Effective Area of Optical Fibres - Definition and Measurement Techniques

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1. Introduction

Optical fibres in telecommunication systems now carry more channels and higher optical powers than ever before. Systems are operating in which the fibre carries such a high optical power density that signals can modify the transmission properties of the fibre. Optical channels can then affect how they and other channels propagate through the fibre - leading to nonlinear effects. By the term nonlinear, we mean that the optical signal leaving the fibre at a given wavelength no longer increases linearly with the input power at that wavelength. Nonlinearity in fibres essentially appears as the conversion of power from one wavelength to another. Examples of nonlinearities include: self-phase modulation (SPM), cross-phase modulation (XPM), four-wave mixing (FWM), stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS).

Nonlinear effects occur more efficiently in optical fibres than in bulk samples of their constituent materials because the optical field is confined to the small fibre core area over long distances. The confinement of the optical field within the core is achieved by the refractive index profile, which determines the field distribution of the fundamental mode. In general, optical power density is given by the optical power divided by the area over which it is distributed. The field of the fundamental mode of a single mode fibre bears little resemblance to the refractive index profile and therefore the area of the doped core region itself does not truly represent the area of the mode field. The effective area, $A_{eff}$, of the mode must be calculated from the field distribution but since this falls gradually to zero away from the fibre axis, some operational definition of effective area is required. A similar problem has been addressed before with respect to defining the mode field diameter (MFD) of single mode fibres. The MFD is used for calculating power losses due to fibre bending and offset splices and definitions have been derived based on the mode field distribution measured using a number of techniques [1]. The same techniques can be used to measure the mode field distribution of a single mode fibre and can therefore be used to calculate the effective area.

This report presents the accepted definition of effective area and describes the relationship of this parameter to the familiar mode field diameter. Four techniques are then described that have been used at NPL to measure the effective areas of real fibres. These techniques are:

- Direct Far-Field (DFF)
- Near-Field Scanning
- Variable Aperture in the Far Field (VAFF)
- Transverse Offset
2. Definition of Effective Area, \( A_{\text{eff}} \)

All nonlinear effects are dependent upon the intensity of the electromagnetic field in the medium. However, it is the total optical power entering and leaving the fibre that is usually measured. Some method is required for converting between the two when comparing theoretical and experimental results. The measured optical power leaving a fibre is simply the integral of the intensity distribution over the entire fibre cross section. For a uniform intensity distribution, \( I \), over a core of area \( A_{\text{core}} \), the intensity could be calculated from the measured power, \( P_{\text{meas}} \), using:

\[
I = \frac{P_{\text{meas}}}{A_{\text{core}}} \tag{2-1}
\]

However, the field in a single mode fibre is not evenly distributed or even fully contained within the core. It is larger at the fibre axis than near the core-cladding interface and extends into the cladding to a degree depending on the actual refractive index profile. Calculating a uniform intensity in the core using equation 2-1 will underestimate the value on the axis of the fibre and overestimate the value near the core-cladding interface.

The effective area parameter has been defined for the purposes of calculating nonlinear effects [2]. It is a single value, based on the modal field distribution, and can be used in equation 2-1 instead of \( A_{\text{core}} \) to calculate a value for the optical intensity. The effective area is defined as:

\[
A_{\text{eff}} = \frac{2\pi \left( \int_0^\infty E_n(r)^2 r dr \right)^2}{\int_0^\infty |E_n(r)|^4 r dr} = \frac{2\pi \left( \int_0^\infty I(r) r dr \right)^2}{\int_0^\infty I^2(r) r dr} \tag{2-2}
\]

where \( E_n(r) \) is the amplitude and \( I(r) \) is the intensity of the near-field of the fundamental mode at radius \( r \) from the axis of the fibre (see section 3).

