NPL National Physical Laboratory

# Monte Carlo Simulation for Solid Angle Calculations in Alpha Particle Spectrometry


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Several discussions with Stefaan Pommé, IRMM, EC-JRC, Geel

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## Brief Overview

- Why is NPL interested in this topic
  - New Defined Solid Angle (DSA) counting system
  - Work in progress
- I have noticed that some laboratories are using "questionable" formulae for simple disc-disc solid angle calculation.
- I have written some simple computer code, to calculate solid angles using Monte Carlo simulation techniques
- Comparisons with published formulae ongoing

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# Introduction
# Defined Solid Angle (DSA) Counting

- NPL's interest in this is to set up a new Defined Solid Angle (DSA) counting system for the Primary Standardisation of certain radionuclides.

- DSA counting is an excellent primary standardisation technique, only suitable for "particles" that:
  - travel in straight lines (are not readily scattered)
  - are heavily absorbed in detector, source substrate, diaphragm

- i.e.:
  - alpha particles (several MeV)
  - Low energy photons, less than 80 keV

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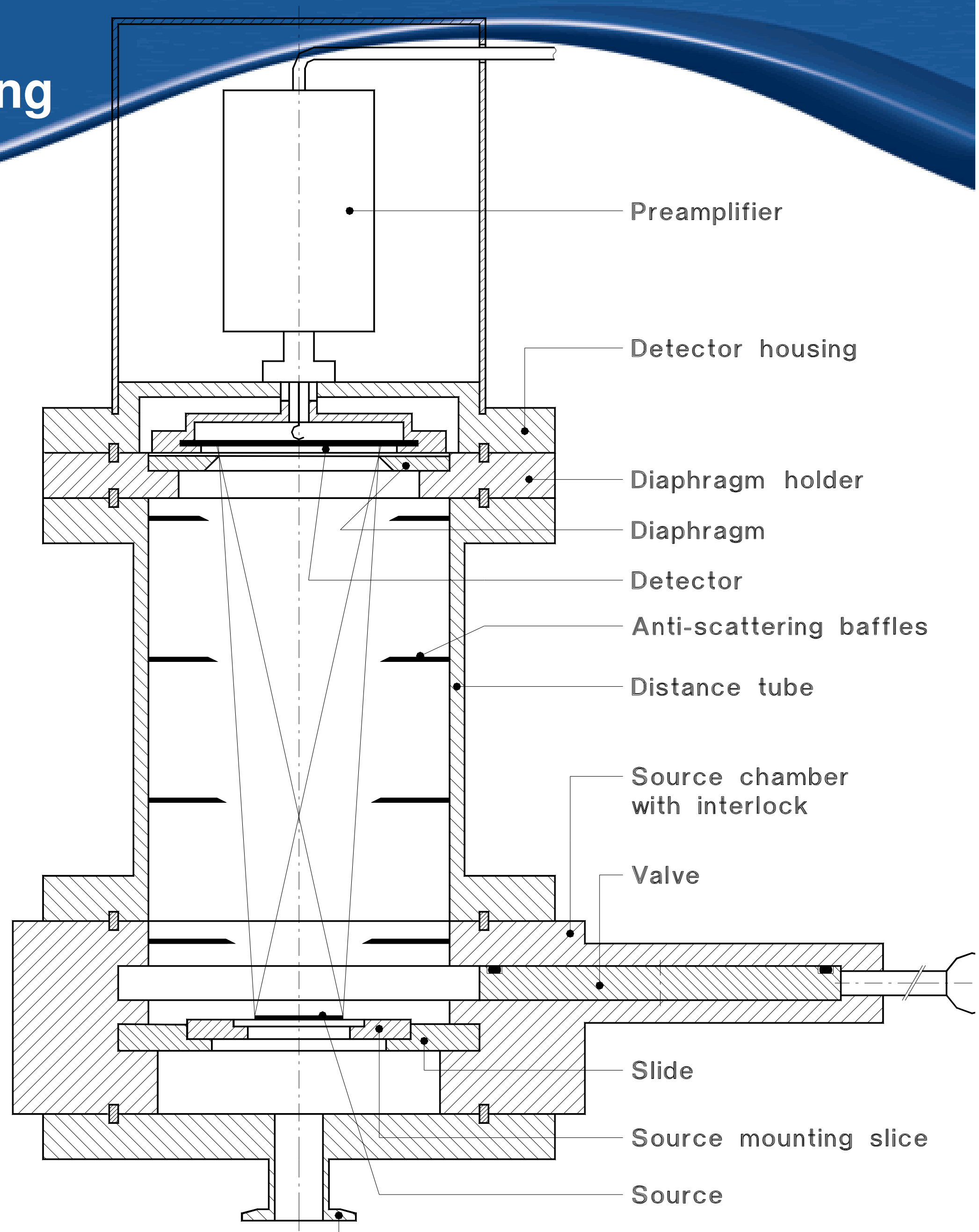
# Introduction
# Defined Solid Angle (DSA) Counting

- The DSA counting system at IRMM >>>>>>
- NPL are in process of building a similar system
- Work on the analysis software has begun
  - Monte Carlo

