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EXECUTIVE COMMITTEE

Proposals for Development in the Mathematics Division

of an Automatic Computing Engine

(ACE)

Report

by Dr. A. M. Turing

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PROPOSED ELECTRONIC CALCULATOR.

PART I.

Descriptive Account.

1. Introductory.

Calculating machinery in the past has been designed to carry out accurately and moderately quickly small parts of calculations which frequently recur. The four processes addition, subtraction, multiplication and division, together perhaps with sorting and interpolation, cover all that could be done until quite recently, if we except machines of the nature of the differential analyser and wind tunnels, etc. which operate by measurement rather than by calculation.

It is intended that the electronic calculator now proposed should be different in that it will tackle whole problems. Instead of repeatedly using human labour for taking material out of the machine and putting it back at the appropriate moment all this will be looked after by the machine itself. This arrangement has very many advantages.

(1) The speed of the machine is no longer limited by the speed of the human operator.

(2) The human element of fallibility is eliminated, although it may to an extent be replaced by mechanical fallibility.

(3) Very much more complicated processes can be carried out than could easily be dealt with by human labour.

Once the human brake is removed the increase in speed is enormous. For example, it is intended that multiplication of two ten figure numbers shall be carried out in 500 μ s. This is probably about 20,000 times faster than the normal speed with calculating machines.

It is evident that if the machine is to do all that is done by the normal human operator it must be provided with the analogues of three things, viz. firstly, the computing paper on which the computer writes down his results and his rough workings; secondly, the instructions as to what processes are to be applied; these the computer will normally carry in his head; thirdly, the function tables used by the computer must be available in appropriate form to the machine. These requirements all involve storage of information or mechanical memory. This is not the place for a detailed discussion of the various kinds of storage available* and the considerations which govern their usefulness and which limit what we can expect. For the present let us only remark that the memory needs to be very large indeed by comparison with standards which prevail in most valve and relay work, and that it is necessary therefore to look for some more economical form of storage.

It/

* See § 16.

It is intended that the setting up of the machine for new problems shall be virtually only a matter of paper work. Besides the paper work nothing will have to be done except to prepare a pack of Hollerith cards in accordance with this paper work, and to pass them through a card reader connected with the machine. There will positively be no internal alterations to be made even if we wish suddenly to switch from calculating the energy levels of the neon atom to the enumeration of groups of order 720. It may appear somewhat puzzling that this can be done. How can one expect a machine to do all this multitudinous variety of things? The answer is that we should consider the machine as doing something quite simple, namely carrying out orders given to it in a standard form which it is able to understand.

The actual calculation done by the machine will be carried out in the binary scale. Material will however be put in and taken out in decimal form.

In order to obtain high speeds of calculation the calculator will be entirely electronic. A unit operation (typified by adding one and one) will take 1 microsecond. It is not thought wise to design for higher speeds than this as yet.

The present report gives a fairly complete account of the proposed calculator. It is recommended however that it be read in conjunction with J. von Neumann's 'Report on the EDVAC'.

2. Composition of the Calculator.

We list here the main components of the calculator as at present conceived:-

(1) Erasible memory units of fairly large capacity, to be known as dynamic storage (DS). Probably consisting of between 50 and 500 mercury tanks with a capacity of about 1000 digits each.

(2) Quick reference temporary storage units (TS) probably numbering about 50 and each with a capacity of say 32 binary digits.

(3) Input organ (IO) to transfer instructions and other material into the calculator from the outside world. It will have a mechanical part consisting of a Hollerith card reading unit, and an electronic part which will be internal to the calculator.

(4) Output organ (OO), to transfer results out of the calculator. It will have an external part consisting of a Hollerith card reproducer and an internal electronic part.

(5) The logical control (LC). This is the very heart of the machine. Its purpose is to interpret the instructions and give them effect. To a large extent it merely passes the instructions on to CA. There is no very distinct line between LC and CA.

(6) The central arithmetic part (CA). If we like to consider LC as the analogue of a computer then CA must be considered a desk calculating machine. It carries out the four fundamental arithmetical processes (with possible exception of division, see p.27), and various others of the nature of copying, substituting, and the like. To a large extent these processes can be reduced to one another by various roundabout means; judgment is therefore required in choosing an appropriate set of fundamental processes.

