe quantum jumps when the clock transition is driven. Despite these difficulties, ions such as $\text{Al}^+$ have strong potential as optical clocks due to their very low sensitivities to systematic shifts. For example, $\text{Al}^+$ has no quadrupole shift, and the smallest blackbody radiation shift coefficient of all the ions and atoms considered for optical clock operation. As already outlined in section 3.2, observation of the 267 nm $\text{Al}^+$ clock transition has been achieved at NIST over the last two years or so, by using a $\text{Be}^+$ logic ion both to sympathetically cool the $\text{Al}^+$ ion and to read out the $\text{Al}^+$ clock transition information by means
of quantum logic operations between the Al" and Be' ions. This quantum logic read-out is a very impressive algorithm adapted from the quantum information processing studies carried out by the same group.

Trap operation for the quantum logic arrangement requires the use of an rf linear trap to accommodate the clock and logic ions along the linear axis. This is a reasonably well-researched arrangement both in Europe and the USA. The $^{27}$Al" ion has a linear Zeeman shift and hence, as for $^{88}$Sr", needs to be probed on two symmetrical Zeeman components and will require mu-metal shielding. The NIST demonstration has so far used Be" as the logic and cooling ion. However, generation of the necessary 313 nm radiation, both for Raman sideband cooling of the Be" ion and for the logic operations has been problematic, requiring the use of frequency doubled dye lasers at 626 nm. This is not viable for space clock operation, and an alternative ion combination of Al" and Mg" has been proposed and would be pursued within this development plan. The volume of the trap package is likely to be similar to the strontium trap package at around 7–10 litres.

The choice of an Al" / Mg" ion combination significantly simplifies the cooling and clock probing arrangements, by concentrating on narrow linewidth high power fibre laser systems to provide both radiations. The Mg" cooling radiation at 280 nm can be produced from a frequency quadrupled fibre laser at 1120 nm, in a similar manner to the quadrupled fibre laser output currently used by NIST to generate the 267 nm clock light. This concentration on a single fibre laser technology brings economy of scale, which would be further enhanced by its application in the Hg atom lattice clock development track.

Local oscillator development in this case is centred around the use of a reference cavity coincident with the frequency doubled output of the 1070 nm fibre laser, prior to further doubling to the UV. This presents no serious difficulty for high finesse cavity coatings in the green. The standard Koheras OEM fibre laser package would give a footprint of 5 litres with an IR enhancement doubling cavity and the ULE cavity / isolator / EOM / AOM combination adding another 5 and 8 litres respectively, giving 18 litres in total.

The necessary algorithm to include the logic operations within the clock probe sequencing is considerably more complex than the quadrupole case; this is partly due to the need for Raman sideband cooling of the logic ion close to the motional ground state, and also to use the quantum logic to prepare the Al" clock ion in its ground state prior to probing. Nevertheless impressive read-out times of a few milliseconds have been achieved, which has little impact on the overall algorithm cycle time. Given the single point of demonstration so far, it will be necessary to rapidly optimise the quantum logic technique during the early part of the five-year prototyping phase.

One area of anticipated difficulty concerns the issue of UV optics in general and the availability of UV-transmitting fibres in particular. At this time, no readily available single mode silica fibre appears to be available for the 260 nm – 280 nm region (the nearest having a cut-off wavelength of 320 nm).

This means that the final frequency doubling stages to the UV for both clock and logic wavelengths will need to take place in close proximity to the trap package (i.e. within the trap package sub-unit). This would add another 6–10 litres to the trap package to give a trap sub-unit total volume of 13–20 litres. Additionally the support structure for the trap sub-unit is likely to be twice the surface area to account for these additional doubling stages.
4.2.1.3 Sr neutral lattice clock

The Sr neutral lattice physics package will comprise a Sr oven, permanent magnet Zeeman slower, beam diverter chamber and double MOT / lattice chamber. With efficient engineering design, this could be limited to dimensions of $100 \times 20 \times 10$ cm$^3$ or 20 litres. If a lower atomic flux could be tolerated, this volume would halve. Three-beam retro-reflecting optics for the 1$^{st}$ and 2$^{nd}$ stage cooling, together with a minimum 1-D lattice arrangement around the MOT / lattice vacuum chamber will increase this volume by around another 10 litres. Assuming two 8 litre s$^{-1}$ ion pumps for differential pumping would add another 7 litres.

The system currently used for generation of the first-stage cooling light at NPL is a commercial high power (~1 W) MOPA diode system at 922 nm, which is frequency doubled to provide 300 mW at 461 nm to provide for the Zeeman slower beam, three retro-reflected MOT beams and additional state preparation and pusher beams. This is reduced to approximately 200 mW by beam conditioning, stabilisation and diagnostic pick-off followed by fibre delivery to the MOT /lattice chamber, sufficient for the above. The MOPA / doubler package has a current volume of $50 \times 30 \times 20$ cm$^3$ (30 litres), though this could realistically be reduced by a factor of 2 by stripping the system down. Second-stage cooling and auxiliary repumper lasers are based on the intercombination line at 689 nm and 707 nm / 688 / 679 nm respectively. Here, a 30 mW extended-cavity diode laser with redundant copy (approximately 2 litres in total) are needed at 689 nm for second-stage cooling, with an additional injection-locking stage to provide for three retro-reflected second-stage MOT cooling beams. Both the IR 922 nm and 689 nm diode lasers need to be stabilised to high finesse tunable reference cavities. A tunable cavity design incorporating two separate wavelength cavities built into the same evacuated ULE spacer would make efficient use of space, occupying a volume of around 3 litres with accompanying optics. DFB diode lasers plus at least one redundant unit at each wavelength (approximately 2 litres) would provide for the repumpers. Overall, the total cooling and repumper package would therefore be in the range 22–37 litres. The optical lattice “magic” wavelength for Sr is 813 nm, and will require a high power (~ 1 W) MOPA diode laser, which has a current (commercial) 10-litre footprint. All repumper and lattice DFB diodes have different frequency control options available, such as tuneable cavity stabilisation, diagnostic wavemeter stabilisation or simple temperature / diode current stabilisation.

The local oscillator needed to probe the clock transition is a 698 nm extended cavity diode laser stabilised to a high finesse evacuated ULE reference cavity. This has direct overlap with the Sr$^+$ ion clock LO technology at 674 nm, and demonstrates the laser synergy between the strontium ion and neutral strontium atom lattice clocks. This synergy also extends to the suite of red diode lasers used for the 2$^{nd}$ stage cooling and repumping.

Fibre delivery, diagnostics and support structure considerations are also similar to the strontium ion clock case, but with the proviso that more diode source lasers and more fibre output coupling beams are required. This would also be reflected in the increased support surface area needed to house the full complement of cooling, repumper and lattice lasers.

4.2.1.4 Hg lattice clock

One of the major systematic frequency shifts for the Sr neutral lattice clock is the blackbody shift, which is approximately 2 Hz for a room temperature apparatus. Neutral Hg is an alternative species for a lattice clock, where the blackbody shift is considerably reduced from this value by a factor of around 30. For the Hg case, there are also some other potential advantages. Only one stage of laser cooling is needed to cool the Hg atoms to the 30 µK level, low enough to load a lattice trap. Secondly, there is no need for a Hg oven, since the Hg
vapour pressure in the MOT /lattice chamber can be readily controlled by means of a Peltier-cooled reservoir. This also removes the need for a Zeeman slower. Under these circumstances, the UHV vacuum system should be much reduced in volume, needing only to contain a 2-D loading MOT some 10 cm from the 3-D MOT /lattice region. This would correspond to a volume of approximately 10 litres for the vacuum package housing the MOT coils / lattice arrangement. Differential pumping via two 8 litre s$^{-1}$ ion pumps would add another 7 litres.