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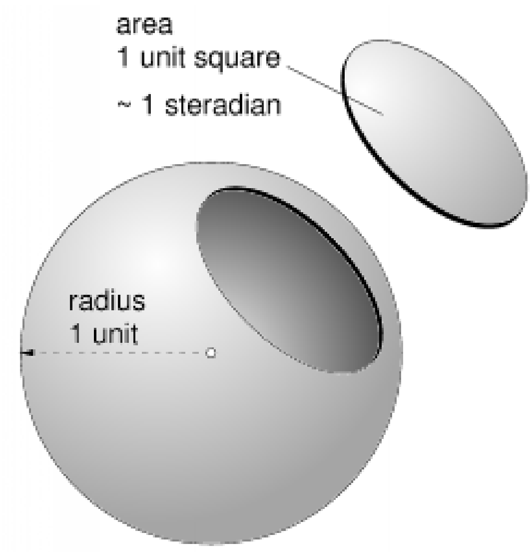
## Definition of Solid Angle (steradian)

- The three dimensional “angle” formed at the vertex of a cone

- When this vertex is the centre of a sphere of radius “r” and the base of the cone cuts out an area “s” on the surface of the sphere, the solid angle in steradians is defined as $\Omega = s/r^2$

- $4\pi$ steradians in a complete sphere

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## Geometry Factor

The Geometry Factor (G) is equal to:

- the ratio of the solid angle to 4 $\pi$ steradians

$$G = \Omega / 4\pi$$

- the fraction of alpha particles emitted directly towards the detector

- the counting efficiency, if detection efficiency is 100%

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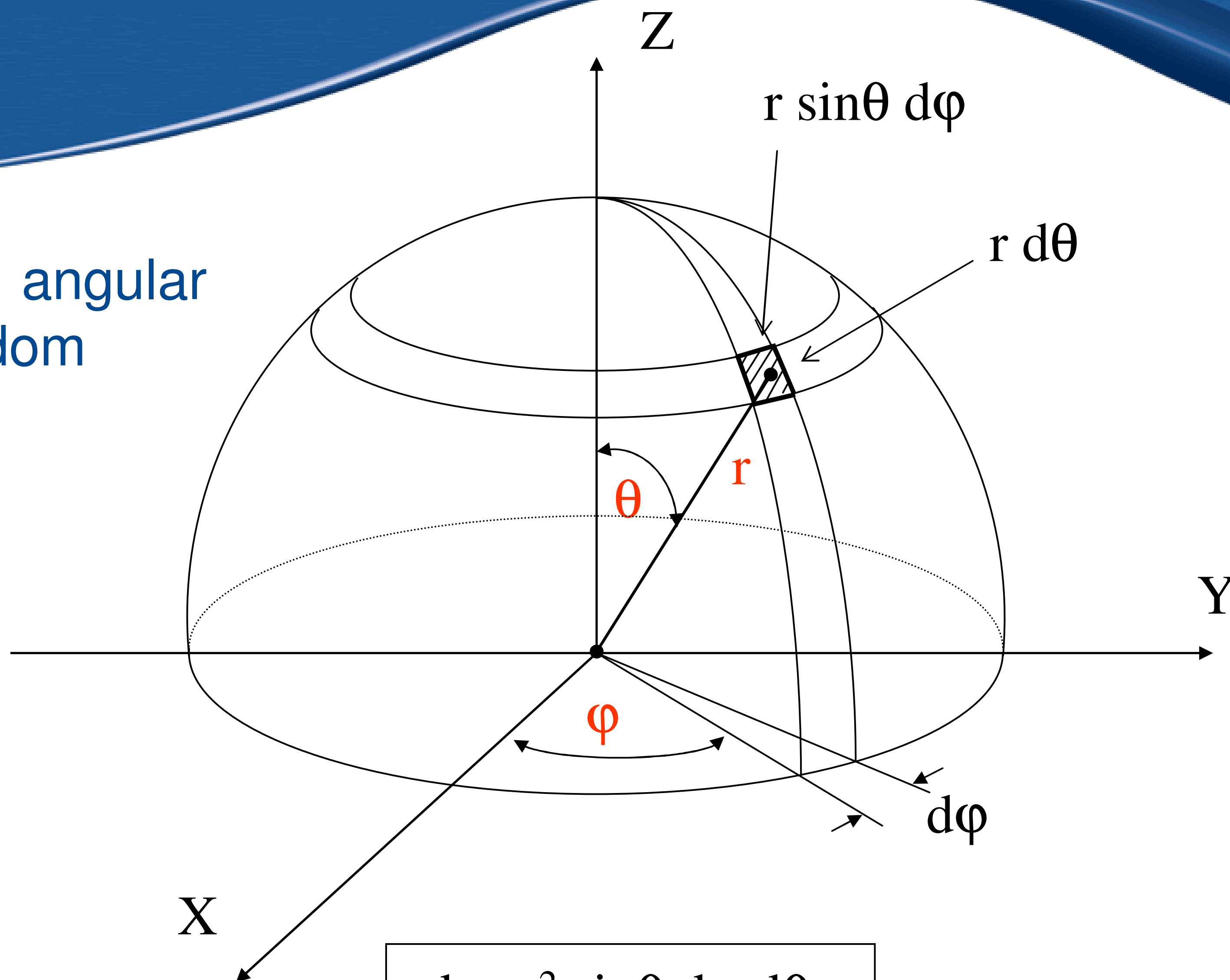
# Spherical Coordinate System