2.1 Relationship of \( A_{\text{eff}} \) to MFD - “The Namihira Relation”

In conventional step-index fibres, the mode field is well-approximated by a Gaussian function of radius \( w \) at the 1/e amplitude points. In this case, the effective area can be shown simply to be:

\[
A_{\text{eff}} = \pi w^2(\lambda) \tag{2-3}
\]

where \( 2w(\lambda) \) is the Petermann-II mode field diameter (MFD) of the fibre at wavelength \( \lambda \). The mode field diameter is defined in the near-field by [1]:

\[ 
\]
\[ MFD_{NF} = 2\sqrt{2} \left\{ \frac{\int_0^\infty |E_a(r)|^2 r dr}{\int_0^\infty [E_a'(r)]^2 r dr} \right\}^{\frac{1}{2}}, \]  
\[ MFD_{FF} = 2\sqrt{2} \left\{ \frac{\int_0^\infty |F(p)|^2 p dp}{\int_0^\infty |F(p)|^2 p^3 dp} \right\}^{\frac{1}{2}}. \]  

2-4 

and in the far-field by:

\[ MFD_{FF} = 2\sqrt{2} \left\{ \frac{\int_0^\infty |F(p)|^2 p dp}{\int_0^\infty |F(p)|^2 p^3 dp} \right\}^{\frac{1}{2}}. \]  
\[ \text{2-5} \]

where \( |F(p)|^2 \) is the angular intensity distribution of the field radiated from the fibre endface and \( p = k \sin(\theta) \). The wavenumber of the radiation, \( k \), is given by the usual relation \( k = 2\pi/\lambda \).

For fibres that do not share the simple step-index geometry of conventional single mode fibre, such as dispersion-shifted and dispersion-flattened fibres, the mode field cannot be approximated by a Gaussian function. The simple relationship between \( MFD \) and \( A_{\text{eff}} \) given by equation 2-3 no longer holds and alternative methods are required to calculate the effective area. An example of this is shown in Figure 1, in which theoretical Gaussian and non-Gaussian field distributions are shown that have the same MFD of 8.00-\( \mu \)m using equation 0-4. However, the effective area of the Gaussian distribution is calculated using equation 2-2 to be 50.3-\( \mu \)m\(^2\) - in agreement with equation 2-3 - whereas that of the non-Gaussian is 52.8-\( \mu \)m\(^2\). It can be seen from these results that apparently small variations in the mode field distribution can lead to a significant difference in effective area.
It has been proposed [3,4,5] that the effective area of various types of non-standard fibres can be calculated using equation 2-3 but including a correction factor, $k_{Nam}$, that depends on wavelength and the type of fibre. This has lead to the so-called “Namihira Relation”:

$$A_{eff} = k_{Nam} w^2(\lambda).$$  \hspace{1cm} 2-6

For the example shown in Figure 1, the Namihira correction factor was calculated to be 1.05. Published values from experimental data have suggested that the correction factor for dispersion-shifted fibres is approximately 0.95 and is only slightly dependent on the actual refractive index profile of the fibre. Similar measurements performed on large effective area fibres gave values of $k_{Nam}$ in the range 1.03 to 1.17 and showed strong variations from one fibre to another [4]. The correction factor is dependent on wavelength and must be characterised as such for each fibre.

The advantage of the Namihira Relation is that mode field diameter is a parameter that is routinely specified for all new fibres and which can be determined accurately by a number of well-established techniques. Most of these techniques do not actually involve calculating the mode field distribution, $I(r)$, as relationships such as equation 2-5 have been derived that define the MFD in terms of other measured parameters. A large amount of test equipment is already installed in the marketplace that automatically measures MFD. Therefore, it would be useful to be able to calculate general correction factors to be applied to the MFD in order to calculate $A_{eff}$ without
the need to invest in new equipment to measure the mode field distribution
directly.