(7) Various 'trees' required in connection with LC and CA for the selection of the information required at any moment. These trees require much more valve equipment than LC and CA themselves.

(8) The clock (CL). This provides pulses, probably at a recurrence frequency of a megacycle, which are applied, together with gating signals, to the grids of most of the valves. It provides the synchronisation for the whole calculator.

(9) Temperature control system for the delay lines. This is a somewhat mundane matter, but is important.

(10) Binary to decimal and decimal to binary converters. These will have virtually no outward and visible form. They are mentioned here lest it be thought they have been forgotten.

(11) Starting device.

(12) Power supply.

3. Storages.

(i) The storage problem. - As was explained in § 1 it is necessary for the calculator to have a memory or information storage. Actually this appears to be the main limitation in the design of a calculator, i.e. if the storage problem can be solved all the rest is comparatively straightforward. In the past it has not been possible to store very large quantities of information economically in such a way that the information is readily accessible. There were economical methods such as storage on five-unit tape, but with these the information was not readily accessible, especially if one wishes to jump from point to point. There were also forms with good accessibility, such as storage on relays and valves, but these were quite prohibitively uneconomical. There are now several possibilities for combining economy with accessibility which have been developed, or are being developed. In this section we describe the one which will most probably be used in the calculator.

(ii) Delay line storage. - All forms of storage depend on modifying in some way the physical state of some storage medium. In the case of 'delay line storage' the medium consists of mercury, water, or some other liquid in a tube or tank, and we modify its state of compression at various points along the tube. This is done by forcing supersonic waves into the tube from one end. The state of the storage medium is not constant as it would be for instance if the storage medium were paper or magnetic tape. The information moves along the tube with the speed of sound. Unless we take some precautions the sound carrying the information will pass out of the end of the tube and be lost. We can effectively prevent this by detecting the sound in some way (some form of microphone) as it comes out, and amplifying it and putting it back at the beginning. The amplifying device must correct for the attenuation of the tube, and must also correct for any distortion of form caused by the transmission through the tube, otherwise after many passages through the tube the form will be eventually completely lost. We can only restore the form of the signal satisfactorily if the various possible ideal signal forms are quite distinct, for otherwise it will not be possible to distinguish between the undistorted form of one signal and a distorted form of another. The scheme actually proposed only recognizes 21024 distinct states of compression of the water medium, these being sequences of 1024 pulses of two different sizes, one of which will probably be zero. The amplifier at the end of the line always reshapes the signal to bring it back to the nearest ideal signal.

Alternatively/

Alternatively we may consider the delay line simply as providing a delay, as its name implies. We may put a signal into the line, and it is returned to us after a certain definite delay. If we wish to make use of the information contained in it when it comes back after being delayed we do so. Otherwise we just delay it again, and repeat until we do require it. This aspect loses sight of the fact that there is still a storage medium of some kind, with a variety of states according to the information stored.

There are, of course, other forms of delay line than those using acoustic waves.

(iii) Technical proposals for delay line. - Let us now be more specific. It is proposed to build 'delay line' units consisting of mercury or water tubes about 5' long and 2" diameter in contact with a quartz crystal at each end. The velocity of sound in either mercury or water is such that the delay will be 1.024 ms. The information to be stored may be considered to be a sequence of 1024 'digits' (0 or 1), or 'modulation elements' (mark or space). These digits will be represented by a corresponding sequence of pulses. The digit 0 (or space) will be represented by the absence of a pulse at the appropriate time, the digit 1 (or mark) by its presence. This series of pulses is impressed on the end of the line by one piezo-crystal, it is transmitted down the line in the form of supersonic waves, and is reconverted into a varying voltage by the crystal at the far end. This voltage is amplified sufficiently to give an output of the order of 10 volts peak to peak and is used to gate a standard pulse generated by the clock. This pulse may be again fed into the line by means of the transmitting crystal, or we may feed in some altogether different signal. We also have the possibility of leading the gated pulse to some other part of the calculator, if we have need of that information at the time. Making use of the information does not of course preclude keeping it also. The figures above imply of course that the interval between digits is 1 μ s.