Integration over angular degrees of freedom

$0 \leq \theta \leq \pi$

$0 \leq \varphi \leq 2\pi$




$$ds = r^2 \sin\theta \, d\varphi \, d\theta$$

$$d\Omega = \sin\theta \, d\varphi \, d\theta$$

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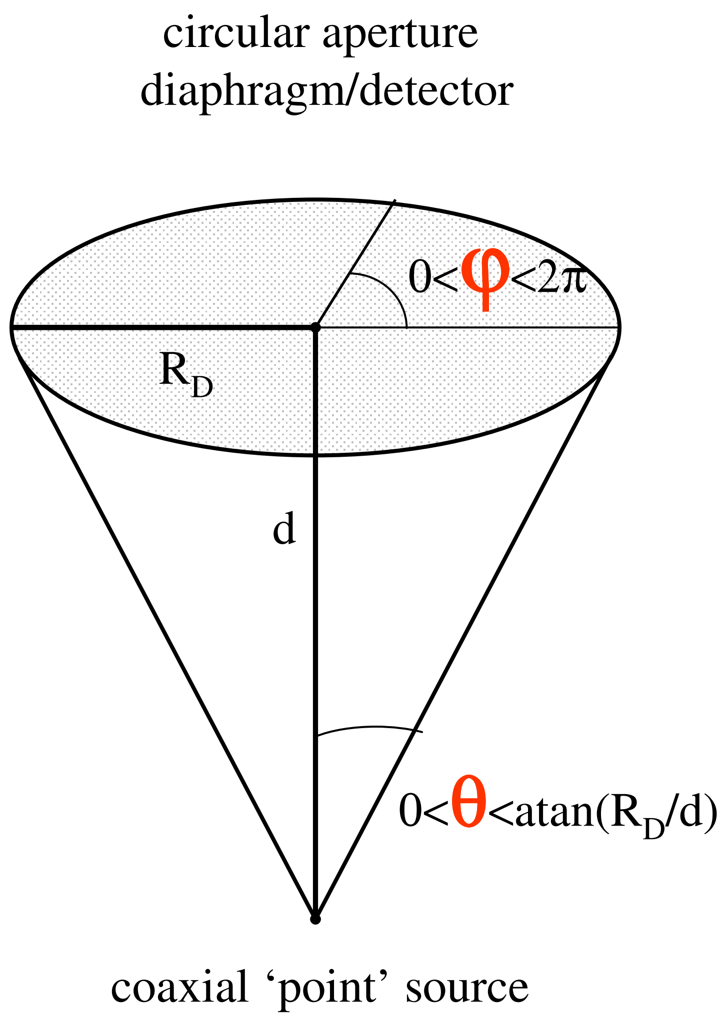
# Point Source on axis of symmetry of a circular detector (diaphragm)

- The **ONLY** geometry with a simple solution for $\Omega$

$$\Omega = 2\pi(1 - \cos\theta)$$

- The maximum value of θ for a "hit" is :

  $\theta_{max} = \text{atan}(R_D/d)$



## Geometric Considerations affecting precision

- In reality :
  - inhomogeneous, off-axis, non-circular source
  - unknown exact detector size
  - diaphragm edge has finite thickness
  - inhomogeneous activity distribution

- Require some means of precise solid angle calculation and uncertainty estimation
  - Monte Carlo technique
  - Numerical Integration

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- Requires good random number generator
  - B.A. Wichmann and I.D. Hill

    *"Generating good pseudo-random numbers"*

    Computational Statistics and Data Analysis,

    51 (3), 2006. 1614 – 1622

- Combination of four linear congruence generators
- Period of generator is approx 2.6 E+36
- Passes "Big Crush" test …
- 2.6 E+6 calls per second (in my C/C++ implementation)

# Wichman-Hill
# Random Number Generator
# (in C/C++)

```
extern "C" __declspec(dllexport) double random()
{

    double W;
    static long int IX = 1234L;
    static long int IY = 5678L;
    static long int IZ = 23456L;
    static long int IT = 56789L;

    IX = 11600L * (IX % 185127L) - 10379L * (IX/185127L);
    IY = 47003L * (IY %  45688L) - 10479L * (IY/ 45688L);
    IZ = 23000L * (IZ %  93368L) - 19423L * (IZ/ 93368L);
    IT = 33000L * (IT %  65075L) -  8123L * (IT/ 65075L);

    if (IX < 0)
       IX += 2147483579L;
    if (IY < 0)
       IY += 2147483543L;
    if (IZ < 0)
       IZ += 2147483423L;
    if (IT < 0)
       IT += 2147483123L;

    W =  (double)(IX) * 0.0000000004656613022697297188506231646486
       + (double)(IY) * 0.0000000004656613100759859932486569933169
       + (double)(IZ) * 0.0000000004656613360968421314794009471615
       + (double)(IT) * 0.0000000004656614011489951998100056779817;
    while (W >= 1.0)
       W -= 1.0;
    return W;
}
```