The drawback of the Hg system is the requirement for deep UV cooling (254 nm) and a 265.6 nm clock transition. However, the clock laser radiation can be generated from a frequency-quadrupled high power narrow linewidth fibre laser. The clock laser wavelength is very close to the Al$^+$ ion clock wavelength, and would benefit from the same technology development. The 254 nm cooling wavelength is currently just outside the lower wavelength end of the commercial quadrupled Yb fibre laser, but has already been generated using a quadrupled solid-sate Yb:YAG thin disc laser providing several watts of fundamental light at 1014 nm [Petersen 2008]. In this case, two-stage doubling in resonant enhancement cavities has produced in excess of 150 mW cw single mode radiation at 254 nm. Alternatively, a 20 mW extended cavity diode laser at 1014 nm boosted to 1 W via a tapered amplifier, has produced 10 mW at 254 nm after two resonant enhancement doubling stages [Hachisu 2008]. Assuming a fundamental cooling laser package at 1016 nm no larger than the fundamental fibre laser package for the clock radiation, and equivalent resonant enhancement doubling cavities, this points to cooling /doubler and clock laser / doubler arrangements generating the intermediate green light with volumes of 8 litres and 13 litres respectively.

An additional wavelength close to 360 nm is required for the Hg lattice trap. Again, this can be generated from a fibre laser system at 1080 nm, doubled and then summed with the doubled output to give 360 nm. This has not yet been achieved, but the capability developed with quadrupled fibre lasers further into the UV should transfer across directly to this requirement. It is envisaged that enhancement cavities will be required for both the doubling and sum frequency stages in order to generate enough UV lattice power, but unlike the quadrupled systems, a more compact fundamental / doubler / sum generator package should be possible given the availability of UV transmitting single mode fibre at 360 nm to deliver the output to the lattice chamber. A complete volumetric footprint within 15 litres is considered feasible.

Similar to the logic ion clock, the cooler and clock doubled outputs in the green would need to be fibre fed to the atom trap / lattice chamber before the final doubling stage. These stages would add an extra volume of 6 – 10 litres to the trap package, giving a total of 23 – 27 litres. Support structure considerations remain the same as for the Sr lattice clock.

4.2.1.5 Optical local oscillators

The optical local oscillator technology development is targeted on a number of separate cavity design issues, including the achievement of minimal sensitivity to vibration levels expected on the spacecraft where the cavity is semi-rigidly fixed to the structure, minimal expansivity by appropriate choice of materials, design and temperature control, and reduced thermal noise limits by trade-off between cavity size, mirror and coating material and temperature, and mode volume. Recent experiments have demonstrated that vibration noise limits in the 1–10 Hz range can be an order of magnitude below thermal noise limits through the use of vibration insensitive cavity geometries supported under gravity via compliant materials [Webster 2008]. Levels of anticipated spacecraft vibration during orbit are at least a factor of ten larger than levels expected in good ground locations, so this is likely to increase the vibration noise to existing thermal noise limits, which has implications for striving for
reduced thermal noise arrangements. Ideally a local oscillator flicker floor reduced by a factor of ten to give $10^{-16}$ relative stability is desirable, but the anticipated on-board vibration levels may make it difficult to achieve this. One of the major targets for the local oscillator technology development will involve a programme to improve both thermal noise limits whilst maximising vibration insensitivity for semi-rigidly supported cavities within the spacecraft environment. This will require developments in support technology, mirror and coating material choices for reduced thermal noise, reduced thermal drift through close-to-zero expansivity cavity material and design choices, and possibly, pulsed tube cryo-cooled cavities.

The overall local oscillator package comprises the ultra-narrow clock laser stabilised to a very high finesse cavity, and this has already been discussed within the local oscillator context for each atomic reference option. Here, it is important to re-iterate that the atomic reference choices allow the local oscillator technology to be divided into only two development areas, namely one based on a low-power extended cavity red diode laser source for the Sr$^+$ ion and Sr neutral clocks, and the other based on narrow-linewidth fibre lasers frequency-quadrupled to the deep UV for both Al$^+$ and Hg. This offers an efficient development approach.

### 4.2.1.6 Optical frequency comb

The optical frequency comb is the sub-unit technology common to all space clock atomic reference options. It is arguably the most advanced sub-unit available at this time, having already been developed as a commercial instrument in modular form, with central processing capability and semi-automatic alignment and control functionality. This functionality, however, needs further development to provide for fully automatic operation. There are a number of key operating characteristics that need to be optimised to give efficient high-accuracy performance. Central to these is the extent of the comb bandwidth and choice of actual visible-to-IR spectral span. This will be designed to provide full overlap with the clock transition frequencies relevant to the atomic reference options under development. This thus includes the red frequencies relevant to the strontium ion and atom cases, and the fundamental frequencies at 1 µm for the Al$^+$ and Hg options.

At present, the most obvious choice for the comb unit is the Er-doped fibre laser comb. This system is organised round a typical 100 – 200 MHz repetition rate fibre laser and associated fibre amplifiers to provide for self referencing and two separate comb spans to cover the spectrum between 500 nm and 2 µm. Such a wide bandwidth is not required for this application. Instead, a single amplifier section feeding a microstructure fibre section to produce the comb span is desirable to limit the sub-unit size. Thus to avoid the need for an extra frequency doubling module to reach the visible wavelengths, this points to a fibre laser centred around the 1 µm region. For this, the Yb-doped fibre laser operating at ~ 1030 nm and pumped by industry-standard 980 nm high-power diodes looks to be a good choice. Such Yb-doped fibre lasers have already generated comb spans with 10 W average output power with ultra-narrow mHz-wide comb mode linewidths [Schibli 2008].

A potentially attractive alternative femtosecond laser source is the Yb:KYW system (section 3.4.3.1). This is a very compact solid-state source pumped by 980 nm diodes. It has an optical-optical conversion efficiency of approximately 50% and, when used together with a 1 m length of microstructure fibre, has produced comb bandwidths of 650 nm – 1450 nm [Meyer 2008].

The location of the clock reference wavelengths just below 700 nm and above 1 µm would also overlap well with the requirement to optimise the comb outputs at either end of the comb span for self referencing purposes. This would then point to a single channel/amplifier for
both self-referencing and clock mode production. In addition, the Yb-doped laser (either fibre or solid-state) is more efficient than the Er-doped system, and this could remove the need for an amplifier stage. Thus, one developmental activity could be a performance comparison between these Yb-doped systems, followed by integration with existing and improved comb opto-electronics (e.g. collinear $f - 2f$ beat generation) and control functions.

One critical development activity will centre on the issue of radiation hardness of the doped fibre or solid-state femtosecond host. There is already evidence of radiation damage to Er-doped fibres that is not fully recoverable. The Yb-doped hosts are considered to be less susceptible, and there may be additional advantage in the solid-state host over the doped-fibre case. Much clearer information is needed in respect of likely particle fluxes and resulting Yb-comb susceptibility and this will form an early integral part of the developmental plan, in order to determine whether additional shielding is needed over and above normal spacecraft requirements. This has a bearing on the comb sub-unit volume, and assuming a fibre-laser system with one amplifier stage and single microstructure broadening stage, this points to an 8-litre optics package, which could potentially be reduced to 6 litres in the case of a Yb-doped:KYW system.