The disadvantage of using the correction factor $k_{Nam}$ is its dependence on the
fibre type. Initial investigations by Namihira using 12 different dispersion-
shifted fibres (ITU-T G.653) at wavelengths between 1540-nm and 1574-nm
showed that the average value of $k_{Nam}$ to be 0.944 with a relatively small
standard deviation of 0.002 [3]. The correction factor was determined
experimentally by using the variable aperture technique to calculate the
MFD $2w$ and the far-field intensity distribution $F(p)$. The far-field was then
transformed to the near-field, $I(r)$, using the inverse Hankel transform. The
near-field intensity distribution was then used to calculated the effective
area, $A_{eff}$, which was then compared with $\pi w^2$. The correction factor for
dispersion-shifted fibres was found to be relatively insensitive to the actual
refractive index profile of the fibre. However, further investigations using
fibres with large effective areas, non-zero dispersion shifted fibres (ITU-T
G.655) and cut-off shifted fibres (ITU-T G.654) found $k_{Nam}$ to be strongly
dependent on the actual refractive index profile [4,5,6]. Therefore the
correction factor should be calculated for each type of G.655, G.654 or large
effective area fibre. Therefore, with the apparent exception of dispersion-
shifted fibres, the same number of measurements are required to give the
correction factor as are needed to find the effective area.

3. The Direct Far-Field (DFF) Technique

3.1 Theory

When the radiation propagating in a single mode fibre reaches a cleaved
endface, it radiates from the fibre into the surrounding medium, which is
usually air. The core of single mode fibres is typically 5 to 10-µm in diameter
and is consequently comparable in size to the wavelength of the radiation -
typically 1.3-µm to 1.65-µm. Under these circumstances, the light diffracts as
it leaves the fibre and the resulting electromagnetic field evolves from a near-
field distribution close to the fibre endface into a far-field distribution further
away. The near-field being usually defined as the region within $w^2/\lambda$ of the
fibre endface and the far-field applies at distances much greater than this.

The amplitude of the radiation pattern in the far-field is related to that in the
near-field by the diffraction integral [1,7]:

$$
\psi(R, p) = O(\theta) \frac{k}{iR} \exp(ikR) \int_0^\infty E_a(r)J_0(rp)rdr
$$
Co-ordinates $R$ and $\theta$ are two of the spherical polar co-ordinates describing the point of observation as shown in Figure 2 and $J_0(rp)$ is the zeroth order Bessel function. The near- and far-field distributions are assumed to independent of the rotational co-ordinate $\phi$ - an assumption that is valid only for spherically-symmetric fibres. $O(\theta)$ is an obliquity factor that should be equal to $\cos(\theta)$ but is often assumed to be unity since the angles of observation for conventional single mode fibres are small [1]. However, fibres with smaller cores, such as dispersion-shifted fibres can generate far-field angular distributions in which the obliquity factor is more significant [7].

![Figure 2. Co-ordinates used to describe the near- and far-field radiation patterns of a single mode fibre.](image)

The observed far-field amplitude is given by the real part of the field distribution and therefore equation 3-1 can be re-written as:

$$F(p) = O(\theta) \int_0^\infty E_a(r)J_0(rp)rdr = O(\theta) \text{HT}\left(E_a(r)\right)$$

3-2

Where $\text{HT}\left(E_a(r)\right)$ is the Hankel transform of the near-field distribution. The inverse Hankel transform is given by [7]:

$$E_a(r) = \int_0^\infty \frac{F(p)}{O(\theta)}J_0(rp)dp = \text{HT}^{-1}\left[\frac{F(p)}{O(\theta)}\right]$$

3-3
Through equations 3-2 and 3-3, the near- and far-field radiation patterns can be related to each other. Although, as pointed out by Wittman and Young [7], the near field is not necessarily the same as the mode field since it applies in the free space just outside the fibre and not actually within the core. To reflect this, a distinction was drawn between the mode-field, $E_m(r)$ and the aperture field, $E_a(r)$.