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## Simulation of a point on a disk source of radius R

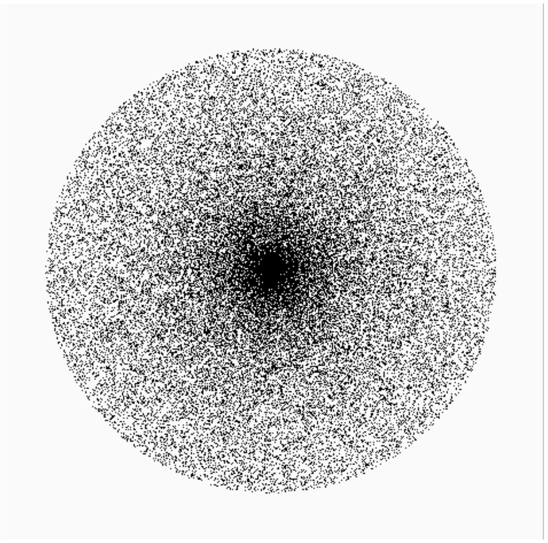
$$\text{LET} : U \in [0,1] = random()$$

- NOT simply:

  angle = U.2π

  distance from centre = U.R

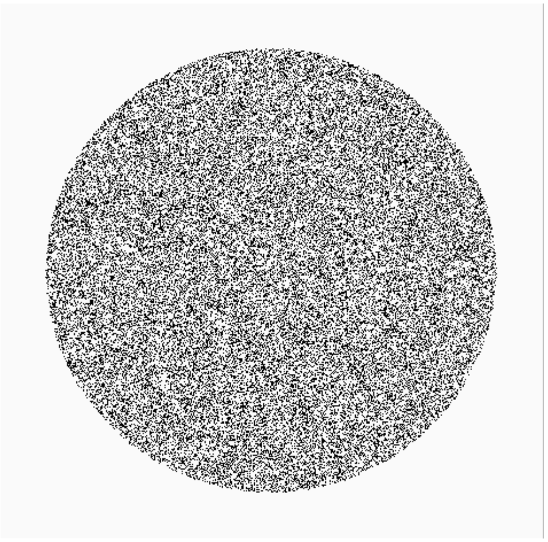
  points are clustered near the centre



- FOR EQUAL AREAS : EQUAL ACTIVITY

  angle = U.2π

  distance from centre = $\sqrt{U}.R$



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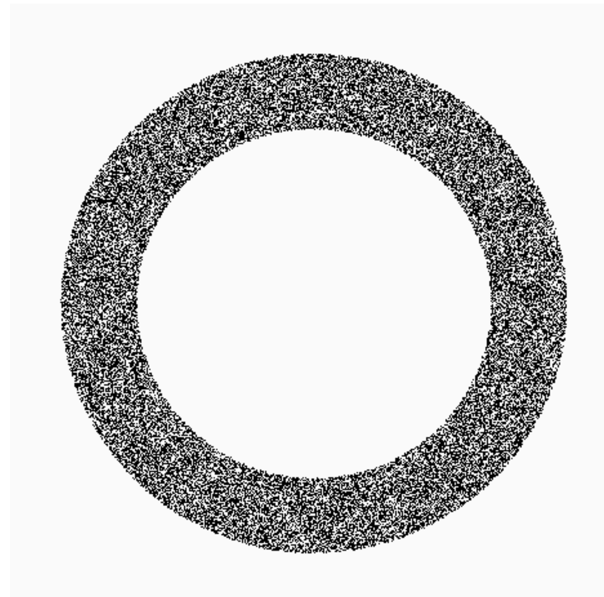
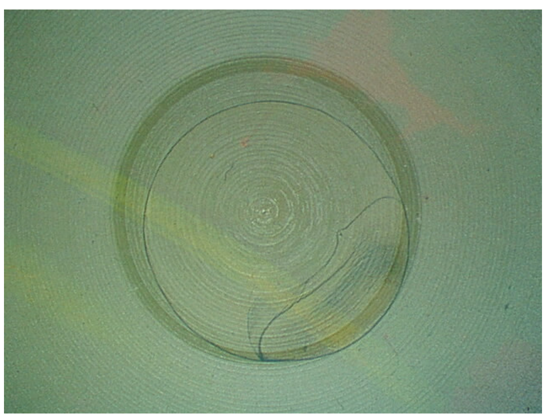
# Simulation of point on source annulus
# Outer radius Ro
# Inner radius Ri

