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Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 14

The Estimation of Uncertainties in Hardness Measurements

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CONTENTS

1. SCOPE

This procedure covers the evaluation of uncertainty in hardness measurement at room temperature, carried out according to the testing standard:

> EN ISO 6506 / 1-3 (1999), EN ISO 6507 / 1-3 (1998), EN ISO 6508 / 1-3 (1999) considering of: ASTM E 10-96, ASTM E 18-97a, ASTM E 92.82(Reapproved 1997), ASTM A 370-97a

Description of Rockwell Test; ASTM A 370 says:
"In this test a hardness value is obtained by determining the depth of penetration of a diamond point or a steel ball into the specimen under certain arbitrarily fixed conditions. A minor load is first applied which causes an initial penetration, sets the penetrator on the material and holds it in position. A major load which depends on the scale being used is applied increasing the depth of indentation. The major load is removed and, with the minor load still acting, the Rockwell number, which is proportional to the difference in penetration between the major and minor loads is determined; this is usually done by the machine and shows on a dial, digital display, printer or other device.**"** Detailed procedure: ASTM E18 and EN ISO 6508 (latest revisions).

Description of Brinell Test; ASTM A 370 says:
"A specified load is applied to a flat surface of the specimen to be tested, through a hard ball of specified diameter. The average diameter of the indentation is used as a basis for calculation of the Brinell hardness number. The quotient of the applied load divided by the area of the surface of the indentation, which is assumed to be spherical, is termed the Brinell hardness number (HB).**"** Detailed procedure: ASTM E10 and EN ISO 6506 (latest revisions).

Description of Vickers Test; ISO 6507 says:
"A diamond indenter in the form of a right pyramid with a square base and with a specified angle between opposite faces at the vertex is forced into the surface of a test piece followed by measurement of the diagonal length of the indentation left in the surface after removal of the test force, F. The Vickers hardness is proportional to the quotient obtained by dividing the test force by the sloping area of the indentation which is assumed to be a right pyramid with a square base, and having at the vertex the same angle as the indenter.**"** Detailed procedure: ASTM E92 and EN ISO 6507 (latest revisions).

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

c_i	sensitivity coefficient
CoP	Code of Practice
d(x)	diameter or diagonal of the indention (i.e. $d_{1(HB)}$)

d_v	divisor used to calculate the standard uncertainty
E	exactness of impact testing machine
F_1	additional force
Fo	preliminary test force
h	permanent increase in depth
HB	Brinell hardness
h_M	measurement for the difference in depth
HR	Rockwell hardness
HV	Vickers hardness
k	coverage factor used to calculate expanded uncertainty (normally corresponding to 95% confidence level)
N	number of input parameters x_i on which the measurand depends
n	number of repeat measurements
p	confidence level
s	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
U	expanded uncertainty
u	standard uncertainty
u_c	combined standard uncertainty
V	value of the measurand
x_i	estimate of input quantity
$\bar{x}$	arithmetic mean of the values of the random variable x_i
y	test (or measurement) mean result

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be controlled more closely.

This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated.
The aim is to produce a series of documents in a common format which is easily understood and accessible to customers, test laboratories and accreditation authorities.
This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into 6 sections as follows, with all the individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing

4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in Reference 1. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in hardness measurement by indirect calibration method.

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTY IN HARDNESS MEASUREMENT BY THE INDIRECT CALIBRATION METHOD

Step 1. Identifying the Parameters for Which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are usually reported in hardness measurements by the indirect calibration method. None of these measurands are measured directly, but are determined from others quantities (or measurements).

Table 1 Measurands, their units and symbols

Measurands	Units and Symbol
Rockwell hardness	HR(scale)
Brinell hardness	HBS(HBW)
Vickers hardness	HV

Table 2 Measurements, their units and symbols

Measurements	Units	Symbol
Permanent increase in depth of penetration under preliminary test force after removal of additional force	0.002mm (regular scale) 0.001mm (superficial scale)	h
Single diameter of the indention	mm	$d_{1(HB)}$, $d_{2(HB)}$
Single diagonal of the indention	mm	$d_{1(HV)}$, $d_{2(HV)}$

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2, the user must identify all possible sources of uncertainty which may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand, as it is associated uniquely with the individual test procedure and apparatus used.

This means that a new list should be prepared each time a particular test parameter changes (for example when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined. Table 3 lists the four categories and gives some examples of sources of uncertainty in each category.