The standard TIA and ITU-T test methods for MFD ignore the difference between the aperture field and the mode field and also the obliquity factor is assumed to be unity [8,9]. This leads to equation 2-5 for the MFD in terms of the far-field, which can also be expressed as:

$$MFD_{FF} = \frac{\sqrt{2} \lambda}{\pi} \left[ \frac{\int_0^{\pi/2} |F(\theta)|^2 \cos(\theta) \sin(\theta) d\theta}{\int_0^{\pi/2} |F(\theta)|^2 \cos(\theta) \sin^3(\theta) d\theta} \right]^{1/2}. \quad 3-4$$

Including the $\cos(\theta)$ obliquity factor, equation 3-4 becomes [7]:

$$MFD_{FF} = \frac{\sqrt{2} \lambda}{\pi} \left[ \frac{\int_0^{\pi/2} |F(\theta)|^2 \tan(\theta) d\theta}{\int_0^{\pi/2} |F(\theta)|^2 \tan(\theta) \sin^2(\theta) d\theta} \right]^{1/2}. \quad 3-5$$

### 3.2 Experimental Technique

The aim of the far-field scan technique is to accurately measure the angular distribution of the intensity of the field from the single mode fibre. The mode-field diameter, as used by ITU-T and the TIA is defined in terms of this intensity distribution and can be calculated directly from it as detailed in the previous section. However, for the purposes of calculating effective area, the far-field distribution must be converted into the near-field distribution for use in equation 2-2.

An experimental system for measuring the far-field radiation pattern from a single mode fibre is shown below in Figure 3.
The chopped output from a laser diode is launched into the fibre under test, which is then passed through a cladding mode stripper to remove any optical power not in the fundamental mode. The angular intensity distribution at radial distance $R \approx 20\text{ mm}$ is then scanned using an InGaAs photodiode with a $100/140\mu\text{m}$ multimode fibre pigtail. The detector fibre is mounted on a precision rotation stage that is stepped in fixed angular increments through a total arc of approximately $80^\circ$. The signal from the photodiode is amplified and passed to a current-to-voltage converter, giving a voltage proportional to the optical power received by the fibre. The acceptance angle of the detector fibre is independent of the scan angle and therefore the received power is linearly proportional to the far-field intensity. The relative optical intensity as a function of angle is then recorded with the computer. An example of a far-field intensity distribution from a matched-cladding fibre at $1549\text{ nm}$ is shown in Figure 4.
Figure 4. Example of a measured far-field intensity distribution, $|F(\theta)|^2$. Note that the intensity is plotted using a log scale and the intensity of the first side-lobe is approximately 45-dB (optical) below the main peak.

4. The Near-Field Scanning Technique

The advantage of scanning the near-field intensity distribution of the test fibre directly is that there is no need for mathematical transformations of the measured data. If the intensity distribution of the aperture field, $|E_a(r)|^2$, is measured then it can be input directly into equation 2-2 to give the effective area. The main difficulty with measuring the near-field is that it extends over a very small area - typically 5-10-\(\mu\)m in diameter. Consequently, optics are required to produce a magnified image of the field that can be scanned radially using a detector with a pinhole in front or a fibre pigtail. The optical system must be carefully constructed to ensure that the magnified image is a faithful representation of the end of the test fibre. A high numerical aperture system is required to avoid smearing the image by truncating of the angular distribution of the field leaving the fibre. This becomes more of a problem for fibres with particularly small cores since the field diverges more quickly outside the fibre.

The near-field scanning system used at NPL is shown schematically in Figure 5. The current from the photodiode detector is proportional to the optical power accepted by the pinhole or detecting fibre. This is linearly converted to a voltage, which is then also proportional to the power and therefore to the near-field intensity. It is the voltage signal as a function of the radius of the near-field sample point that constitutes the measured data. Scanning is
performed by moving the test fibre with stepper motors and using interferometers to measure the position of the fibre. The advantage of this method over moving the detector is that the magnification of the imaging system does not need to be determined.

Initial fibre positioning is performed using a confocal system. The beamsplitter after the objective lens creates two identical optical paths and the pinhole detector and the endface of the confocal single mode fibre are the same distance from the focal point of the objective lens. The test fibre can be positioned precisely at the focal point of the objective lens by switching the laser source to emerge from the confocal fibre. By measuring and maximising the optical power launched backwards into the test fibre, the test fibre can be optimally positioned.

