***Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials***

***Code of Practice No. 11***

# The Determination of Uncertainties in Notched Bar Creep-Rupture Testing

# (to ASTM E292-83 and ESIS TC11 Code of Practice)

**D J Foster**

Mitsui Babcock Technology Centre
High Street, Renfrew PA4 8UW
Scotland
UNITED KINGDOM

**Issue 1**

September 2000

**CONTENTS**

# PROCEDURE FOR THE DETERMINATION OF UNCERTAINTIES IN NOTCHED BAR CREEP RUPTURE TESTING

## 1. SCOPE

The procedure in this document describes the method to evaluate the uncertainties associated with the quantities measured in a notched bar creep rupture test carried out according to the testing standard: -

ASTM E292-83: Conducting Time-for-Rupture Notch Tension Tests of Materials [2]

The use of notched specimens is also covered by the draft "Code of Practice for Conducting Notched-Bar Creep-Rupture Tests and Interpretation of Data", issued by committee TC11 of the European Structural Integrity Society (ESIS) [3]. The terminology, symbols and abbreviations declared in this Code have generally been used throughout the following text for clarity. Certain technical matters discussed in the Code, e.g. analysis of triaxial stress fields and skeletal points, are beyond the scope of current uncertainty procedures. The present document will deal only with standard creep test data.

This Code of Practice is aimed at determination of uncertainties in the results of a single notched specimen creep rupture test, although several plain specimen results at different stresses may be used to estimate representative stress and equivalent plain specimen rupture time. The effect of repeating a notched test under nominally the same conditions, or at a series of net stresses or temperatures, is discussed in Ref. [1] Section 4.

## 2. SYMBOLS

*Specimen Dimensions*

| | |
|---|---|
| $d_{n0}$ | initial diameter at notch plane |
| $d_{nu}$ | minimum diameter after rupture |
| $S_{n0}$ | initial cross-sectional area at notch plane |
| $S_{nu}$ | minimum after-rupture cross-sectional area |

*Loads and Stresses*

| | |
|---|---|
| P | load on test piece |
| $\sigma_{net}$ | initial net stress at notch plane |
| $\sigma_0$ | initial stress in plain specimen creep rupture test |

*Test Results*

| | |
|---|---|
| $R_L$ | notch rupture life ratio |
| $R_S$ | notch strength ratio |
| $t_{nu}$ | rupture life of notched specimen |
| $Z_{nu}$ | percent reduction of area after rupture |

*Other Parameters*

| | |
|---|---|
| A | constant in strain rate vs. stress / temperature equation |
| $c_i$ | ratio of measurand uncertainty to input i uncertainty |
| $d_v$ | divisor to convert Type B error to standard uncertainty |
| n | creep stress exponent |
| Q | creep activation energy |
| R | gas constant |
| $\sigma_{rep}$ | representative stress (giving plain specimen $t_u = t_{nu}$) |
| $t_{n0}$ | time when rupture first recorded |
| $t_{pu}$ | plain specimen rupture life at $\sigma_0=\sigma_{net}$ & same temperature |
| T | test temperature |

## 3. INTRODUCTION

There are requirements for test laboratories to evaluate and report the uncertainty associated with their test results. Such requirements may be demanded by a customer who wishes to know the bounds within which the reported result may be reasonably assumed to lie; or the laboratory itself may wish to understand which aspects of the test procedure have the greatest effect on results so that this may be monitored more closely or improved. This Code of Practice has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4 - CT97-2165, in order to simplify the way in which uncertainties are evaluated. It is hoped to avoid ambiguity and provide a common format readily understandable by customers, test laboratories and accreditation authorities.

This Code of Practice is one of seventeen prepared and tested by the UNCERT consortium for the estimation of uncertainties in mechanical tests on metallic materials. These are presented in Ref. [1] in the following Sections:

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials.

This CoP can be used as a stand-alone document. Nevertheless, for background information on measurement uncertainty and values of standard uncertainties of devices commonly used in materials testing, the user may wish to refer to the relevant section in Reference [1]. A number of sources of uncertainty, such as the reported tolerance of load cells, extensometers, micrometers and thermocouples, are common to several mechanical tests and are included in Section 2 of Ref. [1]. These are not discussed here to avoid needless repetition. The individual procedures are kept as straightforward as possible by following the same structure:

The main procedure
Fundamental aspects and calculation formulae for that test type
A worked example

This document guides the user through several steps to be carried in order to estimate the uncertainties in creep test results. The general process for calculating uncertainty values is described in Ref. [1].

## 4. PROCEDURE FOR ESTIMATING UNCERTAINTIES IN NOTCHED BAR CREEP RUPTURE TESTS

### Step 1. Identification of the Measurands for Which Uncertainty is to be Determined

The first Step consists of setting the measurands, i.e. the quantities which are to be presented as results from the test. Table 1 lists the measurands and the intermediate quantities used in their derivation. In a particular test, not all the quantities will be determined. For example, notch life ratio and notch strength ratio may not be required.

**Table 1.** Intermediate results, measurands, their units, and symbols

| Measurands | Unit | Symbol |
|---|---|---|
| **Intermediate results** | | |
| net stress | MPa | $\sigma_{net}$ |
| initial cross-sectional area at notch plane | mm$^2$ | $S_{n0}$ |
| final cross-sectional area at notch plane | mm$^2$ | $S_{nu}$ |
| **Test measurands** | | |
| creep rupture time | h | $t_{nu}$ |
| % reduction of area at notch plane after creep rupture | | $Z_{nu}$ |
| notch strength ratio | | $R_S$ |
| notch rupture life ratio | | $R_L$ |

The measurands are calculated from the following formulae.

*Cross-sectional Areas at Notch Plane*

$$S_{n0} = \pi\ d_{no}^2 / 4 \qquad S_{nu} = \pi\ d_{nu}^2 / 4$$
(1a, 1b)

*Net Stress* $\sigma_{net}$

$$\sigma_{net} = P / S_{n0} \tag{2}$$

*Percent Reduction of Area* $Z_{nu}$

$$Z_{nu} = 100\ (S_{n0} - S_{nu}) / S_{n0} = 100\ (1 - S_{nu} / S_{n0}) \tag{3}$$

*Creep Rupture Time* $t_{nu}$

If specimen load or displacement is logged at intervals of time $\Delta t$, a rupture recorded at time $t_0$ is equally likely to have occurred at any time within the interval ($t_0 - \Delta t$, $t_0$), with expectation

$$t_{nu} = t_0 - \Delta t/2 \tag{4}$$

This calculation will not arise if the specimen load or extension is recorded continuously.

*Notch Strength Ratio* $R_S$

$$R_S = \sigma_{net} / \sigma_{rep} \tag{5}$$

where $\sigma_{rep}$ is the representative stress, which on a plain specimen gives the same rupture time as the notched specimen at $\sigma_{net}$ .

*Notch Rupture Life Ratio* $R_L$

$$R_L = t_{nu} / t_{pu} \tag{6}$$

where $t_{pu}$ is the plain specimen rupture time under stress $\sigma_0 = \sigma_{net}$.

**Step 2. Identification of all Sources of Uncertainty**

In Step 2, the user identifies all possible sources of uncertainty which may have an effect on the test. This list cannot be exhaustively identified beforehand as it is uniquely linked to the test laboratory's test method and the apparatus used. Therefore a new list shall be drafted each time one test parameter changes (when a plotter is replaced by a computer and printer for example)., Five categories have been defined to help the user list all sources of uncertainty. The following table (Table 2) gives the five categories and examples of sources.

It is important to note that this table is NOT exhaustive. Other sources can contribute to uncertainties depending on specific testing configurations. Users are encouraged to draft their own list corresponding to their own test facilities.

**Table 2** Sources of uncertainty and their likely contribution to uncertainties in measurands

(1 = major contribution, 2 = minor contribution, blank = no contribution)

| Source | Affected Measurand | | | | | | |
|---|---|---|---|---|---|---|---|
| | $S_{no}$ | $S_{nu}$ | $s_{net}$ | $Z_{nu}$ | $t_{nu}$ | $R_S$ | $R_L$ |
| **Test Piece** | | | | | | | |
| initial notch root diameter | 1 | | 1 | 1 | | 1 | 1 |
| notch root diameter after rupture | | 1 | | 1 | | | |
| shape tolerance, notch geometry | | | 2 | 2 | 2 | 2 | 2 |
| **Apparatus** | | | | | | | |
| load cell or lever / weights | | | 1 | 1 | 1 | 1 | 1 |
| specimen temperature | | | | 1 | 1 | 1 | 1 |
| bending stresses | | | | 2 | 2 | 2 | 2 |
| **Environment** | | | | | | | |
| control of creep lab temperature | | | | 2 | 2 | 2 | 2 |
| **Method** | | | | | | | |
| data logger time interval | | | | | 1 | | 1 |
| value for $\sigma_{rep}$ | | | | | | 1 | |
| value for $t_{pu}$ | | | | | | | 1 |
| **Operator** | | | | | | | |

**Step 3. Classification of all Sources According to *type A* or *B***

In accordance with ISO TAG 4 'Guide to the Expression of Uncertainties in Measurement' [4], sources of uncertainty can be classified as *Type A* or *B*, depending on the way their influence is quantified. If a source's influence is evaluated by statistical means (from a number of repeated observations), it is classified *Type A*. If a source's influence is evaluated by any other means (manufacturer's documents, certification, ...), it is classified *Type B*.

Attention should be drawn to the fact that one same source can be classified as *type A* or *B* depending on the way it is estimated. For instance, if the diameter of a cylindrical specimen is measured once, or taken from the drawing, that parameter is considered *Type B*. If the mean value of ten consecutive measurements is taken into account, then the parameter is *Type A*.

**Step 4. Estimation of the Sensitivity Coefficient and Standard Uncertainty for Each Source**

In this step the standard uncertainty of the measurand, $u_i$, for each input source $x_i$, is calculated according to the input quantity uncertainty, the probability distribution and the sensitivity coefficient, $c_i$.

Appendix A1 describes the derivation of the standard uncertainties for the primary measured quantities and sources of uncertainty in a creep test, as listed in Tables 1 and 2.

The standard uncertainty ( $u(x_i)$ ) of the input quantity $x_i$ is defined as one standard deviation and is derived from the uncertainty of the input quantity. For a *Type A* uncertainty, the uncertainty value is not modified. For *Type B*, it is divided by a number, $d_v$, associated with the assumed probability distribution. The divisors for the distributions most likely to be encountered are given in Chapter 2 of Ref. [1].