It is important to note that Table 3 is NOT exhaustive and is for GUIDANCE only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their own test facility and assess the associated significance of the contributions.

Table 3 Sources of uncertainty and their likely contribution to uncertainty in hardness measurement

[1 = major contribution, 2 = minor contribution, 0 = no contribution (zero effect)]

Rockwell	Symbol	Type	h
Testing Means			
Preliminary test force	F_0	B	2
Additional force	F_1	B	1
Measurement for the difference in depth	h_M	B	1
Angle (HRA; HRC, HRD, HRN)	α	B	1
Radius (HRA, HRC, HRD, HRN)	R	B	1
Diameter	D	B	1
Indention velocity	c	B	2
Duration of the preliminary test force	T_{Fo}	B	1-2
Duration of total force application	T_{Ft}	B	1
Test Method			
Zeroing		B	1
Calibration		B	1
Digitizing		B	2
Software		B	2
Anvils		B	1
Test Environment			
Temperature		B	2
Dust, dirt, grease and scale		B	1-2
Operator			
Knowledge and experience		B	1-2
Specimen			
Preparation (heat or cold-working!!!)		B	1
Shape, size and thickness		B	1-2
Parallelism		B	1-2
Surface aspect		B	1-2

[1 = major contribution, 2 = minor contribution, 0 = no contribution (zero effect)]

Brinell and Vickers	Symbol	Type	$d_{1,2(HB)}/d_{1,2(HV)}$
Testing Means			
Test force	F	B	1
Measurement for the diameter / diagonal	d_M	B	1
Angle (HV)	α	B	2
Diameter (HBS/HBW)	D	B	1-2
Indention velocity	c	B	1-2
Duration of total force application	T	B	1-2
Test Method			
Zeroing		B	1
Calibration		B	1
Digitizing		B	2
Software		B	1-2
Anvils		B	1-2
Test Environment			
Temperature		B	2
Dust, dirt, grease and scale		B	1-2
Operator			
Knowledge and experience		B	1-2
Specimen			
Preparation (heat or cold-working!!!)		B	1
Shape, size and thickness		B	1-2
Parallelism		B	1-2
Surface aspect		B	1-2
Quality of the indentation		B	1

This CoP includes the estimation of following sources using the indirect calibration method:

- Uncertainty due to calibration of reference blocks
- Uncertainty of maximum permissible error according the standards
- Uncertainty due to repeatability at certain test conditions

The worked example in Appendix B uses the above categorisation when assessing uncertainties.

Step 3. Classifying the Uncertainty According to Type A or B

In this third step, which is in accordance with Reference 2, *'Guide to the Expression of Uncertainties in Measurement'*, the sources of uncertainty are classified as Type A or B, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified Type A, if it is evaluated by any other means it should be classified as Type B.

The values associated with Type B uncertainties can be obtained from a number of sources including a calibration certificate, manufacturer's information, or an expert's estimation. For Type B uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of Reference 1).

It should be noted that, in some cases, an uncertainty can be classified as either Type A or Type B depending on how it is estimated.

Step 4. Estimating the Standard Uncertainty for Each Source of Uncertainty

In this step the standard uncertainty, u, for each input source is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d_v, associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of Reference 1.

In many cases the input quantity of the measurement may not be in the same units as the output quantity. For example, one contribution to hardness is the surface roughness. In this case the input quantity is roughness (µm), but the output quantity is the hardness which is HRB. In such a case, a sensitivity coefficient, c_T (corresponding to the partial derivative of the hardness / roughness relationship), is used to convert from roughness to HRB.

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} [c_i . u(x_i)]^2} \qquad \textbf{(1)}$$

where c_i is the sensitivity coefficient associated with x_i. This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity. The combined uncertainty has an associated confidence level of 68.26%.

Step 6. Computing the Expanded Uncertainty U

The expanded uncertainty, U, is defined in Reference 2 as “the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand”.

It is obtained by multiplying the combined uncertainty, u_c, by a coverage factor, k, that is selected on the basis of the level of confidence required. For a normal probability distribution, the most generally used coverage factor is 2 that corresponds to a confidence interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty, U, is, therefore, broader than the combined uncertainty, u_c. Where a higher confidence level is demanded by the customer (such as for aerospace and electronics industries), a coverage factor of 3 is often used so that the corresponding confidence level increases to 99.73%.