It is probable that the pulses will be sent down the line as modulation on a carrier, possibly at a frequency of 15 Mc/s.

(iv) Effects of temperature variations. - The temperature coefficient of the velocity of sound in mercury is quite small at high frequencies. If we keep the temperatures of the tanks correct to within one degree Fahrenheit it will be sufficient. It is only necessary to keep the tanks nearly at equal temperatures. We do not need to keep them all at a definite temperature; variations in the temperature of the room as a whole may be corrected by altering the clock frequency.

4. Arithmetical considerations.

(i) Minor cycles. - It is intended to divide the information in the storages up into units, probably of 32 digits or thereabouts. Such a storage will be appropriate for carrying a single real number as a binary decimal or for carrying a single instruction. Each sub-storage of this kind is called a minor cycle or word. The longer storages of length about 1000 digits are called major cycles. It will be assumed for definiteness that the length of the minor cycle is 32 and that of the major 1024, although these need not yet be fixed.

(ii) Use of the binary scale. - The binary scale seems particularly well suited for electronic computation because of its simplicity and the fact that valve equipment can very easily produce and distinguish two sizes of pulse. Apart from the input and output the binary scale will be used throughout in the calculator.

(iii) Requirements for an arithmetical code. - Besides providing a sequence of digits the statement of the value of a real number has to do several other things. All included, (probably), we must:

(a) State the digits themselves, or in other words we must specify an integer in binary form.

(b) We must specify the position of the decimal point.

(c) We must specify the sign.

(d) It would be desirable to give limits of accuracy.

(e) It would be desirable to have some reference describing the significance of the number. This reference might at the same time distinguish between minor cycles which contain numbers and those which contain orders or other information.

None of these except for the first could be said to be absolutely indispensable, but, for instance, it would certainly be inconvenient to manage without a sign reference. The digit requirements for these various purposes are roughly:

(a) 9 decimal digits, i. e. 30 binary,

(b) 9 digits,

(c) 1 digit,

(d) 10 digits,

(e) very flexible.

(iv) A possible arithmetical code. - It is convenient to put the digits into one minor cycle and the fussy bits into another. This may perhaps be qualified as far as the sign digit is concerned: by a trick it can be made part of the normal digit series, essentially in the same way as we regard an initial series of figures 9 as indicating a negative number in normal computing. Let us now specify the code without further beating about the bush. We will use two minor cycles whose digits will be called $i_1 \dots i_{32}$, $j_1 \dots j_{32}$. Of these $j_{24} \dots j_{32}$ are available for identification purposes, and the remaining digits make the following statement about the number ξ .

There exist rational numbers β , γ and an integer n such that

$$\left| \xi - 2^n \beta \right| < \gamma$$

$$|\beta - 2^m \gamma| < \gamma$$

$$\beta = \sum_{s=1}^{31} 2^{s-1} i_s = 2^{31} i_{32}$$

$$m = \sum_{t=1}^9 2^{t-1} j_t = 512$$

$$\gamma = \sum_{u=10}^{17} 2^{u+m-n} j_u$$

$$n = \sum_{v=18}^{23} 2^{v-18} j_v$$

This code allows us to specify numbers from ones which are smaller than 10^{-70} to ones which are larger than 10^{86} , mentioning a value with sufficient figures that a difference of 1 in the last place corresponds to from 2.5 to 5 parts in 10^{10} . An error can be described smaller than a unit in the last place or as large as 30,000 times the quantity itself (or by more if this quantity has its first few 'significant' digits zero).

(v) The operations of CA. - The division of the storage into minor cycles is only of value so long as we can conveniently divide the operations to be done into unit operations to be performed on whole minor cycles. When we wish to do more elaborate types of process in which the digits get individual treatment we may find this form of division rather awkward, but we shall still be able to carry these processes out in some roundabout way provided the CA operations are sufficiently inclusive. A list is given below of the operations which will be included. Actually this account is distinctly simplified, and an accurate picture can only be obtained by reading § 12. The account is however quite adequate for an understanding of the main problems involved. The list is certainly theoretically adequate, i. e. given time and instruction tables any required operation can be carried out. The operations are:

(1) Transfers of material between different temporary storages, and between temporary storages and dynamic storage.