Also critical to space comb operation over extended periods of several years is the issue of high brightness pump diode lifetime and performance within the space environment. This aspect is not only highly relevant to extended comb operation but also to the local oscillator source laser operation in the fibre laser cases (and to a lesser extent, for the low power diode cases). Here, experience from space integration companies will be invaluable, both in respect of standard semiconductor diode performance and laser diode operation. Finally, efficient generation of microwave synthesis from the comb to provide reference for MWL frequencies will form part of the activity plan.

4.2.1.7 Frequency comparison techniques

Various high-accuracy frequency comparison techniques have already been outlined in section 3.5. The primary function of this sub-system development is to provide the metrology infrastructure needed to compare space-borne clocks with ground clock systems at NMIs and other research locations. This is a necessary linkage to realise the space clock applications that address the user requirements itemised in section 2.

Improved microwave links based on the ACES MWL are proposed as one of the space-to-ground comparison methodologies. Here, a MWL link enhancement already suggested in the context of the EGE mission (section 3.5.1.2) comprises an additional frequency for improved ionospheric correction and the use of an on-board optical clock reference, together with carrier phase integration over extended periods or orbits and laser ranging, point to enhanced MWL performance capable of reaching $10^{-17}$ at 10 days averaging time. In order to validate this level of performance, MWL – optical fibre transfer time and frequency comparisons between NMIs would prove invaluable for underpinning MWL comparisons between ground clocks in the field (i.e. not located at institutes within a linked dark fibre network).

The time and frequency comparison requirements are discussed more fully in section 6. Here it is sufficient to outline the technology development necessary to achieve these requirements. The enhanced MWL proposals need further analysis, both through paper study and experimental testing of the concepts. Optical fibre transfer linkages also need to be established between a number of remote metrology institutes, and transfer stability and accuracy over long distances (~1000 km) through phase coherent repeater stations needs to be verified. Finally, the potential capability for ground to satellite optical time and frequency transfer needs to be studied, again drawing on in-flight demonstrations such as Jason 2.
4.2.2 Overall technology development guidelines

Guideline volume, mass and power budgets are given for the optical clock systems and their sub-components in Table 20.

<table>
<thead>
<tr>
<th>Optical clock</th>
<th>Sub-component</th>
<th>Volume (litres)</th>
<th>Mass (kg)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion clock ((^{88}\text{Sr}^+))</td>
<td>Trap package</td>
<td>7</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Cooling / auxiliary lasers (diode lasers)</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>LO (red diode @ 674 nm)</td>
<td>8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Fibre delivery / (\lambda) &amp; (\nu) diagnostics</td>
<td>7</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>32</strong></td>
<td><strong>45</strong></td>
<td><strong>55</strong></td>
</tr>
<tr>
<td>Lattice clock (Sr)</td>
<td>MOT / lattice chamber package</td>
<td>27–37</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Cooling / auxiliary lasers</td>
<td>22–37</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>LO (red diode @ 698 nm)</td>
<td>8</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Lattice laser @ 813 nm</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Fibre delivery / (\lambda) &amp; (\nu) diagnostics</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>74–99</strong></td>
<td><strong>107</strong></td>
<td><strong>95</strong></td>
</tr>
<tr>
<td>Quantum logic clock (Al(^+/\text{Mg}^+))</td>
<td>Trap package</td>
<td>13–20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Cooling / auxiliary lasers (fibre lasers)</td>
<td>8</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>LO (quadrupled fibre laser 267 nm)</td>
<td>18</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Fibre delivery / (\lambda) &amp; (\nu) diagnostics</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>46–53</strong></td>
<td><strong>77</strong></td>
<td><strong>105</strong></td>
</tr>
<tr>
<td>Lattice clock (Hg)</td>
<td>MOT / lattice chamber package</td>
<td>23–27</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Cooling / auxiliary lasers</td>
<td>8</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>LO (quadrupled fibre laser 266 nm)</td>
<td>13</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Lattice laser @ 360 nm</td>
<td>15</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Fibre delivery / (\lambda) &amp; (\nu) diagnostics</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>66–70</strong></td>
<td><strong>107</strong></td>
<td><strong>130</strong></td>
</tr>
<tr>
<td>Frequency comb</td>
<td>Yb-doped fs fibre laser</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yb-doped: KYW fs solid state laser</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Amplifier (if needed) / self referencing unit</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total budget</strong></td>
<td></td>
<td>6–8</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 20. Guideline volume, mass and power budgets for the optical atomic clock sub-units. Figures are for the physics package only; the support structure and electronics mass and volume are not included. Power dissipation assumes a factor of \(\sim 50\%\) efficiency.

4.3 Subsystem integration

The principal sub-components of an optical clock are shown in Figure 42, and are the clock laser, the atomic reference, the cooling and auxiliary lasers and the femtosecond comb, with the addition of an overall control unit.
The light from the clock laser will be delivered to the femtosecond comb and the atomic reference packages using fibre delivery systems. The self-referenced femtosecond comb will be stabilised to the clock laser and used to derive a microwave signal that is the output of the optical clock. The number of cooling and auxiliary lasers required, as well as the details of the atomic reference package, depends on whether a trapped ion optical clock or a neutral atom optical lattice clock is being considered.

For either technology, the optical clock will operate under automatic control, with the control unit providing the software algorithms necessary for cooling, repumping, interrogation of the clock transition, magnetic field switching and stabilisation, and fluorescence monitoring. The control unit will also provide error flags for monitoring of critical processes, and mechanisms for system recovery where necessary. Automatic monitoring of laser power and spectral quality, with signal re-optimisation or laser failure determination, will also be required.

Interfacing of the sub-units of the optical clock will need to be considered and specified at an early stage of the technology development programme in order to prepare properly for subsystem integration into advanced portable prototypes. Following subsystem integration and testing, the stability and accuracy of the advanced portable prototypes will then be assessed by comparison against high performance laboratory-based optical atomic clocks. It is therefore our view that the subsystem integration should take place at the European institutes that have already developed optical atomic clocks of high accuracy, namely NPL, PTB, LNE-SYRTE and the University of Innsbruck.
5 Engineering model design and planning

In this section we discuss how the transition from advanced portable prototype development to engineering model development would be achieved in practice. This is particularly important when one considers the relatively short timescale available to bring optical clock capability to the space-qualified level by 2020.

In the standard development route, shown in Figure 43 (a), the advanced prototype development would be followed sequentially by the development of an engineering model, a qualification model and finally a flight model. This would be unfeasible on the specified timescale. Our recommendation is thus to pursue an alternative “fast-track” approach, illustrated in Figure 43 (b), in which an engineering qualification model (EQM) is developed in parallel with a life test model and a structural and thermal model, before proceeding to a flight model. This could achieve space deployment of an optical atomic clock by 2020. It also has the advantage that the EQM would itself be suitable for a demonstration flight, should a suitable opportunity arise.