The contribution ($u_i$) to the standard uncertainty of the measurand due to the individual input quantity's standard uncertainty $u(x_i)$, is found by multiplying $u(x_i)$ by the sensitivity coefficient $c_i$. This is derived from the relationship between output (measurand, y) and input quantities ($x_i$), and is equal to the partial derivative, i.e.

if $y = F(x_1, x_2,...)$ where F denotes some function
then $c_i = \partial y / \partial x_i$
and $u_i = c_i \, u(x_i)$

**Step 5. Calculation of Combined Uncertainties of Measurands**

Assuming that the N individual uncertainty sources are not correlated, the measurand's combined uncertainty, $U_c(y)$, can be computed in a root sum squares manner:

$$U_c(y) = \sqrt{\sum_{i=1}^{N} [c_i \cdot u(x_i)]^2}$$

This uncertainty corresponds to plus or minus one standard deviation on the normal distribution law representing the studied quantity.

It will be seen that, in some cases, the uncertainty of an input quantity is itself a combined uncertainty from an earlier stage in the calculations. For example, uncertainty of rupture time $u(t_{nu})$ depends on *(inter alia)* the uncertainty in net stress $u(\sigma_{net})$. This in turn depends on load and cross-sectional area, and the latter again on initial diameter, which is one of the primary measured quantities.

In the detailed procedure for calculating combined uncertainties (in Appendix A2) and the illustrative worked example (Appendix B), these successive uncertainties are calculated separately in sequence.

### Step 6. Computation of the Expanded Uncertainty $U_e$

This Step is optional and depends on the requirements of the customer. The expanded uncertainty $U_e$ is broader than the combined uncertainty $U_c$, but in return the confidence level increases. The combined uncertainty $U_c$ has a confidence level of 68.27%. Where a high confidence level is needed (aerospace, electronics, ...), the combined uncertainty $U_c$ is broadened by a coverage factor k to obtain the expanded uncertainty $U_e$. The most common value for k is 2, which gives a confidence level of approximately 95%. If $U_c$ is tripled (K = 3) the corresponding confidence level rises to 99.73%.

Standard worksheets can be used for calculation of uncertainties, and an example is shown in Table 3 below. In creep tests, the sensitivity coefficients giving the uncertainty in a measurand due to the uncertainties in the primary measured quantities (diameter, temperature, load etc.) are rather complicated, and there has to be a series of intermediate steps, as illustrated in the worked example in Appendix B.

The formulae for calculating the combined uncertainty in Table 3 are given in Appendix A2 (Eqs. (10b), (10d), (12)).

**Table 3.** Typical worksheet for uncertainty budget calculations in estimating the uncertainty in percentage reduction of area $Z_{nu}$

| **source of uncertainty (type)**<br>**intermediate result** | **sym-bol** | **value** | **uncer-tainty** | **prob. distrib.** | **divi-sor** | $c_i$ | $u_i$ |
|---|---|---|---|---|---|---|---|
| initial notch root diameter (A) | $d_{n0}$ | | | normal | 1 | $\pi d_{n0}/2$ | |
| initial cross-sectional area | $S_{n0}$ | Eq 1a | | | | | Eq 10b |
| after-rupture diameter (A) | $d_{nu}$ | | | normal | 1 | $\pi d_{nu}/2$ | |
| final cross-sectional area | $S_{nu}$ | Eq.1b | | | | | Eq 10d |
| **test result** | | | | | | | |
| Percent reduction in area<br>combined standard uncertainty | $Z_{nu}$<br>$u_c$ | Eq 3 | | normal | | | Eq 12 |
| expanded uncertainty | $U_e$ | | | normal | | | |

**Step 7. Presentation of Results**

Once the expanded uncertainty has been chosen, the final result can be given in the following format:

$$\mathbf{V = y \pm U \text{ with a confidence level of } X\%}$$

where V is the estimated value of the measurand, y is the test (or measurement) mean result, and U is the expanded uncertainty.

The results would therefore be presented in the form shown below. These are the basic test results, and the Standard lists other information required in the test report.

*The results of a test conducted according to ASTM E292-83 on sample XYZ123, with a confidence interval of 95% are:*

| | |
|---|---|
| *Test Conditions:* | |
| *Net Stress* | $\sigma_{net}$ |
| *Temperature* | $T$ |
| | |
| *Test Results:* | |
| *Notch Rupture Time* | $t_{nu} \pm U_e(t_{nu})$ |
| *Reduction in area* | $Z_{nu} \pm U_e(Z_{nu})$ |
| *Notch Strength Ratio* | $R_s \pm U_e(R_s)$ |
| *Notch Life Ratio* | $R_L \pm U_e(R_L)$ |

## 5. REFERENCES

(1) *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0 946754 41 1, Issue 1, September 2000.

(1a) *Code of Practice no.10: Determination of Uncertainties in Uniaxial Creep Testing of Metallic Materials to European Standard prEN 10291.*

(2) American Society for Testing and Materials - Designation E292-83 (re-approved 1990): *Conducting Time-for-Rupture Notch Tension Tests on Materials.*

(3) European Structural Integrity Society TC11, Eds. G A Webster, S R Holdsworth, and M S Loveday, I J Perrin & H Purper: *Code of Practice for Conducting Notched Bar Creep Rupture Testing*. (March 1998).

(4) BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML: *Guide to the Expression of Uncertainty in Measurement*. ISO (1993) (known as the "TAG4 Guide").

(5) European Creep Collaborative Committee (Working Group 1) *Data Validation and Assessment Procedures* (Vol. 3 Issue 2, Annex 1 Ch. 2.6).

(6) Brown W F, Jones M H, and Newman D P: *Influence of Sharp Notches on Stress-Rupture Characteristics of Several Heat-Resisting Alloys*. ASTM STP 128 p. 25 (Philadelphia Pa. 1953).

(7) ASM International. Ed. H E Boyer: *Atlas of Creep and Stress-Rupture Curves (*1988).

# APPENDIX A1

## Calculation of Uncertainties in Measured Quantities

### A1.1 Introduction

The reason for classifying sources as *type A* or *B* is that each type has it's own method of quantification. By definition, a *type A* source of uncertainty is already a product of statistical computation. The calculated influence is thus left as is.

*Type B* sources of uncertainty can have various origins: a manufacturer's indication, a certification, an expert's estimation or any other means of evaluation. For *type B* sources, it is necessary for the user to estimate the most appropriate (most probable) distribution of the parameter for each source within its uncertainty range (further details are given in Section 2 of Ref. [1]). A correction factor $d_v$ is required dependent on the distribution model chosen in order to compute the standard uncertainty $u_s$ for each *type B* source.

**TYPE A** : Statistically computed influence

A *Type A* uncertainty is most often computed from a set of N repeated measurements of the required quantity x. This then gives a mean value $\mu$ for x, and *Type A* uncertainty estimate:

standard uncertainty $u_s(x)$ = (standard deviation) / $\sqrt{N}$

$$= \frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{N}\frac{(x_i-\mu)^2}{N-1}}$$

The standard deviation is denoted by STDEV in Microsoft Excel, and often by $\sigma_{n-1}$ on hand calculators. (Note the use of $\sigma$ here for standard deviation, and elsewhere for stress).

A *Type A* uncertainty can also be calculated when the value of a dependent variable y is measured at a series of values of an independent variable x, and an estimate of Y, the value of y when x = X, is derived by regression analysis. In the worked example in Appendix B, the representative stress and equivalent plain specimen rupture time, and their uncertainties, are calculated from a series of plain specimen rupture time/stress data points.

**TYPE B**: Standard uncertainty $u_s$ made from the given uncertainty u divided by a correction factor $d_v$ given in Section 2 of Ref. [1] : $u_s = u / d_v$

The most common distribution model for *Type B* uncertainties is the rectangular, which means that the “true” value of a measured quantity is equally likely to have any value within the range ($\mu \pm u$), where $\mu$ is the mean value. The probability of the value lying between x and ($x+\delta x$) is

$\delta x/(2u)$, i.e. independent of x, and from the definition of standard deviation for a continuous variable it can be shown that

$$u_s = u/\sqrt{3}$$

i.e. in this case, $d_v = \sqrt{3}$

Turning now to the Sources of Uncertainty listed in Table 2, we demonstrate how the typical uncertainties in individual measured quantities, differences in apparatus etc. lead to each source's influence. It is assumed that measuring equipment has been appropriately calibrated, and that a test procedure has been written following the standard which minimises typical measurement errors, and that the procedure is being applied by a trained operator.

**Important Note:** Some *Type B* errors have been calculated in the following discussion and the worked example in Appendix B. It will be self-evident to the capable laboratory how *Type A* errors on individual measurements can lead to smaller uncertainties for an individual test piece, or how repeat measurements on several test pieces can lead to the calculation of *Type A* errors directly on the evaluated quantities (see also Section 4 of Ref. [1]).

### A1.2 Uncertainties in Measuring the Test Specimen

The diameter at the notch root, $d_{n0}$ , and its uncertainty, may be taken from the specimen drawing provided the specimen has been checked and certified to be within the machining tolerance of $\pm$ s. ESIS TC11 [3] specifies $\pm$0.03mm. The uncertainty is then *Type B* with rectangular distribution assumed, and its standard uncertainty $u(d_{n0})$ is equal to $s / \sqrt{3}$.

Alternatively, notch root diameter can be measured on a number of different diameters, probably with a travelling microscope. This then gives a *Type A* estimate, $u_1(d_{n0})$, which should be less than the *Type B* value. The method of calculation is shown on the previous page.

The diameter values must also comply with the tolerance requirement of the Standard.

The measuring instrument will also have its own uncertainty $u_m$, available from the manufacturer's specification or periodical calibrations. If this is given as a maximum error, a rectangular distribution may be assumed, giving a *Type B* standard uncertainty

$$u_{sm} = u_m / \sqrt{3}$$

This is combined with the standard uncertainty of the set of measurements by quadratic summation:

$$u(d_{n0}) = \sqrt{[\,(u_1(d_{n0}))^2 + (u_{sm})^2\,]} \qquad (7)$$

Notch root diameter uncertainty affects the uncertainty of cross-sectional area (Eq. (1)), and hence net stress through Eq. (2) and reduction of area through Eq. (3). Uncertainty in net stress is reflected in the uncertainties of notch strength ratio (Eq. (5)) and rupture time, the latter also being a component of notch rupture life ratio (Eq. (6)).

Variation in root diameter affects the net stress and is considered in the previous paragraph. However, the components of the triaxial stress field will not vary in simple proportion to net stress, and will also depend on root radius and notch angle. Notch geometry is specified for a test, for example to match a component with a stress-raising notch operating in the creep range. There is very little information on how small variations in notch parameters affect creep rupture lives, and for the present it will be assumed that variations within the test specification do not add significantly to rupture life uncertainty.