(2) Transfers of material from the DS to cards and from cards to DS.

(3) The various arithmetical operations, addition, subtraction, and multiplication (division being omitted), also 'short multiplication' by numbers less than 16, which will be much quicker than long multiplication.

(4) To perform the various logical operations digit by digit. It will be sufficient to be able to do 'and', 'or', 'not', 'if and only if', 'never' (in symbols $A \& B$, $A \vee B$, $\sim A$, $A \equiv B$, \bar{A}). In other words we arrange to do the processes corresponding to xy , $x + y + xy$, $1 + x$, $1 + (x + y)^2$, 0 digit by digit, modulo 2, where x and y are two corresponding digits from two particular TS (actually TS 9 and TS 10).

5. Fundamental Circuit Elements.

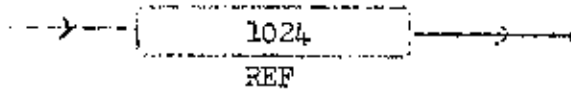
The electronic part of the calculator will be somewhat elaborate, and it will certainly not be feasible to consider the influence of every component on every other. We shall avoid the necessity of doing this if we can arrange that each component only has an appreciable influence on a comparatively small number of others. Ideally we would like to be able to consider the circuit as built up from a number of circuit elements, each of which has an output which depends only on its inputs, and not at all on the circuit into which it is working. Besides this we would probably like the output to depend only on certain special characteristics of the inputs. In addition we would often be glad for the output to appear simultaneously with the inputs.

These requirements can usually be satisfied, to a fairly high accuracy, with electronic equipment working at comparatively low frequencies. At megacycle frequencies however various difficulties tend to arise. The input capacities of valves prevent us from ignoring the nature of the circuit into which we are working; limiting circuits do not work very satisfactorily: capacities and transit times are bound to cause delays between input and output. These difficulties may be best resolved by bending before the storm. The delays may be tolerated by accepting them and working out a time table which takes them into account. Indefiniteness in output may be tolerated by thinking in terms of 'classes of outputs'. Thus instead of saying 'The inputs A and B give rise to the output C', we shall say 'Inputs belonging to classes P and Q give rise to an output in class R'. The various classes must be quite distinct and must be far from overlapping, i. e. topologically speaking we might say that they must be a finite distance apart. If we do this we shall have made a very definite division of labour between the mathematicians and the engineers, which will enable both parties to carry on without serious doubts as to whether their assumptions are in agreement with those of the other party.

For the present we shall merely ignore the difficulties because we wish to illustrate the principles. We shall assume the circuit elements to have all the most agreeable properties. It may be added that this will only affect our circuits in so far as we assume instantaneous response, and that not very seriously. The questions of stable output only involve the mathematician to the extent of a few definitions.

In the present section we shall only be concerned with what the circuit elements do. A discussion of how these effects can be obtained will be given in § 15. The circuit elements will be divided into valve-elements and delay elements.

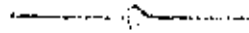
(i) Delay line, with amplifier and clock gate. - This is shown as a rectangle with an input and output lead



the arrow at the input end faces towards the rectangle and at the output end faces away. The name of the delay line, if any, will be written outside and the delay in pulse periods inside.

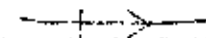
This circuit element delays the input by the appropriate number of pulse periods and also standardizes it, i.e. converts it into the nearest standard form by correcting amplitude shape and time.

(ii) The unit delay. - This is represented by a triangle, thought of as a modified form of arrow



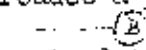
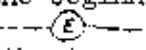
The input to output direction is indicated by the arrow. This delay element ideally provides a delay of one pulse period.