In cases where the probability distribution of u_c is not normal (or where the number of data points used in Type A analysis is small), the value of k should be calculated from the degrees of freedom given by the Welsh-Satterthwaite method (see Reference 1, Section 4 for more details).

Table B1 in Appendix B shows the recommended format of the calculation worksheets for estimating the uncertainty in hardness measurement. Appendix A presents the mathematical formulae for calculating uncertainty contributions.

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U \qquad \textbf{(2)}$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, U is the expanded uncertainty associated with y. An explanatory note, such as that given in the following example should be added (change when appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k = 2, which for a normal distribution corresponds to a coverage probability, p, of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT COP 02:2000.

5. REFERENCES

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APPENDIX A

Mathematical Formulae for Estimating Uncertainties in Hardness Measurement by the Indirect Calibration Method

The estimation of uncertainty by the **Hierarchy scheme** is according to the idea that the value measured by a standard machine should be considered as an approximation of the correct hardness value, within the limits of uncertainties of measurement.

Uncertainty of the results; EN ISO 6506, EN ISO 6507, and EN ISO 6508 says:
"The uncertainty of results is dependent on various parameters that may be separated into two categories:

a) parameters depending on the Rockwell hardness testing machine (including the uncertainty of the verification of the testing machine and of the calibration of the reference blocks);

b) parameters depending on the application of the test method (variation of the operating conditions).

NOTE A complete evaluation of the uncertainty should be carried out according to the *Guide to the Expression of Uncertainty in Measurement* [2]. Indicative values of the extended uncertainty at a confidence level of 95% can be taken equal to the maximum permissible error given in Table 2 of EN ISO 6506-2, Table 5 of EN ISO 6507-2, and Table 5 of EN ISO 6508-2.**"**

Procedure

1. Certified value of the CRM (certified reference material)

X_{CRM} ... Mean value of five measurements
u_{CRM} ... Uncertainty of the mean value calculated by the calibration laboratory. It is the 2-Sigma uncertainty (confidence level: 95%).

2. Determination of users machine repeatability

a) Mean value

$$\overline{H} = \frac{H_1 + H_2 + H_3 + H_4 + H_5}{5} \qquad \textbf{(3)}$$

b) Repeatability according EN ISO 6506, 6507, and 6508

$H_5 - H_1$ (Rockwell; EN ISO 6508) **(4a)**

d_5 - d_1 (Vickers; EN ISO 6507 and Brinell; EN ISO 6506) (**4b**)

c) Empirical standard deviation of a single value

$$s_H = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(H_i - \overline{H}\right)^2} ;n = 5 \qquad (5)$$

3. Assessment according EN ISO 6506, 6507, and 6508

a) Estimation of the error (E) of a laboratory testing device by n repeated measurements on the same CRM

$$\mathrm{E} = \mathrm{X}_{\mathrm{CRM}} - \overline{\mathrm{H}} \qquad (6)$$

The error (E) shall not exceed the values given in Table 2 of EN ISO 6506-2, Table 5 of EN ISO 6507-2, and Table 5 of EN ISO 6508-2.

b) Permissible repeatability values given in Table 2 of EN ISO 6506-2, Table 4 of EN ISO 6507-2, and Table 5 of EN ISO 6508-2.

4. Measure on a material by n_1 repetition

a) Mean Value

$$\overline{x} = \frac{\sum_{i=1}^{n_1} x_i}{n_1} \qquad (7)$$

b) Empirical standard deviation of a single value

$$s_x = \sqrt{\frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \overline{x})^2} \qquad (8)$$

5. In order to estimate the uncertainty of the mean value, $\overline{x}$, following mathematical model has been used

Combined uncertainty:

$$u^2_{C(\overline{x})} = E^2_{1s} + u^2_{CRM(1s)} + u^2_{\overline{H}} + u^2_{\overline{x}} + \ldots \qquad (9)$$