The variability uncertainty in after-rupture notch root diameter $d_{nu}$ is *Type A*, equal to (sample standard deviation of N measurements) / $\sqrt{N}$. To this is added the *Type B* measuring instrument error as described above for $d_{n0}$. This uncertainty affects reduction in area through Eq. (3). The diameter should be measured on different diameters with a travelling microscope or shadowgraph. Separating and reassembling the two pieces each time would ensure independent measurements, but this procedure may be judged to be too time-consuming, and also will cause damage to fracture surfaces which might be needed for scanning electron microscopy. Mounting the specimen pieces in rotating centres would speed up the measurements.

One or two measurements will not allow an estimate of uncertainty in reduction of area. If the fracture is not circular, several measurements of minimum diameter must be made.

### A1.3 Uncertainty in Measuring Rupture Time

If specimen load or displacement is logged at intervals of time $\Delta t$, a rupture recorded at time $t_0$ is equally likely to have occurred at any time within the interval ($t_0$ - $\Delta t$, $t_0$ ), with the expectation:

$$t_{nu} = t_0 - \Delta t/2$$

The maximum error in recording $t_{nu}$ is then a *Type B*:

$$e_r(t_{nu}) = \Delta t/2$$

and the initial determination of standard uncertainty of $t_{nu}$ is:

$$u_m(t_{nu}) = e_r(t_{nu})\sqrt{3} = \Delta t/(2\sqrt{3}) \qquad (8)$$

The uncertainty in $t_0$ is assumed to be negligible.

The subscript m in $u_m(t_{nu})$ denotes that the expression is an initial estimate based on uncertainties in the data used in the direct calculations. The influences of uncertainties in other quantities must then be added.

Rupture time is involved in the calculation of notch life ratio (Eq. (6)).

### A1.4 Uncertainties in Measuring Load

This is a *Type B* estimate derived from the calibration certificate of the load cell or lever system. It may be quoted as a percentage of the test load, or as a percentage of the maximum load (of a load cell). The value must be converted to an absolute uncertainty, or percentage of the test load in the latter case. A rectangular distribution is again assumed, so the standard uncertainty u(P) is $e_P / \sqrt{3}$ , where $\pm$ $e_P$ is the certified maximum error in load.

Load uncertainty affects uncertainties in net stress (Eq. (2)) and rupture time (Appendix F), and hence also the uncertainties in both notch strength ratio and notch life ratio.

### A1.5 Uncertainty in Temperature

Temperature uncertainty u(T) will normally be *Type B*, combining standard uncertainties of the thermocouple and control system, and variation between the thermocouples at different positions on the specimen, by the usual root sum-square method.

Let the maximum errors be:

| | |
|---|---|
| for the main measurement thermocouple | $e_{Tm}$ |
| for the within-specimen uniformity | $e_{Tu}$ |
| for the measuring system | $e_{Tc}$ |

Then the maximum expected error in temperature is

$$e_T = \sqrt{[e_{Tm}^2 + e_{Tu}^2 + e_{Tc}^2]} \qquad (9)$$

with a rectangular distribution, and its standard uncertainty is

$$u(T) = e_T/\sqrt{3} \qquad (9a)$$

Temperature affects rupture time, and the derivation of resulting uncertainties in $t_{nu}$ is given in Appendix F.

## A1.6 Effect of Environment

Variation in ambient temperature may affect specimen temperature control. The extent of this should be determined, and added to temperature uncertainty if it is significant.

## A1.7 Effect of Bending Stresses

Non-coaxiality of gauge length, threads and straining rods causes bending stresses. ECCC WG1 Issue 2 [5] recommends that these be less than 0.2 $\sigma_0$ (on plain specimens implied). The WG recommend that the influence on these stresses be investigated, but to assume that rupture time and ductility will not be affected provided they remain within the stated limit. ASTM E292 require that strain gauge measurements at room temperature give bending stress less than 10% of mean axial stress at the lowest load used in rupture tests.

It is likely that bending stresses will have greater effect with notched specimens than with plain ones. The requirement in various test standards to "minimise bending" should be rigorously observed in notched bar tests.

## A1.8 Effect of Method

The influence of data logging interval on rupture time uncertainty has been considered above. The time interval between data records should be not more than 1% of the expected test duration.

Plain bar rupture time $t_{pu\ at}$ stress $\sigma_0 = \sigma_{net}$ may have been found in separate tests, in which case its uncertainty will also be obtained. It is unlikely that a plain specimen will have been tested at exactly the stress to give the notched bar rupture time, and hence the representative stress. Appendices D and E show how to determine representative stress and plain bar rupture time, together with their uncertainties, from results of creep tests at the same temperature.

# APPENDIX A2

## Formulae for the Calculation of the Measurands' Combined Uncertainties

A worked example is presented in Appendix B to illustrate the following operations

### A2.1 Calculation of Uncertainties in Cross-sectional Areas

Following from Eqs. (1a), (1b):

relative uncertainty: $$\frac{u(S_{n0})}{S_{n0}} = 2\frac{u(d_{n0})}{d_{n0}} \quad (10a)$$

absolute uncertainty: $$u(S_{n0}) = \frac{\pi\, d_{n0} u(d_{n0})}{2} \quad (10b)$$

Also $$\frac{u(S_{nu})}{S_{nu}} = 2\frac{u(d_{nu})}{d_{nu}} \quad \text{or} \quad u(S_{nu}) = \frac{\pi\, d_{nu} u(d_{nu})}{2} \quad (10c, 10d)$$

### A2.2 Calculation of Uncertainty in Net Stress

From Eq. (2), this is the result of uncertainties in load and initial cross-sectional area:

$$\frac{u_c(\sigma_{net})}{\sigma_{net}} = \sqrt{\left(\frac{u(P)}{P}\right)^2 + \left(\frac{u(S_{n0})}{S_{n0}}\right)^2} \quad (11)$$

$u(S_{n0})/S_{n0}$ having been determined in Eq. (10a).

### A2.3 Calculation of Uncertainty in Reduction of Area

The treatment below considers only uncertainties in specimen measurements. In general, increasing temperature or stress gives shorter rupture time and higher ductility and, in principle, an uncertainty component could be calculated for reduction in area due to uncertainties in stress and temperature. However, the effect coefficients $\partial Z/\partial \sigma$ etc. are not generally known or theoretically modelled, but are believed to be relatively small. This contrasts with creep rates and rupture times which have a high stress exponent and exponential temperature dependence.

Since Eq. (3) is not a simple sequence of terms added or multiplied together, the formula for uncertainty is a little more complex (see Appendix F):

$$\frac{u_c(Z_{nu})}{100} = \sqrt{\left(\frac{S_{nu}u(S_{n0})}{S_{n0}^{\;2}}\right)^2 + \left(\frac{u(S_{nu})}{S_{n0}}\right)^2} \qquad (12)$$

### A2.4 Calculation of Uncertainty in Rupture Time

The uncertainty $u_m(t_{nu})$ due to the length of the data-logging period, if applicable, has been given by Eq. (8).

The rupture time $t_{nu}$ is related to stress $\sigma_{net}$ and absolute temperature T (see Appendix F), and the additional components of uncertainty in $t_{nu}$ are then given by:

$$\frac{u_s(t_{nu})}{t_{nu}} = \frac{n\,u(\sigma_{net})}{\sigma_{net}} \qquad \text{for stress} \qquad (13a)$$

and

$$\frac{u_T(t_{nu})}{t_{nu}} = \frac{Q\,u(T)}{RT^2} \qquad \text{for temperature} \qquad (13b)$$

The values of n and Q can be assumed to be the same as for plain specimens, taken from existing data on a metallurgically similar material. Appendix C lists typical values for four classes of alloys, and these may be used if no other data are available.

n and Q can be determined from a series of notch rupture tests at different net stresses and temperatures, and by performing linear regression analysis of $\log(t_{nu})$ against $\log(\sigma_{net})$, or $\log(t_{nu})$ against (1/T).

The sensitivity coefficients for stress and temperature can be found by performing four extra tests, two at temperature T with net stresses above and below $\sigma_{net}$, and two at $\sigma_{net}$ with temperatures above and below T. The varied parameter should differ from the central value by at least five times its standard uncertainty.

The ratio of the change in rupture time to change of parameter (net stress or temperature) gives the sensitivity coefficient c() for the parameter, and:

$$u_s(t_{nu}) = c(\sigma_{net}) * u(\sigma_{net}) \qquad (13c)$$
$$u_T(t_{nu}) = c(T) * u(T) \qquad (13d)$$

The combined uncertainty in rupture life is then the combination of the three components:

$$u_c(t_{nu}) = \sqrt{\{ [u_m(t_{nu})]^2 + [u_s(t_{nu})]^2 + [u_T(t_{nu})]^2 \}} \qquad (14)$$

**A2.5 Calculation of Uncertainty in Notch Strength Ratio**

Calculations of net stress $\sigma_{net}$ and its uncertainty $u(\sigma_{net})$ are given above, in Eqs. (2) and (11).

Methods for determining representative stress $\sigma_{rep}$ and its uncertainty $u(\sigma_{rep})$ are given in Appendix D, and illustrated in the worked example in Appendix B.

From Eq. (5), uncertainty of $R_S$ is given by

$$\frac{u_c(R_S)}{R_S} = \sqrt{\left(\frac{u_c(\sigma_{net})}{\sigma_{net}}\right)^2 + \left(\frac{u(\sigma_{rep})}{\sigma_{rep}}\right)^2} \qquad (15)$$

**A2.6 Calculation of Uncertainty in Notch Life Ratio**

The notched specimen rupture life $t_{nu}$ is compared with the plain specimen rupture life $t_{pu}$ obtained under stress $\sigma_0 = \sigma_{net}$ , at the same temperature.

i.e. $R_L = t_{nu} / t_{pu}$

The uncertainty $u(t_{nu})$ in $t_{nu}$ has been found previously through Eqs. (8), (13) and (14).

If the plain bar rupture life uncertainty is $u(t_{pu})$, the standard uncertainty of $R_L$ is given by:

$$\frac{u_c(R_L)}{R_L} = \sqrt{\left(\frac{u_c(t_{nu})}{t_{nu}}\right)^2 + \left(\frac{u(t_{pu})}{t_{pu}}\right)^2} \qquad (16)$$

If a plain specimen was tested at the required stress and temperature, the rupture life $t_{pu}$ and uncertainty $u(t_{pu})$ are determined by the test procedure and uncertainties Code of Practice for Creep Rupture Testing [1](a).