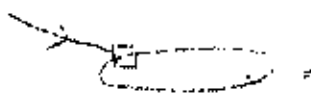
(iii) Limiting amplifier. - Ideally this valve-element is intended to give no output for inputs of less than a certain standard value, and to give a standard pulse as output when the input exceeds a second standard value. Intermediate input values are supposed not to occur. If we combine this with a resistance network in which a number of input signals are combined the condition takes the form that if the input signals are $s_1 s_2 \dots s_n$ there will be zero output unless $\alpha_1 s_1 + \dots + \alpha_n s_n \geq \beta_1$ and a standard or unit output if $\alpha_1 s_1 + \dots + \alpha_n s_n > \beta_2$. This may be simplified by assuming that the inputs $s_1 \dots s_n$ are always either 0 or 1 and the coefficients $\alpha_1 \dots \alpha_n$ either 1 or $-\infty$ and also by requiring the integral parts of β_1 / β_2 to be the same. We represent the valve element by a circle, and the inputs with a line and an arrow facing towards it, the outputs with lines and arrows facing away. (Fig. 1). A coefficient $-\infty$ (inhibitory coupling) is shown with a small circle cutting a large circle (Fig. 2). The smallest total for which an output is obtained (i.e. integral part of β_1 or β_2 plus 1) is shown inside the circle, but is omitted if it is 1. This number we may call the threshold.

When we require coefficients α larger than 1 we may show more than one connection from one source. Negative coefficients may effectively be shown by means of the negation circuit  which interchanges 0 and 1. Thus in the circuit of Fig. 3 the valve element D will be stimulated (i.e. emit a standard pulse) if either A is stimulated or both B and C are not.

(iv) Trigger circuits. - A trigger circuit, which is shown as an ellipse, differs from a limiting amplifier circuit in that once the inputs have reached the threshold so that it emits one pulse, it will continue to emit pulses until it receives an inhibitory stimulus. It is in fact equivalent to a limiting amplifier with a number of excitatory connections from itself with a delay of one unit. Thus for instance the two circuits shown in Fig. 4 are equivalent. We show

the trigger circuits with a different notation partly to simplify the drawing and partly because they will in fact be made up from different circuits. There is also another practical difference. The output from a trigger circuit will be a D.C. voltage, so long as it is not disturbed one way or the other, whereas the output from a limiting amplifier with feedback is more or less pulsiform.

(v) Differentiator circuit and change circuit. - We sometimes wish to indicate an output from a trigger circuit either at the beginning or the end of its stimulation. This would in fact be done with a capacity resistance 'differentiator' circuit. Such a circuit designed to produce a positive (excitatory) pulse at the beginning will be denoted by  and one at the end by . These are understood to be respectively equivalent to the two circuits of Fig. 5. We may also occasionally wish to make connection to a trigger circuit in such a way that stimulus always changes the condition of the trigger circuit, either from stimulation to non-stimulation or vice-versa. This is indicated by a small square at the connection point thus



and is equivalent to Fig. 6.

(vi) The trigger limiter. - Sometimes we wish a continuously varying voltage to initiate a train of pulses, the pulses to be synchronous with the clock and to start approximately when the continuous voltage reaches a certain value. All of the pulses that occur must be of the standard or unit size. There must definitely be no half-size pulses possible. The train of pulses may be stopped by pulses from some other source.

This valve element is indicated by a somewhat squat rectangle containing the letters TL. The continuous voltage input is shown as in an excitatory connection and the stopping pulse as an inhibitory connection, as in Fig. 7.

(vii) The adder and other examples. - We may now illustrate the use of these circuit elements by means of some simple examples.

The simplest circuit perhaps is that for the logical 'or' (cf. p.21). In the circuit of Fig. 8 there is an output pulse from the unnamed element if there is one from any one of A, B, C. We shall find it convenient in such cases to describe this element as $A \vee B \vee C$. The circuits of Fig. 9 are self explanatory in view of our treatment of $A \vee B \vee C$.

An adder network is shown in Fig. 10. It will add two numbers which enter along the leads shown on the left in binary form, with the least significant digit first, the output appearing on the right. An input signal from the top will inhibit any output. The method of operation is as follows. The three valve elements on the left all have stimulation from the same three sources, viz. the two inputs and one corresponding to the carry digit from the last figure, which was formed by the element with threshold 2. We can distinguish the four different possible totals 0, 1, 2, 3 according to which of the valve elements are stimulated. We wish to get an output pulse if the total is 1 or 3. This may be expressed as a pulse if the total is 3 or if it is 1 and not 2 or more. If we write T_n to mean 'the total is n or more' the condition is $T_3 \vee (T_1 \& \sim T_2)$. Using our standard networks for $A \vee B$ and for $A \& \sim B$ and observing that the three valve elements on the left of the adder are stimulated respectively in the cases T_1, T_2, T_3 we finally obtain the circuit given.