(a) Standard development route

(b) “Fast track” development route

Figure 43. Timescales for development of a space-borne optical clock, showing (a) the standard development route and (b) an alternative “fast track” approach.
At the end of the five-year advanced prototyping period, the preferred atomic reference sub-unit(s) will be selected to go forward to the EM / EQM development phase. This selection will be made by considering the technical performance demonstrated by each of the advanced portable prototypes in conjunction with detailed mission possibilities, including the outputs from the expected 2011 Cosmic Vision call.

Although we recommend that the advanced prototyping activities should primarily be carried out by the European metrology research community together with specialist space-oriented time and frequency companies, early involvement will be needed from one or two leading space integration companies. Their input would include system level design, support and engineering for space, together with thermal, structural, radiation and electronics design. Their objectives would include, in addition to the goal of viable advanced prototypes, the development of a concept model of the EQM instrument with attendant design definition documents. This would enable the subsequent EQM development programme to be costed and scheduled with accuracy, and permit a fast start to what is a very challenging goal of a flight model by 2020. It would be expected that a space integrator would take on the prime contractor role as development moved into this second EQM phase.

An approximate cost estimate for the EQM development, based on discussions with a leading space integrator, is €45–55 million for one of the simpler clocks, and €65–85 million for one of the more complex clocks.
6 Infrastructure preparation

In terms of infrastructure, the primary requirement is for methods of carrying out high accuracy frequency comparisons between remotely located optical atomic clocks. The different options for frequency comparison have been reviewed in section 3.5; here we discuss the extent to which existing European infrastructure is adequate to support the optical clock development programme and make recommendations for necessary enhancements. Both ground links (section 6.1.1) and ground to satellite links (section 6.1.2) are discussed. In addition, we consider whether there is a need for an integration centre in which different clock prototype subsystems can be assembled and tested (section 6.2).

6.1 Frequency comparison infrastructure

As already discussed, one of the most critical requirements arising from the development of optical atomic clocks with accuracy at the $10^{-17}$ to $10^{-18}$ level is the need for an improved frequency comparison infrastructure that does not compromise the accuracy of the clocks. The performance of the comparison methods used should ideally meet or exceed the performance of the optical clocks to avoid the need for long integration times.

6.1.1 Optical fibre requirements

For comparison of ground-based clocks in remote locations, the development of a pan-European optical fibre network will be vital in order to assess the performance of the advanced portable prototype clocks during the development programme as well as for testing the accuracy of enhanced ground to space frequency transfer links.

The establishment of a European optical frequency transfer network linking NMIs and other relevant institutions requires access to optically transparent fibre routes with points of access in the optical frequency standards laboratories of each institution. An optically transparent route is one that is free of intermediate electronic interfaces. It may, however, include EDFAs or other all-optical equipment.

Optical fibre links provide the core of commercial telecommunication networks. However, this core is generally surrounded by an interface layer of optoelectronic equipment and hence is not directly accessible. Moreover, data routing over the network is performed using sophisticated packet switching algorithms designed to optimise network usage, with the consequence that the physical path through the network taken by a particular data packet is unpredictable and not reproducible.

An alternative approach is to use dark fibre. Dark fibre is unused installed fibre – an otherwise unexploited asset of the telecommunications companies. A dark fibre network is a bespoke network composed of dark fibre leased from a telecommunications company and equipped by the end-user with transmitting, receiving, and intermediate equipment optimised for their particular application. Dark fibre networks provide the core of national research and education networks such as JANET in the UK and the Deutsches Forschungsnetz in Germany as well as the pan-European research network GÉANT2 (Figure 44). An advantage of international dark fibre networks such as GÉANT2 is that they provide cross-border fibres, i.e. optically transparent links across national boundaries, which may not be readily available through commercial telecommunications networks. However self-build may be required to bridge the gap between research institutes and the nearest point of access of the dark fibre network.
Figure 44. Pan-European research and education network GÉANT2 (http://www.geant2.net/).
6.1.1.1 Fibre specification

As injected optical power and fibre attenuation are key parameters for an optical frequency transmission link, it is desirable to specify fibre with the highest stimulated Brillouin scattering (SBS) threshold and the lowest attenuation at 1550 nm. With the exception of self-build links, it is unlikely that it will be possible to specify a specific fibre from a particular manufacturer. However, the ITU recommendations can be used, with the ITU-T G.652.D standard [ITU 2005] providing the most stringent specification. It should be noted, however that this standard does not specify SBS threshold.

The physical characteristics (at 1550 nm) laid out in ITU-T G.652.D are listed in Table 21, along with values for Corning SMR-28e+ [Corning 2006], an example of a fibre meeting this standard. Attenuation limits the power in the received signal and hence the signal-to-noise ratio. SBS is a nonlinear process whereby back-scattered, frequency-shifted light is generated from the injected light, and effectively limits the optical power that can be launched into the fibre, since excess attenuation is observed above the SBS threshold. Chromatic dispersion causes optical frequency fluctuations at the receiver to translate into optical delay fluctuations and hence additional phase noise at the receiver. Polarization mode dispersion (PMD), i.e. different propagation delays experienced by orthogonal polarisation states of light, is due to birefringence resulting from asymmetry of the fibre core. Fluctuating mechanical stress on the fibre due to vibration and temperature variation causes PMD to vary with time. Temperature variations will induce slow but significant phase drifts.

<table>
<thead>
<tr>
<th>Fibre characteristic</th>
<th>ITU-T G.652.D specification</th>
<th>Corning SMR-28e+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attenuation</td>
<td>&lt; 0.3 dB km⁻¹</td>
<td>0.2 dB km⁻¹</td>
</tr>
<tr>
<td>SBS threshold</td>
<td>Not specified</td>
<td>+13 dBm in 20 MHz optical bandwidth</td>
</tr>
<tr>
<td>Chromatic dispersion coefficient $D$</td>
<td>16.7 ps nm⁻¹ km⁻¹ $\leq D \leq 18.0$ ps nm⁻¹ km⁻¹</td>
<td>17 ps nm⁻¹ km⁻¹</td>
</tr>
<tr>
<td>Polarization mode dispersion</td>
<td>$&lt; 0.20$ ps km⁻¹/²</td>
<td>0.06 ps km⁻¹/²</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>Not specified</td>
<td>$7 \times 10^{-6}$ K⁻¹</td>
</tr>
</tbody>
</table>


The relative importance of external perturbations will of course depend on the environment in which the fibre is situated. Installed fibre links may be buried cables, e.g. alongside or within gas, electricity, or other utility ducting, or may be laid at surface level, e.g. in concrete troughs alongside railway lines. Equally, urban and rural environments will have different noise characteristics. Exchanges or cabinets where fibres are linked or additional equipment installed may present the most problematic environments, as here the fibre may be exposed to relatively rapid temperature cycling due to air conditioning systems as well as to sources of acoustic vibration such as cooling fans.

An extreme example is provided by NASA’s Deep Space Network at Goldstone in the Mojave desert, a location subject to diurnal temperature variations of up to 35 °C. The optical fibre links between the master oscillator and remote antennae are buried at a depth of 1.5 m, at which temperature fluctuations are attenuated by more than four orders of magnitude. However, the fibre passes through a number of steel-lidded vaults, where the temperature variations can be up to 50 °C. Although fibre in the vaults comprises only 0.25% of a total
16 km fibre length, compensation for the temperature-dependent phase delay requires a complex stabilisation system [Calhoun 2002].