Appendices B and E give the method to estimate $t_{pu}$ and $u(t_{pu})$ from data at stresses spanning but not including $\sigma_{net}$.

# APPENDIX B

## Worked Example of Creep Test Uncertainties Calculation

### B1 Test Data

The following data are adapted from Brown et al. [6], with units converted to SI, and typical values for uncertainties assumed. This example is based on the results of a test on a 0.3% C 1.2.5%Cr 0.5% Mo 0.25% V steel, quenched and tempered. The specimen was machined according to a drawing with the following dimensions and tolerances (units mm)

diameter of plain specimens or at notch root 7.59/7.65

Test temperature 538°C

measurement thermocouple error $e_{Tm}$ = ± 0.5°C
along-specimen uniformity error $e_{Tu}$ = ± 1.5°C
measuring system error $e_{Tc}$ = ± 2.0°C

After the initial period of primary creep, specimen parameters were recorded at intervals of 0.1 hours up to 99.5 hours, then at intervals of 1 hour.

Notched specimen load 23530 N
Weights and lever arm certified to give specified load ±1%.
Ruptured at 127.5 h. $\Delta t$ = 1 h
Estimated rupture time = 127h.

Initial notch root diameter ($d_{n0}$) measurements (mm) were made using a travelling microscope with certified maximum error of ±0.003mm
7.610, 7.602, 7.626, 7.620, 7.628, 7.616, 7.654, 7.602, 7.606, 7.630
Mean = 7.619: standard deviation = 0.016
Standard error of mean = $0.016/\sqrt{10}$ = 0.005

Initial cross-sectional area $S_{n0}$ = 45.60mm²
Net stress $\sigma_{net}$ = 516 MPa.

The broken specimen gave the following 10 measurements of minimum diameter $d_{nu}$
7.57, 7.53, 7.61, 7.58, 7.57, 7.55, 7.60, 7.61, 7.52, 7.60
Mean = 7.574: standard deviation = 0.032
Standard error of mean = $0.032/\sqrt{10}$ = 0.010
Final cross-sectional area $S_{nu}$ = 45.05mm².

Tests on plain specimens

| data point number | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| stress $\sigma_0$ MPa | 558 | 516 | 491 | 439 |
| rupture time $t_u$ h | 33.0 | 84.8 | 139 | 372 |
| expanded uncertainty h * | 8 | 20 | 28 | |

* 95% confidence

The uncertainties (assumed values) are given to illustrate the two-points and single point methods. Expanded uncertainties are reported in the test results above, but standard uncertainties, half the expanded values, are used in the calculations below.

Appendix C gives values for (creep activation energy) / R of 43000K, and stress exponent n of 4.7 for this type of steel. The test data from which these results are taken actually give n = 10 by regression (Table B9) for plain specimens, and 10 or 11 between two points (Tables B11 and B12), although the exponent is approximately 4 for notched specimens. The value 4 is used for notched rupture life uncertainty in Tables B5 and B.6, and 10 or 11 in plain specimen regression and interpolation calculations (Tables B9, B10 and B11).

**B2 Calculation of Test Results and Uncertainties**

Note that the net stress of the notched specimen test (516MPa) was used for the second of the plain specimens, and its notched rupture time lies between the figures for the second and third plain specimens. Two-point interpolations therefore used the second and third points for representative stress, and first and third for rupture time. In the latter case, it would be preferred for the two points to be closer together.

These data and the calculation results are shown in the following tables, together with references to the equations used in each uncertainty evaluation.

The initial calculations are set out in Tables B1 to B6 below. These cover uncertainties in cross-sectional area, net stress, reduction in area and notch rupture time.

Tables B7 and B8 show the calculations for notch strength ratio and notch rupture life ratio. These require additional calculations based on the plain specimen data, given above in sections B6 and B7 and the spreadsheets (Table B9, B10, and B11) at the end of this Appendix.)

The results Tables B1 to B8 were completed by manual calculation. The Tables have been incorporated into a spreadsheet in Table B12, where only test data and uncertainties need to be entered; measurands and their uncertainties are calculated automatically. This may be obtained from the author in the same way as for the regression and interpolation spreadsheets, or made up by entering the text and formulae shown into a blank spreadsheet.

## B3. Presentation of Results

To obtain the usual 95% confidence, a coverage factor of 2 should be applied to the standard uncertainties in the table, as follows.

Notched creep rupture test on 0.3% C 1.2.5%Cr 0.5% Mo 0.25% V steel, quenched and tempered, tested at 538°C and net stress 516 MPa.

| | |
|---|---|
| Reduction in area | 1.2 ± 0.6 % |
| Rupture time | 127 ± 26 h |
| Notch strength ratio | 1.05 ± 0.06 (representative stress by linear regression of 4 plain specimen data points) |
| | 1.04 ± 0.06 (by interpolation between two plain specimen points) |
| Notch rupture life ratio | 1.61 ± 0.64 (plain rupture time by linear regression of plain specimen data) |
| | 1.61 ± 0.86 (by interpolation between two plain specimen points) |
| | 1.50 ± 0.46 (single plain specimen rupture life at $\sigma_0$ = notch test net stress) |

## B4 Notes

*B4.1 Relative and Absolute Uncertainties*

The left hand side of some equations for uncertainties of values consists of the ratio of the uncertainty to the value, i.e. u(x) / x = *expression.* In some cases, this ratio can be used in a subsequent calculation without evaluating the uncertainty itself.

For example, uncertainty in net stress (Eq. (10)) gives $u(\sigma_{net}) / \sigma_{net}$ in terms of $u(S_{n0}) / S_{n0}$ derived in Eq. (10a), and u(P) / P, where $S_{n0}$ = original cross-sectional area, P = load.

Uncertainties are given in the worksheet tables as absolute values in the same units as the quantity considered, or ratios (%) as appropriate, or both.

Uncertainties should be given in the final results table using the same units as the measurand.

*B4.2 Comments on Results and Method*

(1) It can be seen that the uncertainty of initial time measurement for rupture is insignificant compared with uncertainties due to stress and temperature variation.

(2) The reduction in area was very low (1.2%) and has a large relative uncertainty due to its dependence on the difference between two similar values, each subject to relatively small uncertainty. A *Type B* uncertainty for initial notch diameter from the machining specification could have made the reduction in area estimate worthless.

The combination of large uncertainties in rupture times also makes notch life ratio of doubtful value.

(3) The importance of extensive information from plain and notched specimens at different stresses has been shown by independent estimates of stress exponents. Tests at different temperatures would have also given the activation energy, which here had to be given an assumed value.

(4) The following Tables B1 to B8 show the results of calculations, which were, performed by hand calculator, except for linear regression analyses and interpolations, which were done via spreadsheets. Tables B1 to B8 have also been combined in a spreadsheet in Table B12, which can be used to calculate many results automatically.

*B4.3 Notes on the Use of Spreadsheets*

(1) The use of spreadsheets simplifies and speeds up calculations. The sheets have been copied into Word tables, and working spreadsheets can be created by entering the text and formulae into an Excel workbook, or obtained from the author.

(2) New data need be entered only in the highlighted cells.

(3) In Table B12, an option is given for relative (percentage) or absolute uncertainties in columns G and H for initial and final diameters, and load. Time and temperature errors will normally be given as absolute values. For computed measurand uncertainties, the first one calculated (absolute or relative) is in column N, the other, if calculated, in column O.

**B5. Notch Creep Test Uncertainties Calculation Sheets**

Some quantities which are used later are represented by upper case letters, e.g. *value* (Z) or = Z. The symbols are used later in explanatory comments in the right hand column.

*Table B1.Uncertainty in Initial Cross-sectional Area*

| **Source of Uncertainty** | | | **Uncertainty in Source** | | | | | **Affected Measurand** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| initial diameter | $d_{n0}$ (mm) | 7.620 | A | 0.005 | | 1 | 0.005 | | | | |
| microscope error | $u_m$ (mm) | | B | 0.003 | rect. | √3 | 0.0017 | $d_{n0}$ | 7.620 | 0.0053<br>0.07% (A) | (7) |
| | | | | | | | | $S_{n0}$ ($mm^2$) | 45.60 | 0.06<br>0.14% (B) | B=2A (10a) |

*Table B2. Uncertainty in Net Stress*

| **Source of Uncertainty** | | | **Uncertainty in Source** | | | | | **Affected Measurand** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| load | P (N) | 23530 | B | 1% | rect. | √3 | 0.58% (C) | | | | |
| initial cross-sectional area | $S_{n0}$ ($mm^2$) | 45.60 | A | | - | | 0.14% (=B) | $\sigma_{net}$ (MPa) | 516 | 0.59% (D) | $D=\sqrt{(B^2 + C^2)}$ (11) |

*Table B3. Uncertainties in Final Minimum Cross-sectional Area and Reduction of Area*

| Source of Uncertainty | | | Uncertainty in Source | | | | | Affected Measurand | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| final minimum diam. | $d_{nu}$ (mm) | 7.574 | A | 0.010 | | 1 | 0.010<br>0.13% | | | | |
| microscope error | | | B | 0.003 | rect. | √3 | 0.0017 | $d_{nu}$ | 7.574 | 0.01<br>0.13% (E) | |
| | | | | | | | | $S_{nu}$ (mm²) | 45.05 | 0.12<br>0.26% (F) | F=2E (10c) |
| initial cross-sectional area | $S_{n0}$ (mm$^2$) | 45.60 | | | | | 0.06<br>0.14% (B) | reduction in area | | | |
| | | | | | | | | $Z_{nu}$ (%) | 1.2 | 0.3 | (12) |

*Table B4. Temperature*

| Source of Uncertainty | | | Uncertainty in Source | | | | | Affected Measurand | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol, (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| measurement thermocouple | $T_m$ (K) | | B | 0.5 | | | | | | | |
| specimen uniformity | $T_u$ (K) | | B | 1.5 | | | | | | | |
| measuring system error | $T_c$ (K) | | B | 2.0 | | | | | | | |
| total temperature error | (K) | | B | 2.55 | rect. | √3 | 1.47 | T (K) | 811 | 1.47 | (9), (9a) |

*Table B5. Data from Appendix C, and Factors for Effects of Stress and Temperature Uncertainties on Uncertainty in Rupture Time.*

| | | | | | | |
|---|---|---|---|---|---|---|
| | n = 4 | Q / R = 45000K | n U($\sigma_{net}$) / $\sigma_{net}$ = n D = 0.024 (L) | u(T) Q / RT² =0.101 (M) | | (13a, b) |