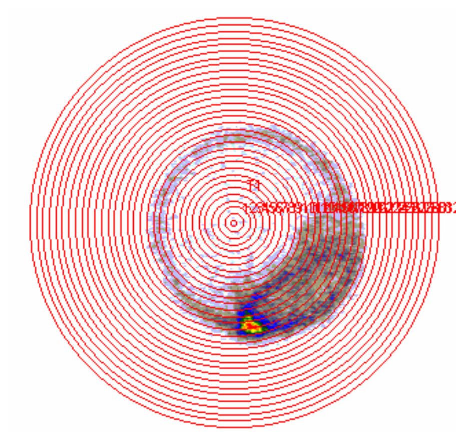


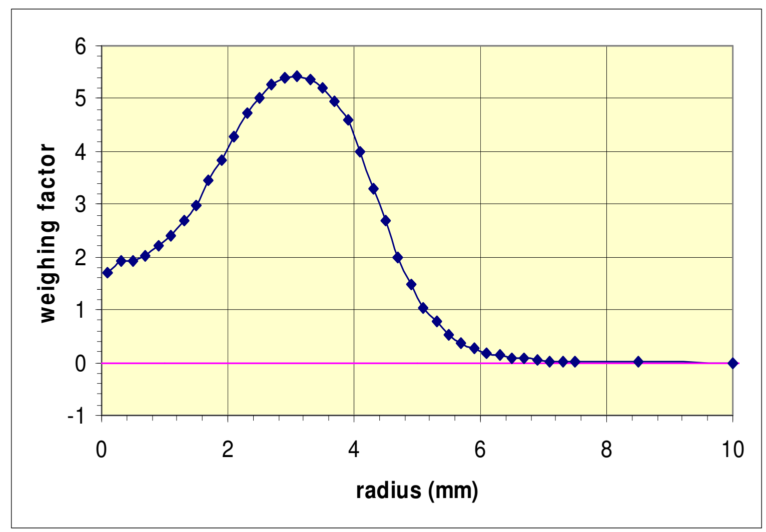
angle = U.2π

distance from centre =

$$\sqrt{Ri^2 + U.(Ro^2 - Ri^2)}$$





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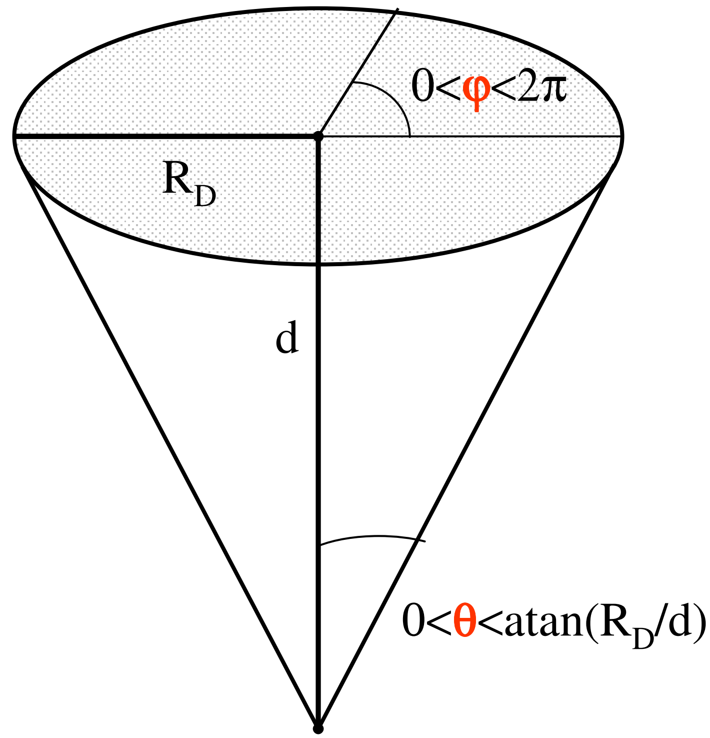
# Simulation of particle direction

φ : azimuthal angle

$\varphi = U.2\pi$

θ : elevation angle

$\theta = U.\pi$




But, we may wish to limit angle for simulations ($\theta_1 < \theta < \theta_2$) to avoid wastage of random numbers

$$\theta = \mathrm{acos}\left(\cos(\theta_1) - U\left(\cos(\theta_1) - \cos(\theta_2)\right)\right)$$