An example of a measured near-field from a standard matched-cladding fibre is shown below in Figure 6.

![Figure 5](image)

**Figure 5.** Confocal near-field scanning system used at NPL to measure the near-field of a single mode fibre.
5. The Variable Aperture in the Far-Field (VAFF) Technique

5.1 Theory

The direct far-field scanning technique measures the angular distribution of the far-field intensity distribution from the single mode fibre, $|F(p)|^2$, (see section 3). The variable aperture technique also measures power in the far-field but rather than making a continuous measurement of the intensity distribution, the total power passing through a set of circular apertures is measured. It is assumed that the far-field and therefore the fibre are rotationally symmetric. The apertures are centred on the optical axis of the fibre so that they are also centred on the axis of rotational symmetry of the far-field pattern.

For a circular aperture of radius $a$ located a distance $D$ from the fibre endface, the half angle of the cone subtended by the aperture at the fibre endface, $\theta_{aperture}$, is given by:

$$\theta_{aperture} = \arctan \left( \frac{a}{D} \right).$$  \hspace{1cm} 5-1

The optical power passing through the aperture from a curved wavefront centred on the fibre endface is given by [1]:

![Figure 6. Example of a measured near-field intensity distribution from a matched-cladding fibre.](image)
\[ P(v) = 2\pi \int_{0}^{\nu} F^2(p) pdp \quad 5-2 \]

where \( \nu = k \sin(\theta_{\text{aperture}}) \). Since \( p = k \sin(\theta) \), this equation can be re-written as:

\[ P(\theta_{\text{aperture}}) \propto \int_{0}^{\theta_{\text{aperture}}} F^2(\theta') \sin(\theta') \cos(\theta') d\theta'. \quad 5-3 \]

Equation 5-2 forms the basis for the EIA/TIA reference test method for mode-field diameter measurement by the VAFF technique [10]. However, it has been suggested that equation 5-2 is incorrect and that the appropriate version of equation 5-3 is [11]:

\[ P(\theta_{\text{aperture}}) \propto \int_{0}^{\theta_{\text{aperture}}} F^2(\theta') \sin(\theta') d\theta'. \quad 5-4 \]

where \( R \) is the distance from the fibre endface to the edge of the aperture. This is the expression that has recently appeared in ITU-T draft recommendations for an effective area test method [12]. The difference between the two expressions can be understood from Figure 7, which shows the parameters used to derive equations 5-3 and 5-4. The discrepancy arises from which incremental length of the phase front is integrated to give the total power passing through the aperture. Equation 5-4 uses the arc length along the curved wavefront, given by \( R \delta \theta \). However, 5-3 uses the projection of this curved section onto a plane parallel to that of the aperture, \( R \delta \theta \cos(\theta) \). Clearly, for small angles \( \cos(\theta) \approx 1 \) and the difference between the two formulations is minimised. It has been shown that they lead to a difference in calculated MFD of typically less than 0.5% - even for dispersion shifted fibres with the smallest cores and broadest far-fields [11].
Once the aperture power function has been determined for a number of apertures, the relative far-field power distribution can be calculated from its gradient. Analysis of the data then follows the same method as that of the direct far-field method, i.e. transformation to the near-field and calculation of the effective area from the field distribution (see section 3). The far-field power distribution can be calculated using either

\[ F^2(p) \propto \frac{1}{p} \frac{dP(p)}{dp} \quad 5-5 \]

or

\[ F^2(\theta) \propto \frac{1}{\sin(\theta)} \frac{dP(\theta)}{d\theta} \quad 5-6 \]

depending on whether the aperture power function is assumed to follow from equation 5-2 or 5-4 respectively.