*Table B6. Uncertainty in Rupture Time*

| **Source of Uncertainty** | | | **Uncertainty in Source** | | | | | **Affected Measurand** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| rupture time | $t_{nu}$ (h) | 127 | B | 0.5 | rect. | √3 | 0.3 (N) | | | $u_m$ ($t_{nu}$)=0.3 | = N (8) |
| net stress | $\sigma_{net}$ (MPa) | 516 | | | | | 0.59% | | | $u_s$ ($t_{nu}$)=3.0 | = P = L * $t_{nu}$ (13a) |
| temperature | T (K) | 811 | | | | | 1.4 | rupture time | | $u_T$ ($t_{nu}$)=12.7 | = S = M * $t_{nu}$ (13b) |
| | | | | | | | | $t_{nu}$ (h) | 127 | u($t_{nu}$)=13.1 | = $\sqrt{N^2 + P^2 + S^2}$ (14) |

*Table B7. Uncertainty in Notch Strength Ratio*

| Source of Uncertainty | | | Uncertainty in Source | | | | | Affected Measurand | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| net stress | $\sigma_{net}$ (MPa) | 516 | | | | | 0.59% (D) | | | | |
| representative stress by | | | | | | | | | | | |
| regression analysis | $\sigma_{rep}$ (MPa) | 492 | A | 13<br>2.7% | | 1 | 14<br>2.7% (V) | $R_S$ | 1.05 | 0.03<br>2.9% | $= \sqrt{(D^2 + V^2)}$ (15) |
| 2-point interpolation | $\sigma_{rep}$ (MPa) | 495 | A | 15<br>3.0% | | 1 | 15<br>3.0% (W) | $R_S$ | 1.04 | 0.03<br>3.1% | $= \sqrt{(D^2 + W^2)}$ (15) |

*Table B8. Uncertainty in Notch Life Ratio*

| Source of Uncertainty | | | Uncertainty in Source | | | | | Affected Measurand | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source quantity | symbol (unit) | value | type | value | prob. distrbn. | divisor | standard uncertainty | measurand | value | standard uncertainty | equation |
| notch rupture life | $t_{nu}$ (h) | 127 | | 13<br>10% | | | 13<br>10% (X) | | | | |
| plain bar rupture life by: | | | | | | | | | | | |
| regression analysis | $t_{pu}$ (h) | 79 | A | 14<br>18% | | 1 | 14<br>18% (Y) | $R_L$ | 1.62 | 21%<br>0.33 | $= \sqrt{(X^2+Y^2)}$ (16) |
| 2-point interpolation | $t_{pu}$ (h) | 79 | A | 18<br>23% | | 1 | 18<br>23% (Z) | $R_L$ | 1.61 | 25%<br>0.40 | $= \sqrt{(X^2+Z^2)}$ (16) |
| single test point | $t_{pu}$ (h) | 84.8 | A | 10<br>12% | | 1 | 10<br>12% ($Z_1$) | $R_L$ | 1.50 | 16%<br>0.23 | $= \sqrt{(X^2+Z_1^2)}$ (16) |

**B6. Determination of Representative Stress and its Uncertainty**

*(1) By Regression Analysis of Four Data Points*

The plain specimen results are placed in the Excel spreadsheet (Table B9) C5:F6, and the notched specimen results in E15, F15, E21, F21. E11:E13 contains the slope, intercept and standard error of estimated value, for the regression line with log(rupture time) as dependent variable. F11:F13 contains the same for log(stress) as dependent variable.

It is not necessary to work through the steps in Table D1 by using the spreadsheet (Appendix D).

representative stress $\sigma_{rep}$ = 492 MPa
standard uncertainty = 12.7 MPa

*(2) By Interpolation between Two Points*

This is taken from the spreadsheet in Table B10, and again it is not necessary to use Table D1 from Appendix D.
Note that from two data points, this spreadsheet can give both a representative stress if notched rupture time is entered, and plain specimen rupture life if notched net stress is entered. Here, only the representative stress is used, since the notched bar rupture time lies between the data points, but the net stress does not. Different data are used for plain specimen rupture life.

representative stress $\sigma_{rep}$ = 495 MPa
standard uncertainty = 15.5 MPa

**B7. Determination of Plain Specimen Rupture Time and its Uncertainty**

*(1) By Regression Analysis of Four Data Points*

Figures are obtained from the spreadsheet in Table B9.

plain specimen rupture time $t_{pu}$ = 78.6 h
standard uncertainty = 13.5 h

*(2) By Interpolation Between Two Data Points*

Data (1) and (3) in Section B1 are used. The results are taken from the spreadsheet in Table B11, it is not necessary to use Table E1 from Appendix E.

plain specimen rupture time $t_{pu}$ = 79.5 h
standard uncertainty = 19.7 h

**Table B9. Excel Spreadsheet Calculations of Representative Stress and Plain Specimen Rupture Time by Regression Analysis**

| A | B | C | D | E | F | G | H | I | J | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Unnotched test results | | | | | | | | | |
| | $\sigma_0$ MPa | 558 | 516 | 491 | 439 | | | | | |
| 4 | $t_u$ h | 33 | 84.8 | 139 | 372 | | | | | |
| 5 | | | | | | | | | | |
| 6 | log($\sigma_0$) | 2.7466 | 2.7126 | 2.6911 | 2.6425 | | | | | |
| 7 | log($t_u$) | 1.5185 | 1.9284 | 2.1430 | 2.5705 | | | | | |
| 8 | regression analysis | | dependent variable | log($t_u$) | log(stress) | | | | | C6=IF(C3<>"", LOG10(C3), "") etc. |
| 9 | | | slope | -10.0098 | -0.0992 | | | | | E9=SLOPE(C7:J7, C6:J6) F9=SLOPE(C6:J6, C7:J7) |
| 10 | | | intercept | 29.0486 | 2.9005 | | | | | E10=INTERCEPT(C7:J7, C6:J6) F10=INTERCEPT(C6:J6, C7:J7) |
| 11 | | | std error | 0.0458 | 0.0046 | | | | | E11=STEYX(C7:J7, C6:J6) F11=STEYX(C6:J6, C7:J7) |
| 12 | | | | | | | | | | |
| 13 | notched rupture time & uncertainty h | | | 127 | 13.2 | | | | | |
| 14 | log(repres. stress) | | | 2.69 | | | | | | E14=F10+F9*LOG10(E13) |
| 15 | uncertainty due to u($t_{nu}$) | | | 0.0104 | | | | | | E15=F13/E13/ABS(E9) |
| 16 | total uncertainty in log($\sigma_{rep}$) | | | 0.0113 | | | | | | E16=SQRT(E15^2+F11^2) |
| 17 | repres. stress & uncertainty | | | 492 | 12.8 | | | | | E17=10^E14 E18=2.3*E15*E14 |
| 18 | | | | | | | | | | |
| 19 | net stress $\sigma_{net}$ & relative uncertainty | | | 516 | 0.59% | | | | | |
| 20 | calculated log(plain rupt. time) | | | 1.90 | | | | | | E20=E10+E9*LOG10(E19) |
| 21 | uncertainty due to u($\sigma_{net}$) | | | 0.0591 | | | | | | E21=ABS(E9)*F19 |
| 22 | total uncertainty in log($t_{pu}$) | | | 0.0747 | | | | | | E22=SQRT(E21^2+E11^2) |
| 23 | plain rupt. time $t_{pu}$ & uncertainty h | | | 78.6 | 13.5 | | | | | E23=10^E19 F23=2.3*E23*E22 |

## Table B10. Excel Spreadsheet Calculations of Representative Stress and Equivalent Plain Specimen Rupture Time by Interpolation Between Two Points

### Representative Stress from Data (2) and (3) in Section B1

| | C | D | E | F | G | |
|---|---|---|---|---|---|---|
| 2 | NOTCHED BAR net stress | $\sigma_{net}$ | | | | |
| 3 | relative uncertainty | $u(\sigma_{net})/\sigma_{net}$ | | | | |
| 4 | log(stress) | $\log(\sigma_{net})$ | | | | E4=IF(E2<>"", LOG10(E2), "") |
| 5 | rupture time h | $t_{nu}$ | 127 | | | |
| 6 | uncertainty h | $u(t_{nu})$ | 13.2 | | | |
| 7 | log(rupture time) | | 2.1038 | | | E7=IF(E5<>"", LOG10(E5), "") |
| 8 | | | | | | |
| 9 | PLAIN BAR stress MPa | $\sigma_1$ = | 516 | $\sigma_2$= | 491 | |
| 10 | rupture time h | $t_1$ = | 84.8 | $t_2$ = | 139 | |
| 11 | rupture time uncertainty h | $u_1$ = | 10 | $u_2$ = | 14 | |
| 12 | | | | | | |
| 13 | stress exponent | n = | 9.9507 | | | E13=(G15-E15)/(E17-G17) |
| 14 | | | | | | |
| 15 | log(rupture time) | | 1.9284 | | 2.143 | E15=LOG10(E10) G15=LOG10(G10) |
| 16 | uncertainty | | 0.1179 | | 0.1007 | E16=E11/E10 G16=G11/E10 |
| 17 | log(stress) | | 2.7126 | | 2.6911 | E17=LOG10(E9) G17=LOG10(G9) |
| 18 | uncertainty | | 0.0119 | | 0.0101 | E18=E16/E13 G18=G16/E13 |
| 19 | REPRESENTATIVE STRESS | | | | | |
| 20 | $L=\log(\sigma_{rep})$ | 2.6950 | | | | D20=IF(E5<>"", E17 -(E7-E15)/E13, "") |
| 21 | $u_1(L)$ | 0.0086 | | | | D21=IF(D20<>"",SQRT(E18^2*(G15-E7) |
| 22 | $u_2(L)$ | 0.0104 | | | | ^2+G18^2*(E7-E15)^2)/(G15-E15), "") |
| 23 | U(L) | 0.0135 | | | | D22=IF(D20<>"", E6/E13/E5, "") |
| 24 | | | | | | D23= IF(D20<>"", SQRT(D21^2+D22^2) , |
| 25 | representative stress MPa | 495.5 | | | | D25=IF(D20<>"", 10^D20, "") |
| 26 | uncertainty MPa | 15.4 | | | | D26=IF(D20<>"", 2.3*D25*D23, "") |
| 27 | | | | | | |
| 28 | | | | | | |
| 29 | PLAIN BAR RUPTURE LIFE | | | | | D30=IF(E2<>"", E15+E13* |
| 30 | $M=\log(t_{pu})$ | | | | | (E17-LOG10(E2)), "") |
| 31 | $u_1(M)$ | | | | | D31=IF(D30<>"",SQRT(E16^2*(E4-G17) |
| 32 | $u_2(M)$ | | | | | ^2+G16^2*(E17-E4)^2)/(E17-G17), "") |
| 33 | U(M) | | | | | D32=IF(D30<>"",E13*E3, "") |
| 34 | | | | | | D33= IF(D30<>"",SQRT(D31^2+D32^2), “”) |
| 35 | plain bar rupture life h | | | | | D35=IF(D30<>"",10^D30, "") |
| 36 | uncertainty h | | | | | D36=IF(D30<>"",2.3*D35*D33, "") |
| 37 | | | | | | |