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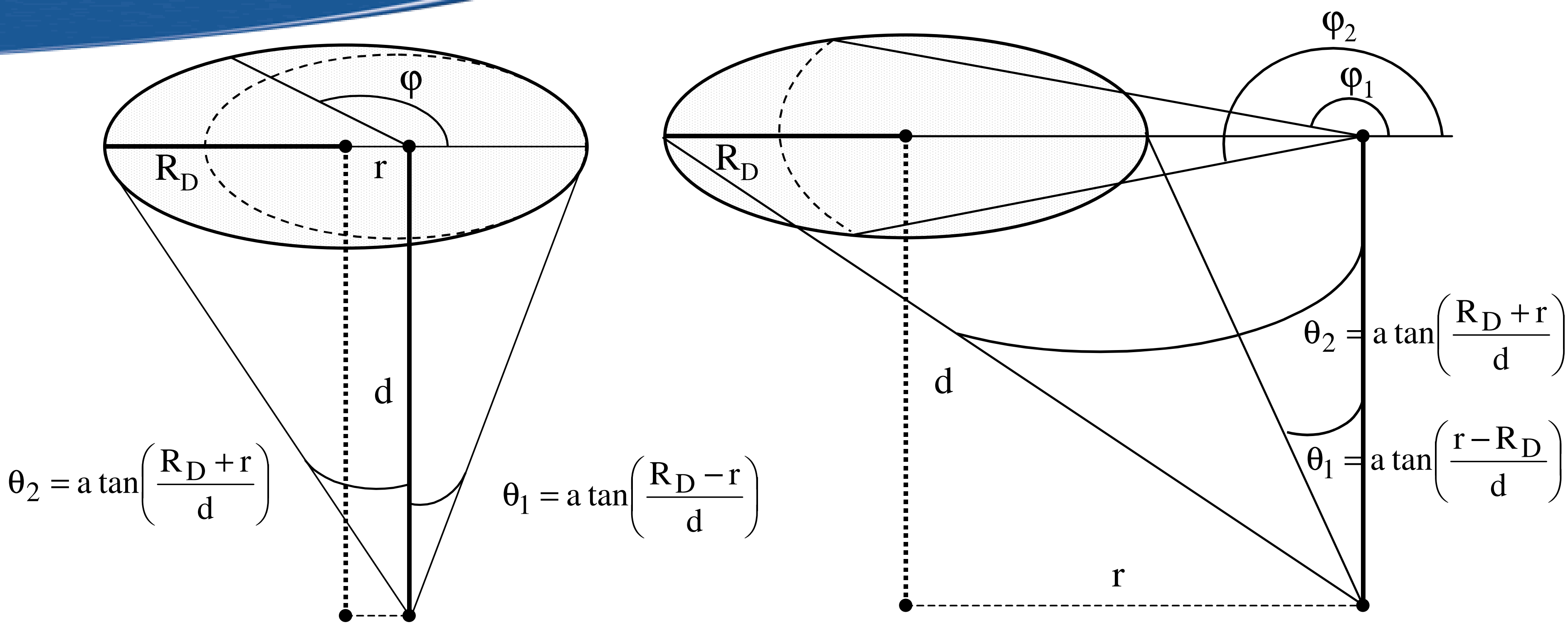
# Eccentric Point Source
# with circular diaphragm/detector




$$\Omega_1 = \begin{cases} 2\pi(1-\cos\theta_1) & for\ \ r < R_D \\ 0 & for\ \ r \geq R_D \end{cases}$$

$$\Omega_2 = \int_{\theta_1}^{\theta_2} 2(\pi - \varphi(\theta))\ \sin\theta\ d\theta$$

$$\Omega = \Omega_1 + \Omega_2$$

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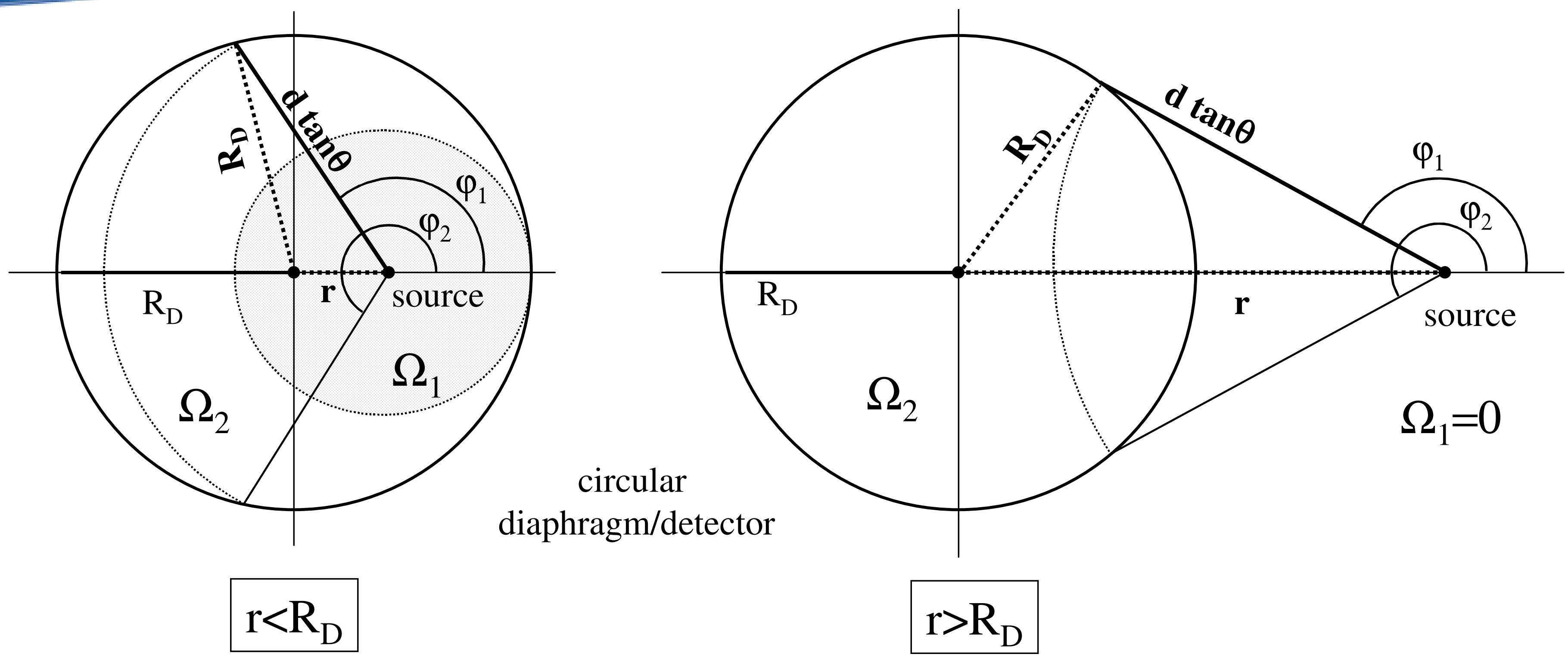
$$\varphi(\theta) = a\cos\left(\frac{R_D^2 - \{r^2 + d^2\tan^2\theta\}}{2\,r\,d\tan\theta}\right)$$