Expanded uncertainty:

$$u_{e(\bar{x})} = k\, u_{C(\bar{x})} \;;\; k = 2 \qquad \textbf{(10)}$$

a) The permissible error $(E_{2\sigma})$ is a 2-Sigma error (normal distribution is assumed), therefore

$$E_{1s} = \frac{E_{2s}}{2} \qquad \textbf{(11)}$$

b) The uncertainty u_{CRM} is a 2-Sigma uncertainty, therefore

$$u_{CRM(1s)} = \frac{u_{CRM(2s)}}{2} \qquad \textbf{(12)}$$

c) The uncertainty of $\overline{H}$ (mean value of users machine on CRM)

$$u_{\overline{H}} = \frac{t\, s_H}{\sqrt{n}} \qquad \textbf{(13)}$$

t ... Student coefficient for ‘n’ and P = 68.27%

d) The uncertainty of $\bar{x}$ (measurements on a material),

$$u_{\bar{x}} = \frac{t\, s_x}{\sqrt{n_1}} \qquad \textbf{(14)}$$

t ... Student coefficient for ‘n_1’ and P = 68.27%

n_1 ... at least 5 repetitions; Eqn. 14 would be valid for $n_1 \geq 2$

APPENDIX B

A Worked Example for Estimating Uncertainty in Hardness Measurement According Rockwell B Scale

B1. Introduction

A customer asks the laboratory to obtain the Rockwell B hardness of a cold rolled steel sheet tested according to Eurocode EN ISO 6508-1. The laboratory has a certified Rockwell hardness testing machine, which has been verified both with the direct and indirect verification according to Eurocode EN ISO 6508-2. The sources of the uncertainty measured on the standardized hardness blocks are known, see B3 Item 1.

B2. Testing conditions

Temperature: 23°C
Testing machine: Verified Rockwell hardness tester according EN ISO 6508
Automatic mode (zeroing, preload, holding time etc.)
CRM: No. 43 2001 7 97 - MPA NRW 12116.497
Test specimen: Cold rolled steel; thickness: 2.1mm

B3. Example of Uncertainty Calculations and Reporting of Results

All calculations are based on the formulae in Appendix A. Every table is produced for a certain measured or evaluated quantity. The worked example shows the procedure concerning Rockwell B hardness.

1. Certified value of the CRM

X_{CRM} = 63.4 HRB
u_{CRM} = ± 0.5 HRB (2-Sigma uncertainty)

2. Determination of users machine repeatability

Results of VASL-Rockwell test machine

a) Mean value (Eqn. 3)

$$\overline{H} = \frac{63.0 + 63.0 + 63.2 + 63.2 + 63.3}{5}$$

$$\overline{H} = 63.1\ \text{HRB}$$

b) Repeatability according EN ISO 6508 (Eqn. 4a)

$$H_5 - H_1 = 0.3\,\text{HRB}$$

c) Empirical standard deviation of a single measurement (Eqn. 5)

$$s_H = 0.13\,\text{HRB}$$

3. Assessment According EN ISO 6508

Hardness range of the reference block: >45 HRB to ≤ 80 HRB
Permissible error Rockwell units: ± 3 HRB (2-Sigma)
Permissible repeatability of testing machine: $\leq 0.04 * (130 - \overline{\text{H}})$ or $1.2\,\text{HRB}$

a) Estimation of the error (E) of the VASL-Rockwell test machine (Eqn. 6)

$$E = 63.4 - 63.1$$
$$E = 0.3HRB$$

$$E \leq E_{2s}$$
$$E < 3HRB$$
Table 5 of EN ISO 6508-2

b) Estimation of the repeatability value of the VASL-Rockwell test machine

$$H_5 - H_1 \leq 0.04 \times (130 - \overline{H}) \text{ or } 1.2HRB$$
$$H_5 - H_1 \leq 0.04 \times (130 - 63.1)$$
$$H_5 - H_1 \leq 2.68HRB$$

$$H_5 - H_1 = 63.3 - 63.0 = 0.3HRB < 2.68HRB$$

The VASL-Rockwell test machine satisfies the indirect verification requirement of EN ISO 6508-2. For this example we inspect only the range >45 HRB to ≤ 80 HRB.