6.1.1.2 Potential fibre routes

As a minimum, the four “core” laboratories with existing high-accuracy laboratory-based optical clocks (NPL, PTB, LNE-SYRTE and Innsbruck) should be linked by pairs of optical fibres. Although the availability of dark fibre is in principle not a problem, critical issues will be the identification of national dark fibre providers that will support the laboratories in establishing national link capabilities with suitable access points to a European fibre network, and the costs of renting dark fibre over the 5 – 10 year development programme.

A short dark fibre link is already in operation between PTB and the University of Hannover in Germany. Negotiations with a national fibre provider have recently secured access to an additional 900 km fibre link between PTB in Braunschweig and the Max Planck Institute for Quantum Optics (MPQ) in Garching (Figure 45), and the first experiments on this link are due to begin shortly. In order to compensate for the unavoidable fibre losses, it will be necessary to install erbium-doped fibre lasers at intermediate container stages along the length of the route, to which access can be gained for short periods of time. At larger distances it may also be necessary to implement intermediate transportable lasers as active noise filters in order to remove accumulated phase noise due to residual uncompensated fibre noise.

Figure 45. Routing of the German dark fibre link between PTB in Braunschweig and MPQ in Garching, showing the intermediate container stations where additional equipment such as erbium-doped fibre amplifiers can be installed. Also shown is the much shorter link between the University of Hannover and PTB. The location of laboratories developing optical atomic clocks are shown in green.
Elsewhere, only access to local test networks has so far been negotiated, for example the 43 km link between LNE-SYRTE and LPL in Paris [Amy-Klein 2004], and the JANET Aurora network (http://www.ja.net/services/aurora/), which links research groups at the universities of Cambridge, Essex and UCL within the UK. Further negotiations are therefore required with telecommunications providers and/or national research and education networks in order to secure access to the necessary dark fibre links.

One potential network configuration is shown in Figure 46. This is based on extending the existing German network to link the four “core” laboratories that currently operate high-accuracy optical clocks. The loop linking NPL, LNE-SYRTE and PTB (via Hannover and Düsseldorf) is important in order that the accuracy of the dark fibre frequency comparison technique can be verified by comparing alternative routes. This network configuration could then be expanded to other locations within Europe, for example as indicated for the Italian laboratories at INRIM and LENS where optical clocks are also under development.

![Figure 46. Potential dark fibre network configuration between European laboratories with existing high accuracy optical atomic clocks (marked in red) and with optical atomic clocks under development (marked in blue). The blue line shows the existing German dark fibre route to which access has already been negotiated, whilst the red line indicates how this could be extended to the other “core” laboratories at NPL, LNE-SYRTE and Innsbruck. Other potential future extensions are shown in yellow.](image)

6.1.2 Enhanced satellite frequency transfer techniques

The optical fibre networks discussed in the preceding section are expected to play a crucial role in assessing the performance of the advanced portable prototype clocks during the development programme. However for most applications of space-borne optical clocks, it will be necessary to develop improved methods for ground to satellite clock comparison that do not compromise the stability or accuracy of the high performance optical atomic clocks. Our recommendation is to pursue two parallel approaches.
The first approach is to develop improved microwave links based on an enhanced version of the microwave link that is currently being developed for the ACES mission. The proposed upgrades were discussed in section 3.5.1.2, and involve the addition of a bi-directional link operating in the Kα-band to reduce noise levels and determine higher-order ionospheric corrections, as well as the generation of the microwave carrier frequencies directly from the microwave output of the femtosecond optical frequency comb sub-unit of the optical clock. These proposals require further detailed analysis and “proof-of-concept” experimental tests.

The second approach is to explore all-optical techniques for ground to satellite frequency transfer, for example drawing on in-flight demonstrations such as the T2L2 experiment on board the Jason-2 ocean observation satellite. Such techniques have the potential to achieve even higher accuracy than microwave-based techniques, and provide a complementary method that is subject to different performance limitations and causes of instability.

6.2 Integration centre

One idea that has been raised in the past is that of a Technology Integration Centre as a focal point for optical atomic clock technology developments, where hardware outputs from the separate activities could be further developed in order to integrate them with other optical atomic clock sub-components. With a co-ordinated technology development programme such as the one proposed here, we do not consider that an integration centre is necessary during the first five-year phase of the work. Instead we recommend that the individual sub-systems of the optical clocks should be integrated into advanced portable prototypes at metrology institutes where they can readily be tested against existing high performance laboratory optical clocks. This is considered to represent a more efficient use of resources.
7 Manpower

7.1 Detailed development plan

The proposed development schedule for the first five-year phase of the technology development programme is shown in Table 22. The clock sub-systems and the frequency comb each include a design activity for low-mass high-stiffness support structures (e.g. space-qualifiable carbon-fibre-matrix platforms), which should be capable of being of being integrated into a common payload structure. They also include design activity to define a common processor control and electronics packaging architecture, facilitating full integration into engineering models during the second EM/FM phase of the technology development programme. Analysis of the thermal management requirements will also be carried out during the first five-year phase, in preparation for EM build and qualification. These activities will involve the large space integrator companies, so that good co-ordination and efficient time management is maintained between the two phases. However, it should be stressed that the prototypes will most likely be built on standard breadboards during phase 1, with the breadboard footprint corresponding to the final required size and shape, and the sub-units being linked by fibres and cables but not fully integrated into a final structure. Additionally, it is expected that each clock system will use its own processor and servo-electronics package, since the integrated versions will not become fully available until the second phase of the programme. It should be recognised that the added complexity of the Al$^+$ quantum logic clock and the Hg lattice clock mean that they will not have reached such a high technology readiness level (TRL) than the Sr$^+$ ion clock and the Sr lattice clock by the end of the first five-year development phase. It is considered that the Sr$^+$ / Sr clock options should reach TRL 5/6 by the end of the 5-year phase, whereas the Al$^+$ / Hg options will target TRL 4.

<table>
<thead>
<tr>
<th>Optical clock sub-system</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ion clock ($^{88}$Sr$^+$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of miniature trap package</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of cooling /auxiliary diode lasers</td>
<td></td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of LO (red diode @ 674 nm)</td>
<td></td>
<td></td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre delivery and $\lambda$ &amp; $\nu$ diagnostics package</td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integrated electronics/control package design</td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td></td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lattice clock (Sr)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of MOT / lattice chamber</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of cooling /auxiliary diode lasers</td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Development of LO (red diode @ 698 nm)</td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Development of lattice laser @ 813 nm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Optical clock sub-system</td>
<td>2009</td>
<td>2010</td>
<td>2011</td>
<td>2012</td>
<td>2013</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td><strong>Local oscillator</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cavity designs for low vibration, drift &amp; thermal noise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cavities build and test with existing red diodes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incorporation into clock option with red LOs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incorporation into clock options with UV LOs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Frequency comb</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study of Yb-doped fibre vs solid-state fs laser</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selection and build of Yb-doped fs laser</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Build of fs comb (self referencing / amplifier)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study of radiation hardness properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High power pump diode lifetime tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integratable support structure design &amp; build</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integrated electronics/control package design</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Frequency comparison</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introduction of enhanced MWL performance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre link repeater provenance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre link demo between research institutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground-satellite optical link development</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Quantum logic clock (Al&quot;)/Mg&quot;)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of miniature trap package</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of cooling /auxiliary fibre lasers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develop LO (×4 fibre laser      UV 267 nm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development and test of quantum logic algorithms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integrated electronics/control package design</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lattice clock (Hg)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of MOT / lattice chamber</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of cooling /auxiliary fibre lasers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develop LO (fibre laser ×4        UV 266 nm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of lattice laser @ 360 nm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integrated electronics/control package design</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 22. Five-year multi-track development schedule for optical clock advanced prototypes.
7.2 Manpower

The delegation of sub-unit tasks to research institutes and companies within the advanced prototype development (2009 – 2013) has been made by reference to existing capabilities of the various teams. In particular, the capabilities to provide the sub-unit components in an efficient and synergistic way have been considered. This can be seen in part in the correspondence of effort between the Sr and Sr systems, and between the quantum logic and Hg clocks. For the integratable support structure, it is envisaged that platform and payload space integrator companies would provide a co-ordinated input here, interacting both with the appropriate teams in each clock sub-system, and with each other. Electronics and control design in advance of the EM builds would also involve a co-ordinated approach on the same basis. Thermal management models are more likely to be specific to individual space companies close to the major sub-system team.