# Eccentric Point Source with circular diaphragm/detector (top view)




*a "hit" is defined as:*

$$R_D^{\,2} > r^2 + (d\tan\theta)^2 - 2r(d\tan\theta)\cos(\pi - \varphi_1)$$

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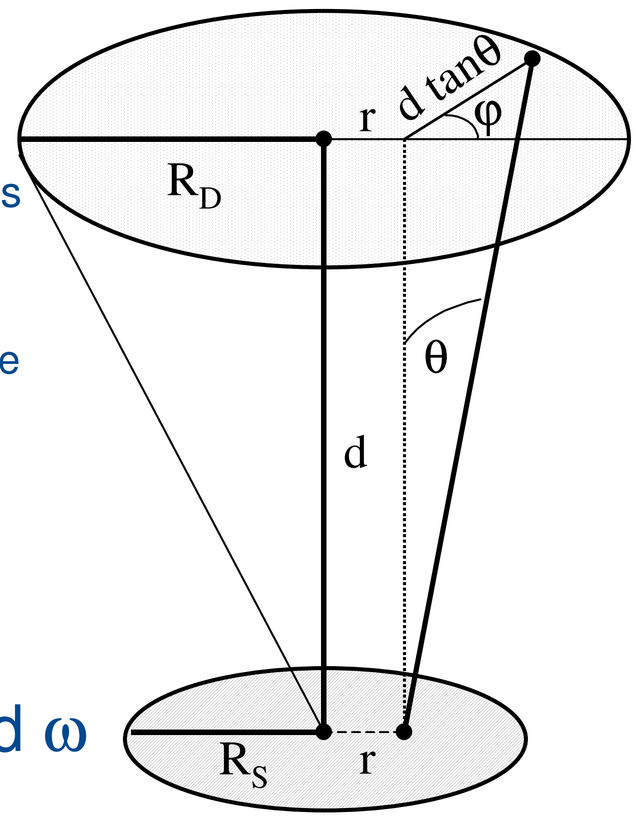
# Coaxial circular source

$\psi = R_S/d$

$\omega = R_D/d$

- Attempts to derive formula:
- Ruby
    - NIM 58 (1968), 345
    - NIMA, 337, (1994), 531
  - Expressions as integrals as products of Bessel Functions
- Tsoulfanidis
    - "Measurement and Detection of Radiation", Hemisphere (1983). Chapter 8.
  - Simple algebraic approximation to 8th order in $\psi$ and $\omega$
- Serial Expansions by Pommé and Conway …
- Appear to derail for large values of $\psi$ and $\omega$