4. Measure on a Cold Rolled Material by 5 Repetitions

a) Mean value (Eqn. 7)

$$\overline{x} = \frac{64.4 + 64.8 + 64.9 + 65.1 + 65.3}{5}$$
$$\overline{x} = 64.9HRB$$

b) Empirical standard deviation of a sinlge value (Eqn. 8)

$$s_x = 0.34 \text{ HRB}$$

5. Uncertainty Estimation

a) Permissible error (Eqn. 11)

$$E_{1s} = \frac{E_{2s}}{2}$$
$$E_{1s} = \frac{3}{2}$$
$$E_{1s} = 1.5HRB$$

b) Uncertainty u_{CRM} (Eqn. 12)

$$u_{CRM(1s)} = \frac{u_{CRM(2s)}}{2}$$
$$u_{CRM(1s)} = \frac{0.5}{2}$$
$$u_{CRM(1s)} = 0.25HRB$$

c) Uncertainty of $\overline{\text{H}}$ (Eqn. 13)

$$u_{\overline{H}} = \frac{t\ s_H}{\sqrt{n}}$$
$$u_{\overline{H}} = \frac{1.15 \times 0.13}{\sqrt{5}}$$ Probability, P = 68.27%
$$u_{\overline{H}} = 0.07HRB$$

d) Uncertainty of $\overline{\text{x}}$ (Eqn. 14)

$$u_{\bar{x}} = \frac{t\ s_x}{\sqrt{n_1}}$$
$$u_{\bar{x}} = \frac{1.15 \times 0.34}{\sqrt{5}}$$ Probability, P = 68.27%
$$u_{\bar{x}} = 0.17HRB$$

Combined uncertainty (Eqn. 9)

$$u^2_{C(\bar{x})} = E^2_{1s} + u^2_{CRM(1s)} + u^2_{\bar{H}} + u^2_{\bar{x}}$$
$$u^2_{C(\bar{x})} = 1.5^2 + 0.25^2 + 0.07^2 + 0.17^2$$
$$u^2_{C(\bar{x})} = 2.35$$
$$u_{C(\bar{x})} = \pm 1.53 HRB$$

Expanded uncertainty (Eqn. 10)

$$u_{e(\bar{x})} = k\ u_{C(\bar{x})}$$
$$u_{e(\bar{x})} = 2 \times 1.53$$
$$u_{e(\bar{x})} = \pm 3.06 HRB$$

Comments:

- In this CoP "E" is not used as a systematic error - therefore no correction of "$\bar{x}$" by "E". It is important for the assessment according the standards and for long-term studies (e.g. control chart according ISO 8258 "*Shewhart control charts*"
- In the case of a long-term study the mean value of "E" exists, and its empirical standard deviation can be used for uncertainty studies and for correction of systematic errors. $\bar{x}_{Corr} = \bar{x} + \bar{E}$ and $u^2_{C(\bar{x}_{corr})} = u^2_{\bar{E}} + u^2_{CRM(1s)} + u^2_{\bar{H}} + u^2_{\bar{x}} + ...$
- In both cases "E" shall not exceed the maximum permissible values given in the standards. Otherwise the test machine satisfies not the requirement of the standards and uncertainty studies are irrelevant.

TABLE B1 Uncertainty Budget Calculations for Rockwell hardness, HRB
(sensitivity coefficient is not dimensionless - see appendix A)

Source of uncertainty											
Symbol	**Measurands or evaluated quantities**	**Value**	**Symbol of uncertainty**	**Value**	**Type**	**Probability Distribution**	**Divisor dv**	**u(xi)**	**Sensitivity coefficient c_i**	**u(Xi)**	**v_i of v_{eff}**
X_{CRM}	Certified hardness value	63.4 HRB	u_{CRM}	±0.5 HRB	B	normal	2	±0.25 HRB	1	±0.25 HRB	∞
$\overline{H}$	Mean value of users machine on reference hardness block	63.1 HRB	$u_{\overline{H}}$	±0.07 HRB	A	normal	1	±0.07 HRB	1	±0.07 HRB	∞
E	Estimated error block	0.3 HRB	$E_{2\sigma}$	±3 HRB	B	normal	2	±1.5 HRB	1	±1.5 HRB	∞
$\overline{x}$	Mean value of a measured material	64.9 HRB	$u_{\overline{x}}$	0.17 HRB	A	normal	1	±0.17 HRB	1	±0.17 HRB	∞
			$u_{C(\overline{x})}$	**Combined uncertainty**	A+B	normal	±2.36%			**±1.53 HRB**	∞
			$u_{e(\overline{x})}$	**Expanded uncertainty**	A+B	normal	**k = 2**		±4.71%	**±3.06 HRB**	∞

B4. Reported Results

$$\overline{x} = 64.9 \text{ HRB} \pm 3.06 \text{ HRB} \quad (\pm 4.71\ \%)$$

The above reported expanded uncertainties are based on standard uncertainties multiplied by a coverage factor k=2, providing a level of confidence of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT recommendations.