## Table B11. Excel Spreadsheet Calculations of Representative Stress and Equivalent Plain Specimen Rupture Time by Interpolation Between Two Points

### Equivalent Plain Specimen Rupture Time from Data (1) and (3) in Section B1

| | C | D | E | F | G |
|---|---|---|---|---|---|
| 2 | NOTCHED BAR net stress | $\sigma_{net}$ | 516 | | |
| 3 | relative uncertainty | $u(\sigma_{net})/\ \sigma_{net}$ | 0.59% | | |
| 4 | log(stress) | $\log(\sigma_{net})$ | 2.7126 | | |
| 5 | rupture time h | $t_{nu}$ | | | |
| 6 | uncertainty h | $u(t_{nu})$ | | | |
| 7 | log(rupture time) | | | | |
| 8 | | | | | |
| 9 | PLAIN BAR stress MPa | $\sigma_1$ = | 558 | $\sigma_2$= | 491 |
| 10 | rupture time h | $t_1$ = | 33 | $t_2$ = | 139 |
| 11 | rupture time uncertainty h | $u_1$ = | 4 | $u_2$ = | 14 |
| 12 | | | | | |
| 13 | stress exponent | n = | 11.242 | | |
| 14 | | | | | |
| 15 | log(rupture time) | | 1.5185 | | 2.143 |
| 16 | uncertainty | | 0.1212 | | 0.1007 |
| 17 | log(stress) | | 2.7466 | | 2.6911 |
| 18 | uncertainty | | 0.0108 | | 0.009 |
| 19 | REPRESENTATIVE STRESS | | | | |
| 20 | $L=\log(\sigma_{rep})$ | | | | |
| 21 | $u_1(L)$ | | | | |
| 22 | $u_2(L)$ | | | | |
| 23 | U(L) | | | | |
| 24 | | | | | |
| 25 | representative stress MPa | | | | |
| 26 | uncertainty MPa | | | | |
| 27 | | | | | |
| 28 | | | | | |
| 29 | PLAIN BAR RUPTURE LIFE | | | | |
| 30 | $M=\log(t_{pu})$ | 1.9006 | | | |
| 31 | $u_1(M)$ | 0.0775 | | | |
| 32 | $u_2(M)$ | 0.0663 | | | |
| 33 | U(M) | 0.1020 | | | |
| 34 | | | | | |
| 35 | plain bar rupture life h | 79.5 | | | |
| 36 | uncertainty h | 18.7 | | | |
| 37 | | | | | |

Note: The formulae are the same as in Table B10.

**Table B12. Spreadsheet for Entering Test Data and Automatic Computation of Measurands and Uncertainties**

*Spreadsheet Formulae*

For users to prepare their own versions, formulae are given below for Table 12, and on the spreadsheet in Tables 9 and 10. Formatting of cell borders and number display needs to be set by the user. Layout can be modified as required.

K5=IF(H5="", G5/J5, E5*H5/J5) K6=G6/J6 M6=E5 N6=SQRT(K5^2+K6^2) O6=N6/M6 M7=PI()*M6^2/4 N7=O6*2 K10=IF(G10="", H10/J10, G10/E10/J10) E11=M7 H11=N7 K11=H11/J11 M11=E10/E11 N11=SQRT(K10^2+K11^2) K14=IF(H14="", G14/J14, E14*H14/J14) K15=G15/J15 M15=E14 N15=SQRT(K14^2+K15^2) O15=N15/M15 M16=PI()*E14^2/4 N16=O15*2 O16=N16*M16 E17=M7 K17=N7*M7 M18=100*(1-M16/E17) N18=100*SQRT((M16*K17/E17^2)^2+(O16/E17)^2) G24=SQRT(G21^2+G22^2+G23^2) K24=G24/J24 M24=E24 N24=K24 K28=D28*N11 M28=K24*G28/E24^2 K31=G31/J31 N31=K31 E32=M11 K32=N11 N32=E31*K28 E33=E24 K33=K24 N33=E31*M28 M34=E31 N34=SQRT(N31^2+N32^2+N33^2) E37=M11 K37=N11 H39=G39/E39 K39=H39/J39 M39=E37/E39 N39=SQRT(K37^2+K39^2) O39=N39*M39 H40=G40/E40 K40=H40/J40 M40=E37/E40 N40=SQRT(K37^2+K40^2) O40=N40*M40 E43=E31 G43=N34 H43=G43/E43 K43=H43 H45=G45/E45 K45=H45 M45=E43/E45 N45=SQRT(K43^2+K45^2) O45=N45*M45 H46=G46/E46 K46=H46 M46=E43/E46 N46=SQRT(K43^2+K46^2) O46=N46*M46 H47=G47/E47 K47=H47 M47=E43/E47 N47=SQRT(K43^2+K47^2) O47=N47*M47

**Table B12.** Upper Section

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Measurand for uncertainty evaluation** | **Source of Uncertainty** | | | **Uncertainty in Source** | | | | | | **Affected Measurand** | | | |
| | | Source quantity | symbol (unit) | value | type | value absolute | relative | prob. distrbn. | divisor | standard uncer'ty | measurand | value | standard uncertainty | |
| 4 | | | | | | | | | | | | | | |
| 5 | Initial cross-sectional area | initial diameter | $d_{n0}$ (mm) | 7.620 | A | 0.005 | | | 1 | 0.005 | | | | |
| 6 | | microscope error | $u_m$ (mm) | | B | 0.003 | | rect. | 1.73 | 0.002 | $d_{n0}$ (mm) | 7.620 | 0.0053 | 0.069% |
| 7 | | | | | | | | | | | $S_{n0}$ (mm$^2$) | 45.60 | 0.14% | |
| 8 | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | |
| 10 | Net stress | load | P (N) | 23530 | B | | 1% | rect. | 1.73 | 0.58% | | | | |
| 11 | | initial cross-sect. | $S_{n0}$ (mm$^2$) | 45.60 | A | | 0.14% | - | 1 | 0.14% | $\sigma_{net}$ (MPa) | 516.0 | 0.59% | |
| 12 | | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | | |
| 14 | Final minimum cross section & Reduction in area | final min. diam. | $d_{nu}$ (mm) | 7.574 | A | 0.01 | | | 1 | 0.010 | | | | |
| 15 | | microscope error | $u_m$ (mm) | | B | 0.003 | | rect. | 1.73 | 0.0017 | $d_{nu}$ (mm) | 7.574 | 0.010 | 0.13% |
| 16 | | | | | | | | | | | $S_{nu}$ (mm$^2$) | 45.05 | 0.27% | 0.12 |
| 17 | | initial cross-sect. | $S_{n0}$ (mm$^2$) | 45.60 | | | | | | 0.063 | | | | |
| 18 | | | | | | | | | | | $Z_{nu}$ (%) | 1.2 | 0.3 | |
| 19 | | | | | | | | | | | | | | |
| 20 | | | | | | | | | | | | | | |
| 21 | Temperature | measuring th'couple | $T_m$ (K) | | B | 0.5 | | | | | | | | |
| 22 | | specimen uniformity | $T_u$ (K) | | B | 1.5 | | | | | | | | |
| 23 | | controller | $T_c$ (K) | | B | 2 | | | | | | | | |
| 24 | | total temp. error | T (K) | 811 | B | 2.55 | | rect. | 1.73 | 1.47 | T (K) | 811 | 1.47 | |
| 25 | | | | | | | | | | | | | | |

**Table B12.** Lower Section

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Measurand for uncertainty evaluation** | **Source of Uncertainty** | | | **Uncertainty in Source** | | | | | | **Affected Measurand** | | | |
| | | Source quantity | symbol (unit) | value | type | value absolute | relative | prob. distrbn. | divisor | standard uncer'ty | measurand | value | standard uncertainty | |
| 27 | | *Data from Appendix C, and Factors for Effects of Stress and Temperature Uncertainties on Uncertainty of Rupture Time.* | | | | | | | | | | | | |
| 28 | | stress exponent n | 4 | | Q/R (K) | 45000 | | | $nu(\sigma_0)/\sigma_0$ | 0.024 | u(T) Q / RT² | 0.101 | | |
| 29 | | | | | | | | | | | | | | |
| 30 | | | | | | | | | | | | | | |
| 31 | Rupture time | rupture time | $t_{nu}$ (h) | 127 | B | 0.5 | | rect. | 1.73 | 0.3 | | | 0.3 | $u_m(t_{nu})$ |
| 32 | | net stress | $\sigma_{net}$ (MPa) | 516.0 | | | | | | 0.59% | | | 3.0 | $u_s(t_{nu})$ |
| 33 | | temperature | T (K) | 811 | | | | | | 1.47 | | | 12.8 | $u_T(t_{nu})$ |
| 34 | | | | | | | | | | | $t_{nu}$ (h) | 127 | 13.2 | |
| 35 | | | | | | | | | | | | | | |
| 36 | | | | | | | | | | | | | | |
| 37 | Notch strength ratio | net stress | $\sigma_{net}$ (MPa) | 516.0 | | | | | | 0.59% | | | | |
| 38 | | represent. stress by: | | | | | | | | | | | | |
| 39 | | regression analysis | $\sigma_{rep}$ (MPa) | 492 | A | 14 | 2.8% | | 1 | 2.8% | $R_S$ | 1.049 | 2.9% | 0.030 |
| 40 | | 2-point interpolation | $\sigma_{rep}$ (MPa) | 495 | A | 15 | 3.0% | | 1 | 3.0% | $R_S$ | 1.042 | 3.1% | 0.032 |
| 41 | | | | | | | | | | | | | | |
| 42 | | | | | | | | | | | | | | |
| 43 | Notch life ratio | notch rupture life | $t_{nu}$ (h) | 127 | A | 13.2 | 10.4% | | | 10.4% | | | | |
| 44 | | plain bar rupture life by: | | | | | | | | | | | | |
| 45 | | regression analysis | $t_{pu}$ (h) | 79 | A | 14.0 | 18% | | 1 | 18% | $R_L$ | 1.61 | 21% | 0.33 |
| 46 | | 2-point interpolation | $t_{pu}$ (h) | 79 | A | 18 | 23% | | 1 | 23% | $R_L$ | 1.61 | 25% | 0.40 |
| 47 | | single test point | $t_{pu}$ (h) | 84.8 | A | 10 | 12% | | 1 | 12% | $R_L$ | 1.50 | 16% | 0.24 |
| | | | | | | | | | | | | | | |

## APPENDIX C

### Values of Stress Exponent and (Activation Energy) / R

If it is assumed that rupture time $t_{nu}$ varies in the same manner as for plain specimens over a small temperature or stress range i.e.

$$t_{nu} = A\,\sigma_{net}^{-n} \exp(Q / RT) \qquad \text{(C1)}$$

then the values of A, n and Q/R could be obtained by regression analysis of the results of a series of notched rupture tests at different stresses and temperatures.