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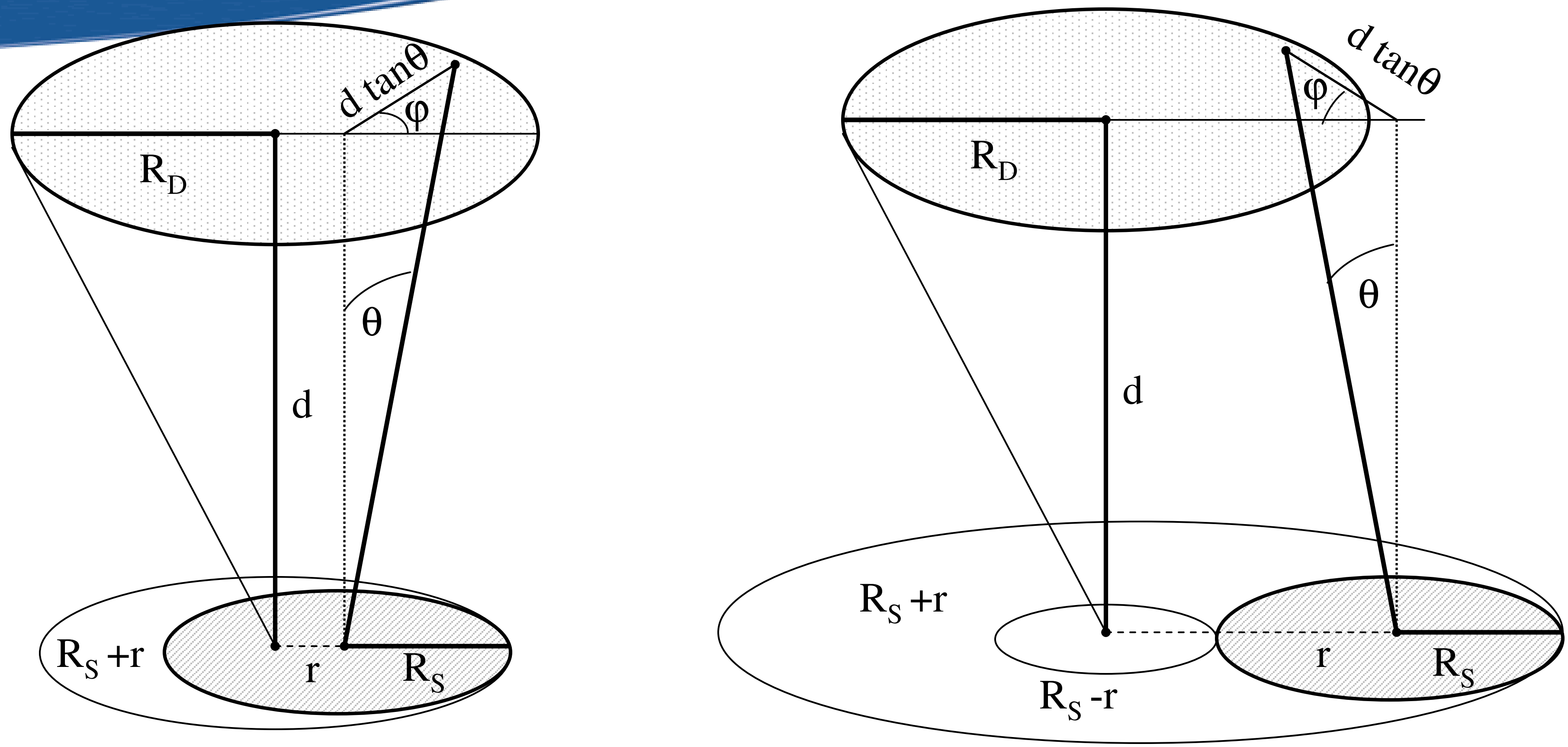
# Eccentric circular source



## Eccentric Sources
## Useful References on expressions

- Pommé
  - NIMA, 505 (2003), 286-289
  - NIMA, 579 (2007), 272-274)
- Conway
  - NIMA, 562 (2006), 146-153
  - NIMA, 583, (2007), 382-393
  - NIMA, 589 (2008), 20-33
  - NIMA, 614 (2010), 17-27

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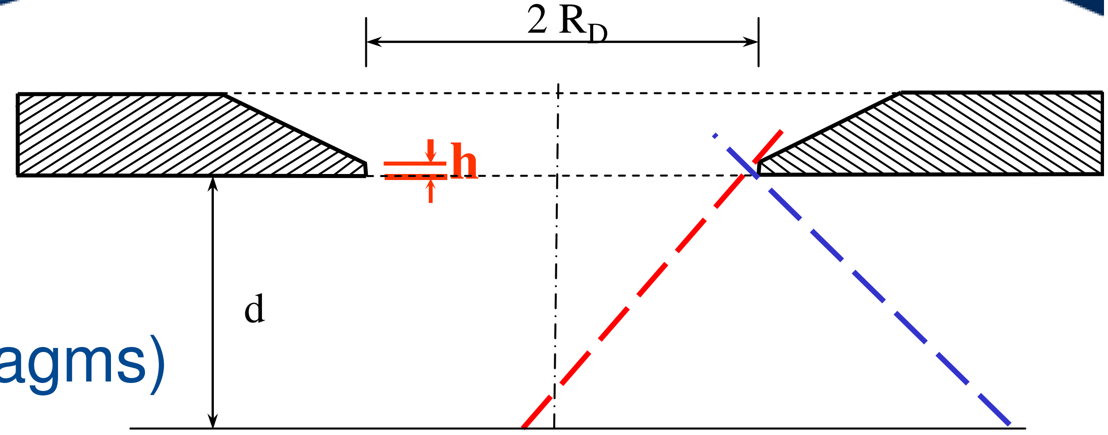
# Ongoing work …

Incorporate Realistic Diaphragms:
(I.e.: two theoretical diaphragms)




Continue verification of formulae (serial expansions) of Pommé and Conway

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## Conclusions

- Depending on the source – detector geometrical configuration, the derivations of expressions for solid angles can be COMPLICATED !
- Numerical integration for solid angle calculations from can be difficult to program.
- Rather simpler to use Monte Carlo technique
  - Acts as useful validation tool for expressions
  - Code is still under development, but is in use for "rough" calculations, as further validation is required

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