<table>
<thead>
<tr>
<th>Optical clock system development</th>
<th>Responsible Institute</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Management &amp; scientific co-ordination</strong></td>
<td>NPL</td>
</tr>
<tr>
<td><strong>Ion clock (⁶⁸Sr⁺)</strong></td>
<td></td>
</tr>
<tr>
<td>Development of miniature trap package</td>
<td>PTB</td>
</tr>
<tr>
<td>Development of cooling/auxiliary diode lasers</td>
<td>NPL</td>
</tr>
<tr>
<td>Development of LO (red diode @ 674 nm)</td>
<td>NPL</td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td>PTB, NPL</td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td>NPL, PTB, UK space integrator co.</td>
</tr>
<tr>
<td>Integratable control/control package design</td>
<td>PTB, NPL, UK space integrator co.</td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>UK space integrator co.</td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td>NPL, PTB</td>
</tr>
<tr>
<td><strong>Lattice clock (Sr)</strong></td>
<td></td>
</tr>
<tr>
<td>Development of MOT / lattice chamber</td>
<td>LNE-SYRTE, LENS</td>
</tr>
<tr>
<td>Development of cooling/auxiliary diode lasers</td>
<td>LENS, Düsseldorf</td>
</tr>
<tr>
<td>Development of LO (red diode @ 698 nm)</td>
<td>PTB</td>
</tr>
<tr>
<td>Development of lattice laser @ 813 nm</td>
<td>LNE-SYRTE</td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td>LENS, Düsseldorf</td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td>LNE-SYRTE, LENS, French/Italian space integrator co.</td>
</tr>
<tr>
<td>Integratable electronics/control package design</td>
<td>LNE-SYRTE, French/Italian space integrator co.</td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>French/Italian space integrator co.</td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td>LNE-SYRTE, LENS</td>
</tr>
<tr>
<td><strong>Local oscillator</strong></td>
<td></td>
</tr>
<tr>
<td>Cavity designs for low vibration, drift &amp; thermal noise</td>
<td>NPL, PTB</td>
</tr>
<tr>
<td>Cavities build and test with existing red diodes</td>
<td>PTB, NPL</td>
</tr>
<tr>
<td>Incorporation into clock option with red LOs</td>
<td>NPL, PTB (see Sr⁺ and Sr clocks)</td>
</tr>
<tr>
<td>Incorporation into clock options with UV LOs</td>
<td>NPL, Duesseldorf</td>
</tr>
<tr>
<td><strong>Optical clock system development</strong></td>
<td><strong>Responsible Institute</strong></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Frequency comb</strong></td>
<td></td>
</tr>
<tr>
<td>Study of Yb-doped fibre vs solid-state fs laser</td>
<td>Menlo Systems</td>
</tr>
<tr>
<td>Selection and build of Yb-doped fs laser</td>
<td>Menlo Systems</td>
</tr>
<tr>
<td>Build of fs comb (self referencing /amplifier)</td>
<td>Menlo Systems</td>
</tr>
<tr>
<td>Study of radiation hardness properties</td>
<td>Menlo Systems, German space integrator co.</td>
</tr>
<tr>
<td>High power pump diode lifetime tests</td>
<td>German space integrator co.</td>
</tr>
<tr>
<td>Integratable support structure design &amp; build</td>
<td>German space integrator co., Menlo Systems</td>
</tr>
<tr>
<td>Integratable electronics/control package design</td>
<td>Menlo Systems, German space integrator co.</td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>German space integrator co.</td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td>Menlo Systems, PTB</td>
</tr>
<tr>
<td><strong>Frequency comparison</strong></td>
<td></td>
</tr>
<tr>
<td>Introduction of enhanced MWL performance</td>
<td>Timetech</td>
</tr>
<tr>
<td>Fibre link repeater provenance</td>
<td>PTB, LNE-SYRTE, NPL</td>
</tr>
<tr>
<td>Fibre link demo between research institutes?</td>
<td>PTB, LNE-SYRTE, NPL, Innsbruck</td>
</tr>
<tr>
<td>Ground-satellite optical link development</td>
<td>LNE-SYRTE, Cote d’Azur, Timetech</td>
</tr>
<tr>
<td><strong>Quantum logic clock (Al⁺/Mg⁺)</strong></td>
<td></td>
</tr>
<tr>
<td>Development of miniature trap package</td>
<td>Innsbruck</td>
</tr>
<tr>
<td>Development of cooling /auxiliary fibre lasers</td>
<td>Innsbruck</td>
</tr>
<tr>
<td>Develop LO (×4 fibre laser    UV 267 nm)</td>
<td>NPL, Duesseldorf</td>
</tr>
<tr>
<td>Development and test of quantum logic algorithms</td>
<td>Innsbruck</td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td>Innsbruck or NPL</td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td>Innsbruck, Austrian + UK space integrator companies</td>
</tr>
<tr>
<td>Integratable electronics/control package design</td>
<td>Austrian space co.</td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>UK space integrator co.</td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td>Innsbruck, PTB</td>
</tr>
<tr>
<td><strong>Lattice clock (Hg)</strong></td>
<td></td>
</tr>
<tr>
<td>Development of MOT / lattice chamber</td>
<td>LNE-SYRTE</td>
</tr>
<tr>
<td>Development of diode/solid state cooling laser</td>
<td>LNE-SYRTE</td>
</tr>
<tr>
<td>Develop LO (×4 fibre laser    UV 266 nm)</td>
<td>NPL, Duesseldorf</td>
</tr>
<tr>
<td>Development of lattice laser @ 360 nm</td>
<td>LNE-SYRTE</td>
</tr>
<tr>
<td>Fibre delivery and λ &amp; ν diagnostics package</td>
<td>LNE-SYRTE</td>
</tr>
<tr>
<td>Integratable support structure design</td>
<td>LNE-SYRTE, French/Italian space integrator co.</td>
</tr>
<tr>
<td>Integratable electronics/control package design</td>
<td>LNE-SYRTE, French/Italian space integrator co.</td>
</tr>
<tr>
<td>Development of thermal management model</td>
<td>French/Italian space integrator co.</td>
</tr>
<tr>
<td>Sub-system integration &amp; test against lab standard</td>
<td>LNE-SYRTE</td>
</tr>
</tbody>
</table>

Table 23. Delegation of responsibilities between metrology and research institutes and companies, and large space integrator companies.
In drawing up this responsibility table, consideration has been given to the respective capabilities of the research partners (both research institutes and specialist time and frequency companies such as Menlo Systems and Timetech) in order to reach a coherent development plan which makes best use of these capabilities. In addition, consultation with the partners has been undertaken in order to establish their broad commitment to this plan. In addition, early discussions with some of the major European space integrator companies have been undertaken in order to take advantage of pre-existing space integration experience, which is critical to the development plan in order to avoid re-invention of generic space technology, or worse, unfeasible development routes that could critically hinder the rapid transfer of advanced prototype capability to the EM design and build. Thus, it is considered that the space platform and payload integrator companies would provide design and consultancy within the first 5-year phase of the programme, particularly in the areas of thermal management, electronics and integrated control design, and instrument integrated support structure and materials choice. Possible space integrator companies are given below, but the list is not exclusive and no order of importance is implied.