It was noted in Section A2.4 that sensitivity coefficients $\partial t_{nu}/\partial\sigma_{net}$ , $\partial t_{nu}/\partial T$, can be found from four additional tests at net stresses and temperatures slightly displaced from $\sigma_{net}$ and T. This provides an alternative route to the combined uncertainty of $t_{nu}$.

If the data is not available for this approach then it will be assumed that stress exponent n and activation energy Q have similar values to those for plain specimens the for uncertainty evaluation in a notched bar rupture test.

The figures in the Tables below are derived from stress-rupture data in Atlas of Creep and Stress-Rupture Curves (ASM 1988, [7]). They give the values of n and Q/R in the Eq. (C1) above which best fit the data in the Atlas for a typical material in four classes. Least-squares regression analysis was used, after transforming Eq. (C1) to

$$\log\ t_u = \log A - n \log \sigma + Q / RT$$

Other materials in a class would give slightly different figures from the example chosen, but the figures given are representative, and sufficient for estimation of uncertainty arising from tolerance in stress and temperature.

| | | | | |
|---|---|---|---|---|
| Ferritic Pipe Steel | Temperature °C | 500 | 540 | 580 |
| 2.25Cr 1Mo | n | 7.1 | 5.4 | 4.3 |
| Atlas p. 19.35 | Q/R K | 48000 | 50000 | 47000 |

| | | | | |
|---|---|---|---|---|
| Ferritic Rotor Steel | Temperature °C | 480 | 530 | 580 |
| 0.5Cr 0.5Mo 0.25 V | n | 6.7 | 4.7 | 4.0 |
| Atlas p. 19.22 | Q/R K | 50000 | 43000 | 45000 |

| | | | | |
|---|---|---|---|---|
| Austenitic Stainless | Temperature °C | 600 | 650 | 700 |
| Type 316 | n | 6.9 | 4.7 | 3.7 |
| Atlas p. 11.39 | Q/R K | 48000 | 49000 | 48000 |

| | | | | |
|---|---|---|---|---|
| Ni Base Superalloy | Temperature °C | 700 | 815 | 870 |
| Nimonic 90 | n | 4.9 | 4.3 | 4.1 |
| Atlas p. 5.68 | Q/R K | 45000 | 52000 | 58000 |

# APPENDIX D

## Representative Stress and its Uncertainty

*D1 Theoretical Model Basis*

The Representative Stress can be determined from a series of creep-rupture tests with plain specimens at the same temperature as the notched specimen test, and with rupture times $t_u$ spanning the target rupture time $t_{nu}$. Provided the experimental range is not too large, the results will follow a power law relationship of the form

$$t_u = A\ \sigma_0^{-n}$$

$$\text{or} \quad \log \sigma_0 = (\log A) / n - (\log t_u) / n \qquad \text{(D1)}$$

where A = a constant (a function of temperature), n = creep stress exponent, logs to base10.

The equation (D1) above is the inverse of the normal, with $\log \sigma_0$ as the dependent variable.

*D2 Spreadsheet Calculations*

Appendices D and E demonstrate the derivation of the formulae used in the calculations, but working through the steps in Tables D1 and E1 with numerical data is complex and prone to error. To simplify the calculations described in the two procedures below, the formulae have been incorporated in spreadsheets, which are included in the worked example in Appendix B. Working spreadsheets can be obtained from the author, or by entering the text and formulae into an Excel workbook.

There are notes on using the spreadsheets in Appendix B.

*D3 Regression Analysis*

If there are enough data points, the values of log A and n giving the best-fit line can be found by linear regression analysis. By putting $t_u = t_{nu}$ (the rupture time for the notched specimen), the corresponding value of $\log \sigma_{rep}$ is calculated. These operations can be carried out easily with computer software or on many hand calculators.

$$\text{Let} \quad L = \log \sigma_{rep}$$

$$\text{i.e.} \quad \sigma_{rep} = 10^{L}$$

The first component of standard error or uncertainty of the estimate of log $\sigma_{rep}$, due to scatter of data points about the "best" straight line, is given by

$$u_1(L) = \sqrt{\frac{1}{m(m-2)}\left(m\Sigma\left(y^2\right) - (\Sigma y)^2 - \frac{\left[m\Sigma(xy) - \Sigma x \Sigma y\right]^2}{m\Sigma\left(x^2\right) - (\Sigma x)^2}\right)}$$

where m = number of points and the x and y values are provided by log $t_u$ and log $\sigma_0$ respectively.

This function is also available in computer software, for example STEYX in Microsoft Excel.

The effect of uncertainty in $t_{nu}$ is next added by taking this value of $u_1(L)$ , and proceeding from line (6c) in Table D1 at the end of this Appendix. The stress exponent n, which is required at this point, is given by the slope of the regression line.

*D4 Interpolation Between 2 Points*

If only two ($\sigma_0$ ,$t_u$ ) values are known, at ($\sigma_1$ ,$t_1$) and ($\sigma_2$, $t_2$) , with the $t_u$ values at either side of $t_{nu}$ , then $\sigma_{rep}$ can be estimated by interpolation, assuming a power law relationship as in the previous section. This is illustrated in the diagram below. The calculation proceeds according to Table D1.

To work with positive numbers, the data should be arranged so that $\sigma_1 > \sigma_{rep} > \sigma_2$ and $t_2 > t_{nu} > t_1$. Logarithms are to base 10.

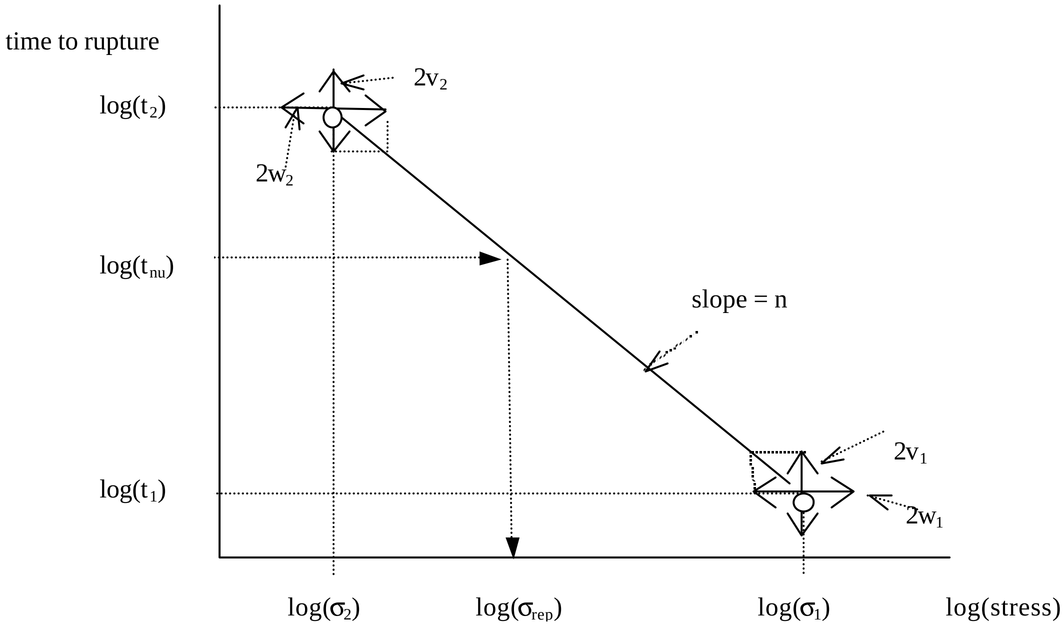

Figure D1. Logarithmic interpolation to determine $\sigma_{rep}$ and notched rupture time $t_{nu}$ from two data points ($\sigma_1$, $t_1$) and ($\sigma_2$, $t_2$).
The plain specimen rupture times $t_1$ and $t_2$ have uncertainties $u_1$ and $u_2$, determined by the Procedure for creep rupture testing [1](a). The uncertainties $v_1$ and $v_2$ in log($t_1$) and log($t_2$) are calculated from these in Table lines (4) and (5), then uncertainties $w_1$ and $w_2$ in log($\sigma_1$) and log($\sigma_2$) are calculated. The w values require the stress exponent n, derived in line (3).

Log($\sigma_{rep}$) is found by linear interpolation (6a), and the uncertainty of this value as a weighted combination of $w_1$ and $w_2$ (6b). The uncertainty contribution due to $t_{nu}$ (u($t_{nu}$)) is then added. Finally, the inverse logs give $\sigma_{rep}$ and its uncertainty, the latter in an approximated form.

### *D4 Calculation Sequence for Representative Stress and its Uncertainty*

Interpolating between two data points, start at line (1).

If n, log($\sigma_{rep}$) , i.e. L, and $u_1$(L) , have been obtained by linear regression, enter at line 6(c).

**Table D1**

| | | |
|---|---|---|
| (1) stress | $\sigma_1$ | $\sigma_2$ |
| (2) rupture time $t_u$<br>uncertainty | $t_1$<br>$u_1$ | $t_2$<br>$u_2$ |
| (3) stress exponent n | $n = \frac{\log(t_2) - \log(t_1)}{\log(ó_1) - \log(ó_2)}$ | |
| (4) log(rupture time)<br>uncertainty | = log($t_1$)<br>$v_1 = u_1 / t_1$ | = log($t_2$)<br>$v_2 = u_2 / t_2$ |
| (5) log(stress)<br>uncertainty | = log($\sigma_1$)<br>$w_1 = v_1 / n$ | = log($\sigma_2$)<br>$w_2 = v_2 / n$ |
| (6a) log(repres. stress) | $L = \log(\sigma_{rep}) = \log(\sigma_1) - [\log(t_{nu}) - \log(t_1)] / n$ | |
| (6b) interpolation uncertainty | $u_1(L) = \frac{\sqrt{w_1^2[\log(t_2) - \log(t_{nu})]^2 + w_2^2[\log(t_{nu}) - \log(t_1)]^2}}{\log(t_2) - \log(t_1)}$ | |
| (6c) addition for u($t_{nu}$) uncertainty in $t_{nu}$ | $u_2(L) = \frac{u(t_{nu})}{nt_{nu}}$ | |
| (6d) total uncertainty in L | $u(L) = \sqrt{[u_1(L)^2 + u_2(L)^2]}$ | |
| (7) representative stress<br>uncertainty | $\sigma_{rep} = 10^L$<br>$u(\sigma_{rep}) = 2.3\ \sigma_{rep}\ u(L)$ | |

# APPENDIX E

## Plain Specimen Rupture Life and its Uncertainty

*E1 Theoretical Model Basis*

This procedure is required when there are no test data from plain specimens tested at stress $\sigma_0 = \sigma_{net}$ , and the corresponding plain specimen rupture time $t_{pu}$ has to be estimated from other stress-rupture data. The starting point is the same relation between $t_u$ and $\sigma$ as used in Appendix D:

$$t_u = A\ \sigma_0^{-n}$$

where A = a constant (a function of temperature), n = creep stress exponent, logs to base10.