5.2 Experimental Technique

The basic experimental setup for the variable aperture technique is shown below in Figure 8. Apertures of different diameters are usually precision mounted on a wheel so that they can be easily changed and the throughput power measured for each one. Figure 9 shows an example of measured VAFF data for a matched-cladding fibre.
The equipment used in our investigations was a PK Technology S25 measurement system including an internal tungsten halogen lamp filtered using a monochromator with 10-nm spectral width. The wavelength calibration uncertainty is estimated to be less than ±2-nm. Twenty three apertures were used, giving a range of collection numerical apertures \( = \sin(\theta_{\text{aperture}}) \) from 0.0066 to 0.4948.
6. The Transverse Offset Technique

6.1 Theory

When two single mode fibres are butted together to form a joint, the fundamental mode field of the source fibre is coupled into the modes of the recipient fibre. These recipient modes consist of the fundamental mode, lossy cladding modes and radiation modes. The magnitude of the source mode field coupled into each recipient mode is given by the overlap integral of the two modal electric fields. In the transverse offset technique, we are concerned with the transfer of optical power from the source mode into the same mode of an identical fibre when their axes are parallel but laterally offset from each other.

The power coupled from the fundamental mode of the source fibre into the same mode of an identical recipient fibre is given by the square of the field overlap integral, \( C(u) \). With reference to Figure 10, the overlap integral can be expressed as [1]:

\[
C(u) = \int \int_{S} E_{a}(|r|)E_{a}(|r - u|)\,d\mathbf{r}
\]

where \( u = |u| \) is the magnitude of the separation of the fibre axes. The integral should be performed over the region \( S \), which extends over all space.

![Figure 10](image)

**Figure 10.** Geometry of the overlap integral in the transverse offset technique.
The overlap integral can be expressed as a convolution in polar co-ordinates, such that:

$$C(u) = \int_{0}^{2\pi} \int_{0}^{\infty} E_a(r)E_a(r')rdrd\phi = \left[ E_a(r) \ast E_a(r) \right]_{r=u}$$ \hspace{1cm} (6-2)

where $r^2 = u^2 + r^2 - 2ru \cos(\phi)$, $r' = |r'|$, $r = |r|$ and the $\ast$ represents a 2-dimensional convolution. The Hankel transform of the convolution is \cite{14}:

$$H\{C(u)\} = H\{E_a(r) \ast E_a(r)\} = F^2(p)$$ \hspace{1cm} (6-3)

where $F^2(p)$ is the far-field power distribution of the fibre. Through equation 6-3, the measured power transfer function for the offset splice, $C^2(u)$, can be related to the near-field of the fundamental mode via the far-field power distribution. The near-field distribution can then be used to calculate the effective area, as described in section 2.

6.2 Experimental Technique

The transverse offset system developed at NPL is shown below in Figure 11. A sample of test fibre was cleaved and checked with an interferometer to ensure that the cleaved endfaces were perpendicular to the fibre axis to within 0.5°. The fibre samples were then mounted in vacuum chucks facing each other and actively aligned using an xyz flexure stage to maximise the detected signal. The endfaces were brought as close together as possible without making contact. The lateral offset between the fibre axes was then scanned using the piezo actuator attached to one of the vacuum chucks.

The position of the scanning fibre was monitored by measuring the output from an interferometer. Scanning was automated by using the lock-in amplifier to drive the piezo controller and read the output signals from the interferometer and the throughput power detector. The range of travel of the piezo stack was approximately 15-μm - meaning that at least two scans were required to cover enough range to measure a complete transmission curve.
Figure 11. Transverse offset system used at NPL to record the power transferred between two cleaved samples of fibre.

An example of a power transmission curve starting from a point just to one side of the peak transmission is shown below in Figure 12.

Figure 12. Example of measured transverse offset power transmission curve for a Lucent All Wave™ fibre.
7. References


5 Namihira Y., “Measurement Results of Effective Area (Aeff) and MFD and their Correction Factor for Non-Zero Dispersion Shifted Fibres (NZFs, G.655) and Dispersion Shifted Fibres (DSFs, G.653) by Using Variable Aperture Technique”, ITU Com 15-53-E, (1997).