UK: EADS Astrium UK
Germany: Kayser Threde, EADS Astrium Germany
France / Italy: Thales Alenia, Sodern
Austria: Schrack Aerospace
Switzerland: RUAG
8 Summary of recommendations

With the overall objective of supporting future missions in the areas of Science, Earth Observation and Navigation, the technology development programme presented in this document aims to achieve space deployment of an optical atomic clock by 2020.

A multi-stranded development programme is proposed, with the aim of enabling the most viable technical solution for a specific mission scenario to be selected, based on actual performance. This approach will satisfy the recommendations regarding technological readiness for Cosmic Vision proposals in the shorter term, whilst assuring development to the highest possible specification in the longer term.

Our recommendation is that advanced portable prototypes of four different optical atomic clocks should be developed in parallel:

1. A trapped ion optical clock based on $^{88}\text{Sr}^+$;
2. A strontium atom optical lattice clock;
3. A quantum-logic-based trapped ion optical clock using $^{27}\text{Al}^+$;
4. A mercury atom optical lattice clock.

This multiple-clock development approach will allow for the highest performance specifications to be achieved during the prototyping phase, whilst at the same time undertaking the engineering developments necessary to prepare the clocks for space integration. It will lead to a number of different clock options with individual advantages of compactness, stability or accuracy, from which the best option for any particular mission scenario can be selected with a high degree of confidence and minimum technological risk. Some elements of the clock are common to all four systems, in particular the optical frequency comb and many aspects of the optical local oscillator.

In parallel, it will be necessary to develop an improved frequency comparison infrastructure that does not compromise the stability or accuracy of the high performance optical atomic clocks. Both ground links and ground to satellite links must be addressed.

On the ground, the development of a pan-European optical dark fibre network will be vital in order to assess the performance of the advanced portable prototype clocks during the development programme, as well as for verifying the performance of enhanced ground-space frequency transfer links. For the ground to satellite links, our recommendation is to pursue two parallel approaches. The first would be improved microwave links based on an enhanced version of the microwave link that is currently being developed for the ACES (Atomic Clock Ensemble in Space) mission. The second is to explore all-optical techniques. These have the potential to achieve even higher accuracy, and provide a complementary method that is subject to different performance limitations and causes of instability.

The proposed programme is a partnership between ESA member states, including the UK, Germany, France, Austria, Italy and Switzerland. Europe has several world-leading groups in optical frequency metrology, and our recommendation is that, together with specialised space-oriented time and frequency companies, they should take the lead in the advanced prototyping phase. However the need for early inputs on the space integration side will require consultancy and outline platform design from leading space integration companies as a
precursor to the engineering model development phase. Our recommendation is that one of these space integrators should then take on the prime contractor role for this second phase.

Within these terms of reference, the technology development necessary for achieving complete optical clock sub-unit technologies at the advanced prototype level can be broken down into packages to be pursued by different metrology institutes and companies. However strong links between the prime contractor and the groups leading the development of each primary sub-unit will be vital to ensure that proper coordination is maintained across the breadth of the development plan. The sub-units should be integrated into complete optical clock prototypes at one of the metrology institutes, where they can readily be tested against existing high performance laboratory optical clocks.

With suitable investment, Europe will be able to accelerate the development of high performance optical clocks for applications ranging from tests of fundamental physical theories to geodesy and satellite-based navigation systems, and focus efforts on achieving the goal of a space-borne optical clock by 2020.
9 References


[Oskay 2005] W. H. Oskay, W. M. Itano and J. C. Bergquist, “Measurement of the $^{199}\text{Hg}^* \ ^{5}d_{\frac{9}{2}} ^{6}s_{\frac{2}{2}} ^{2}D_{\frac{5}{2}}$ electric quadrupole moment and a constraint on the quadrupole shift”, Phys. Rev. Lett. 94, 163001 (2005).