In this case the logarithmic version retains $t_u$ as the subject:

$$\log t_u = \log A - n \log \sigma_0 \qquad \text{(a)}$$

*E2 Spreadsheet Calculations*

Refer to Section D2 in the previous Appendix.

*E3 Regression Analysis*

If there are enough data points, the values of log A and n giving the best-fit line can be found by linear regression analysis. By putting $\sigma = \sigma_{net}$ (the net stress for the notched specimen), the corresponding value of $\log t_{pu}$ is calculated. These operations can be carried out easily with computer software or on many hand calculators.

$$\text{Let} \quad M = \log t_{pu}$$
$$\text{i.e.} \quad t_{pu} = 10^{M}$$

The first component of standard error or uncertainty of the estimate of $\log t_{pu}$, due to scatter of data points about the "best" straight line, is given by:

$$u_1(M) = \sqrt{\frac{1}{m(m-2)}\left(m\Sigma\left(y^2\right) - (\Sigma y)^2 - \frac{\left[m\Sigma(xy) - \Sigma x \Sigma y\right]^2}{m\Sigma\left(x^2\right) - (\Sigma x)^2}\right)}$$

where m = number of points and the x and y values are provided by $\log \sigma_0$ and $\log t_u$ respectively, i.e. the reverse of the order in estimating $\sigma_{rep}$ in the previous Appendix. This function is also available in computer software, for example STEYX in Microsoft Excel.

The effect of uncertainty in $\sigma_{net}$ is next added by taking the calculated value of $u_1(M)$, and proceeding from line (5c) in Table E1 at the end of this Appendix.

*E3 Interpolation Between Two Points*

If only two ($t_u$, $\sigma_0$ ) values are known, at ($t_1$, $\sigma_1$) and ($t_2$, $\sigma_2$) , with the $\sigma_0$ values at either side of $\sigma_{net}$, $t_u$ at $\sigma_0 = \sigma_{net}$, i.e. $t_{pu}$, can be estimated by interpolation, assuming a power law relationship as in the previous section. The calculation proceeds according to Table E1 on the following page.

To work with positive numbers, the data should be arranged so that $\sigma_1 > \sigma_{net} > \sigma_2$ and $t_2 > t_1$. Logarithms are to base 10.

The plain specimen rupture times $t_1$ and $t_2$ have uncertainties $u_1$ and $u_2$, determined by the Procedure for creep rupture testing [1](a). The uncertainties $v_1$ and $v_2$ in log ($t_1$) and log ($t_2$) are calculated from these in Table line (3). The stress exponent n is derived in line (4).

Log ($t_{pu}$) is found by linear interpolation (5a), and the uncertainty of this value as a weighted combination of $v_1$ and $v_2$ (5b). The uncertainty contribution due to $\sigma_{net}$ ($u(\sigma_{net})$ ) is then added. Finally, the inverse logs give $t_{pu}$ and its uncertainty, the latter in an approximated form.

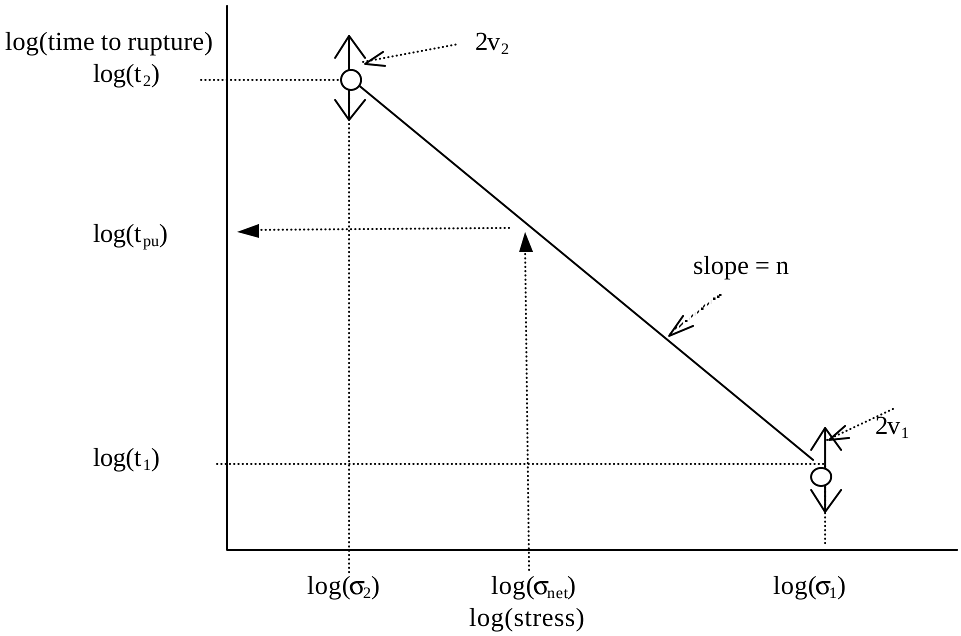


Figure E1. Logarithmic interpolation to determine $t_{pu}$ and notched specimen net stress $\sigma_{net}$ from two data points ($\sigma_1$ ,$t_1$) and ($\sigma_2$, $t_2$).

*E4 Calculation Sequence for Plain Bar Rupture Life and its Uncertainty*

Interpolating between two data points, start at line (1).

If n, $\log(t_{pu})$ , i.e. M, and $u_1(M)$ , have been obtained by linear regression, enter at line 5(c).

**Table E1**

| | | |
|---|---|---|
| (1) stress<br>log(stress) | $\sigma_1$<br>$= \log(\sigma_1)$ | $\sigma_2$<br>$= \log(\sigma_2)$ |
| (2) rupture time $t_u$<br>uncertainty | $t_1$<br>$u_1$ | $t_2$<br>$u_2$ |
| (3) log(rupture time)<br>uncertainty | $= \log(t_1)$<br>$v_1 = u_1 / t_1$ | $= \log(t_2)$<br>$v_2 = u_2 / t_2$ |
| (4) stress exponent n | $n = \frac{\log(t_2) - \log(t_1)}{\log(\sigma_1) - \log(\sigma_2)}$ | |
| (5a) $\log(t_{pu})$ | $M = \log(t_{pu}) = \log(t_1) + n[\log(\sigma_1) - \log(\sigma_{net})]$ | |
| (5b) interpolation uncert-ainty | $u_1(M) = \frac{\sqrt{v_1^2[\log(\sigma_{net}) - \log(\sigma_2)]^2 + v_2^2[\log(\sigma_1) - \log(\sigma_{net})]^2}}{\log(\sigma_1) - \log(\sigma_2)}$ | |
| (5c) addition for $u(\sigma_{net})$<br>uncertainty in $\sigma_{net}$ | $u_2(M) = \frac{n u(\sigma_{net})}{\sigma_{net}}$ | |
| (5d) total uncertainty in M | $u(M) = \sqrt{[u_1(M)^2 + u_2(M)^2]}$ | |
| (6) predicted plain bar rupture life at $\sigma_0 = \sigma_{net}$<br><br>uncertainty | $t_{pu} = 10^M$<br><br>$u(t_{pu}) = 2.3\, t_{pu}\, u(M)$ | |

# APPENDIX F

## Derivation of Formulae for Uncertainties

*F1*

When a measurand Y is a function of N measured quantities $x_1$ , $x_2$ , $x_3$.....$x_N$, and each $x_i$ is subject to uncertainty $u(x_i)$, then the resulting uncertainty in Y is given by:

$$u(Y) = \sqrt{\sum \left( u(x_i) \frac{\partial Y}{\partial x_i} \right)^2} \qquad \text{(see Steps 4 and 5 in Procedure)}$$

*F2 Reduction in Area*

This is calculated from the final and initial cross-sectional areas $S_{nu}$ and $S_{n0}$ using Eq. (3), i.e.

$$Z_{nu} = 100 \, [1 - (S_{nu} / S_{n0})]$$

Then

$$\frac{1}{100} \frac{\partial Z_{nu}}{\partial S_{n0}} = \frac{S_{nu}}{S_{n0}^{\;2}} \qquad \frac{1}{100} \frac{\partial Z_{nu}}{\partial S_{nu}} = -\frac{1}{S_{n0}}$$

hence

$$\frac{u(Z_{nu})}{100} = \sqrt{\left( \frac{S_{nu} \, u(S_{n0})}{S_{n0}^{\;2}} \right)^2 + \left( \frac{u(S_{nu})}{S_{n0}} \right)^2}$$

*F3 Rupture Time - Effect of Stress and Temperature Uncertainties*

It is assumed that rupture time $t_{nu}$ varies in the same manner as for plain specimens over a small temperature or stress range, i.e.

$$t_{nu} = A \, \sigma_{net}^{\;-n} \exp(Q / RT)$$

where n is the stress exponent, Q the creep activation energy, and A a constant.

The partial derivatives are

$$\frac{\partial t_{nu}}{\partial \sigma_{net}} = -A \, n \, \sigma_{net}^{\;-n-1} \exp(Q / RT) = -\frac{n}{\sigma_{net}} t_{nu}$$

$$\frac{\partial t_{nu}}{\partial T} = -\frac{AQ}{RT^2} \, \sigma_{net}^{\;-n} \exp(Q / RT) = -\frac{Q}{RT^2} t_{nu}$$

and hence

$$u(t_{nu}) = \sqrt{\left( \frac{n \, u(\sigma_{net})}{\sigma_{net}} t_{nu} \right)^2 + \left( \frac{Q \, u(T)}{RT^2} t_{nu} \right)^2}$$