### Annex A  Abbreviations and acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACES</td>
<td>Atomic Clock Ensemble in Space</td>
</tr>
<tr>
<td>A-D</td>
<td>analogue-digital</td>
</tr>
<tr>
<td>ADEV</td>
<td>Allan deviation</td>
</tr>
<tr>
<td>AHM</td>
<td>active hydrogen maser</td>
</tr>
<tr>
<td>AM</td>
<td>amplitude modulation, amplitude modulated</td>
</tr>
<tr>
<td>AOM</td>
<td>acousto-optic modulator</td>
</tr>
<tr>
<td>APD</td>
<td>avalanche photodiode</td>
</tr>
<tr>
<td>A-SCOPE</td>
<td>Advanced Space Carbon and Climate Observation of Planet Earth</td>
</tr>
<tr>
<td>ASTROD</td>
<td>Astrodynamical Space Test of Relativity using Optical Devices</td>
</tr>
<tr>
<td>AU</td>
<td>astronomical unit</td>
</tr>
<tr>
<td>BEC</td>
<td>Bose-Einstein condensate</td>
</tr>
<tr>
<td>BIPM</td>
<td>Bureau International des Poids et Mesures</td>
</tr>
<tr>
<td>CHAMP</td>
<td>Challenging Mini-satellite Payload for Geophysical Research and Application</td>
</tr>
<tr>
<td>CIPM</td>
<td>International Committee for Weights and Measures</td>
</tr>
<tr>
<td>CP</td>
<td>Carrier phase</td>
</tr>
<tr>
<td>cw</td>
<td>continuous wave (laser)</td>
</tr>
<tr>
<td>D-A</td>
<td>digital-analogue</td>
</tr>
<tr>
<td>DCF</td>
<td>dispersion compensation fibre</td>
</tr>
<tr>
<td>DFB</td>
<td>distributed feedback</td>
</tr>
<tr>
<td>DSF</td>
<td>dispersion shifted fibre</td>
</tr>
<tr>
<td>DSN</td>
<td>Deep Space Network</td>
</tr>
<tr>
<td>EDFA</td>
<td>erbium-doped fibre amplifier</td>
</tr>
<tr>
<td>EEP</td>
<td>Einstein Equivalence Principle</td>
</tr>
<tr>
<td>EGE</td>
<td>Einstein Gravity Explorer</td>
</tr>
<tr>
<td>EIT</td>
<td>electromagnetically-induced transparency</td>
</tr>
<tr>
<td>EM</td>
<td>engineering model</td>
</tr>
<tr>
<td>Envisat</td>
<td>Environmental satellite</td>
</tr>
<tr>
<td>EOM</td>
<td>electro-optic modulator</td>
</tr>
<tr>
<td>EQM</td>
<td>engineering qualification model</td>
</tr>
<tr>
<td>ERS</td>
<td>European Remote Sensing</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>EU</td>
<td>European Union</td>
</tr>
<tr>
<td>FM</td>
<td>flight model</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
<tr>
<td>Acronym</td>
<td>Full Form</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
</tr>
<tr>
<td>GEO</td>
<td>geostationary orbit</td>
</tr>
<tr>
<td>GEOSAT</td>
<td>geodetic satellite</td>
</tr>
<tr>
<td>GLONASS</td>
<td>GLObalnaya Navigatsionnaya Sputnikovaya Sistema</td>
</tr>
<tr>
<td>GNSS</td>
<td>global navigation satellite system</td>
</tr>
<tr>
<td>GOCE</td>
<td>Gravity field and steady state Ocean Circulation Explorer</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GR</td>
<td>general relativity</td>
</tr>
<tr>
<td>GRACE</td>
<td>Gravity Recovery And Climate Experiment</td>
</tr>
<tr>
<td>GST</td>
<td>Galileo system time</td>
</tr>
<tr>
<td>GSTP</td>
<td>General Support Technology Programme</td>
</tr>
<tr>
<td>HNLF</td>
<td>highly nonlinear fibre</td>
</tr>
<tr>
<td>IGS</td>
<td>International GNSS Service</td>
</tr>
<tr>
<td>ILRS</td>
<td>International Laser Ranging Service</td>
</tr>
<tr>
<td>INRIM</td>
<td>L’Istituto Nazionale di Ricerca Metrologica</td>
</tr>
<tr>
<td>IR</td>
<td>infra-red</td>
</tr>
<tr>
<td>ISS</td>
<td>International Space Station</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
</tr>
<tr>
<td>JILA</td>
<td>Joint Institute for Laboratory Astrophysics</td>
</tr>
<tr>
<td>KRISS</td>
<td>Korea Research Institute of Standards and Science</td>
</tr>
<tr>
<td>KTP</td>
<td>potassium titanyl phosphate</td>
</tr>
<tr>
<td>LASSO</td>
<td>Laser synchronisation from stationary orbit</td>
</tr>
<tr>
<td>LENS</td>
<td>European Laboratory for Non-linear Spectroscopy</td>
</tr>
<tr>
<td>LEO</td>
<td>low Earth orbit</td>
</tr>
<tr>
<td>LIDAR</td>
<td>laser induced differential absorption radar</td>
</tr>
<tr>
<td>LISA</td>
<td>Laser Interferometer in Space Antenna</td>
</tr>
<tr>
<td>LKB</td>
<td>Laboratoire Kastler-Brossel</td>
</tr>
<tr>
<td>LLI</td>
<td>local Lorentz invariance</td>
</tr>
<tr>
<td>LNE-SYRTE</td>
<td>Laboratoire national de métrologie et d'essais - Système de Références Temps-Espace</td>
</tr>
<tr>
<td>LO</td>
<td>local oscillator</td>
</tr>
<tr>
<td>LPI</td>
<td>local position invariance</td>
</tr>
<tr>
<td>LPL</td>
<td>Laboratoire de Physique des Lasers</td>
</tr>
<tr>
<td>LTT</td>
<td>Laser Time Transfer</td>
</tr>
<tr>
<td>MEO</td>
<td>medium Earth orbit</td>
</tr>
<tr>
<td>MOPA</td>
<td>master oscillator power amplifier</td>
</tr>
<tr>
<td>MORE</td>
<td>Mercury Orbiter Radioscience Experiment</td>
</tr>
<tr>
<td>MOT</td>
<td>magneto-optical trap</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>MPQ</td>
<td>Max Planck Institute for Quantum Optics</td>
</tr>
<tr>
<td>MWL</td>
<td>Microwave link</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NICT</td>
<td>National Institute of Information and Communications Technology</td>
</tr>
<tr>
<td>NIST</td>
<td>National Institute of Standards and Technology</td>
</tr>
<tr>
<td>NMI</td>
<td>National Metrology Institute</td>
</tr>
<tr>
<td>NMIJ/AIST</td>
<td>National Metrology Institute of Japan/Advanced Industrial Science and Technology</td>
</tr>
<tr>
<td>NPL</td>
<td>National Physical Laboratory</td>
</tr>
<tr>
<td>NRC</td>
<td>National Research Council (of Canada)</td>
</tr>
<tr>
<td>OEM</td>
<td>original equipment manufacturer</td>
</tr>
<tr>
<td>OICETS</td>
<td>Optical Inter-orbit Communications Engineering Test Satellite</td>
</tr>
<tr>
<td>OISL</td>
<td>Optical Inter-Satellite Link</td>
</tr>
<tr>
<td>PBS</td>
<td>polarizing beamsplitter</td>
</tr>
<tr>
<td>PD</td>
<td>photodiode</td>
</tr>
<tr>
<td>PHARAO</td>
<td>Project d’Horloge Atomique par Refroidissement d’Atomes en Orbite</td>
</tr>
<tr>
<td>PLL</td>
<td>phase-locked loop</td>
</tr>
<tr>
<td>PMD</td>
<td>polarisation mode dispersion</td>
</tr>
<tr>
<td>PMT</td>
<td>photomultiplier tube</td>
</tr>
<tr>
<td>PPN</td>
<td>parameterized post-Newtonian</td>
</tr>
<tr>
<td>PPS</td>
<td>pulse per second</td>
</tr>
<tr>
<td>PRARE</td>
<td>Precise Range And Range-rate Equipment</td>
</tr>
<tr>
<td>PRN</td>
<td>pseudo-random noise</td>
</tr>
<tr>
<td>PTB</td>
<td>Physikalisch-Technische Bundesanstalt</td>
</tr>
<tr>
<td>PTF</td>
<td>Precise Timing Facility</td>
</tr>
<tr>
<td>RAFS</td>
<td>rubidium atomic frequency standard</td>
</tr>
<tr>
<td>rf</td>
<td>radio frequency</td>
</tr>
<tr>
<td>SAGAS</td>
<td>Search for Anomalous Gravity with Atomic Sensors</td>
</tr>
<tr>
<td>SBS</td>
<td>stimulated Brillouin scattering</td>
</tr>
<tr>
<td>SESAM</td>
<td>semiconductor saturable absorber</td>
</tr>
<tr>
<td>SFG</td>
<td>sum frequency generation</td>
</tr>
<tr>
<td>SHM</td>
<td>space hydrogen maser</td>
</tr>
<tr>
<td>SI</td>
<td>Système International</td>
</tr>
<tr>
<td>SLR</td>
<td>satellite laser ranging</td>
</tr>
<tr>
<td>SMF</td>
<td>single mode fibre</td>
</tr>
<tr>
<td>SPHM</td>
<td>space passive hydrogen maser</td>
</tr>
<tr>
<td>T2L2</td>
<td>time transfer by laser link</td>
</tr>
</tbody>
</table>
TDEV  time deviation, or time Allan deviation
TDRSS  tracking and data relay satellite system
TRL   technology readiness level
TWSTFT  two-way satellite time and frequency transfer
UCL  University College London
UGR  universality of the gravitational redshift
UHV  ultra-high vacuum
UK  United Kingdom
ULE  ultra-low expansion
USA  United States of America
USNO  United States Naval Observatory
UTC  Coordinated Universal Time
UV  ultra-violet
WALES  Water vapour lidar experiment in space
WEP  Weak Equivalence Principle