The adder will be shown as a single block as in Fig. 11. The input with the inhibiting circle being of course that shown at the top in the complete diagram.

6. Outline of Logical Control.

A simple form of logical control would be a list of operations to be carried out in the order in which they are given. Such a scheme can be made to cover quite a number of jobs, e.g. calculations from explicit formulae, and has been used in more than one machine. However it lacks flexibility. We wish to be able to arrange that the sequence of orders can divide at various points, continuing in different ways according to the outcome of the calculations to date. We also wish to be able to arrange for the splitting up of operations into subsidiary operations. This should be done in such a way that once we have written down how an operation is to be done we can use it as a subsidiary to any other operation.

These requirements can largely be met by having the instructions on a form of erasible memory, such as the delay lines. This gives the machine the possibility of constructing its own orders; i.e. there is always the possibility of taking a particular minor cycle out of storage and treating it as an order to be carried out. This can be very powerful. Besides this we need to be able to take the instructions in an order different from their natural order if we are to have the flexibility we desire. This is sufficient.

It is convenient to divide the instructions into two types A and B. An instruction of type A requires the central arithmetic part CA to carry out certain operations. Such an instruction, translated from its symbolic form into English might run:-

Instruction 491. A. Multiply the content of TS 23 by the content of TS 24 and store the result in TS 25. Then proceed to carry out the next instruction (i.e. No. 492).

Instructions of type merely specify the number of the next instruction.

Instruction 492. B. Proceed with instruction 301.

We must now explain in more detail how it comes about that we can branch the sequence of instructions and arrange for subsidiary operations. Let us take branching first. Suppose we wish to arrange that at a certain point instruction 33 will be applied if a certain digit is 0 but instruction 50 if it is 1. Then we may copy down these two instructions and then do a little calculation involving these two instructions and the digit D in question. One form the calculation can take is to pretend that the instructions were really numbers and calculate

$$D \times \text{Instruction } 50 + (1-D) \times \text{Instruction } 33.$$

The result may then be stored away, let us say in a box which is permanently labelled 'Instruction 1'. We are then given an order of type B saying that instruction 1 is to be followed, and the result is that we carry out instruction 33 or 50 according to the value of D.

When we wish to start on a subsidiary operation we need only make a note of where we left off the major operation and then apply the first instruction of the subsidiary. When the subsidiary is over we look up the note and continue with the major operation. Each subsidiary operation can end with instructions for this recovery of the

note. How is the burying and disinterring of the note to be done? There are of course many ways. One is to keep a list of these notes in one or more standard size delay lines (1024), with the most recent last. The position of the most recent of these will be kept in a fixed CS, and this reference will be modified every time a subsidiary is started or finished. The burying and disinterring processes are fairly elaborate, but there is fortunately no need to repeat the instructions involved each time, the burying being done through a standard instruction table BURY, and the disinterring by the table UNBURY.

7. External organs.

(i) General - It might appear that it would be difficult to put information into the calculator and to take it out, on account of the high speeds associated with the calculator, and the slow speeds associated with mechanical devices; but this difficulty is not a real one. Let us consider for instance the output organ. We will allow the mechanical part of the output organ to work at whatever pace suits it, to take its own time in fact. However we will require it to give out signals stating when it is ready to accept information. This signal provides a gate for the feeding of the information out to the output organ, and also signifies to the calculator that it may note that information as recorded and proceed to feed out some more. The preparation for feeding the information out consists merely in transferring it from dynamic storages onto trigger circuits.

In the case of the output arrangements we have the full power of the calculator behind us, i.e. we can do the conversion of the information into the required form as an I/O. In the case of the input organ we must go more warily. If we are putting the instruction tables into delay lines, then when the power has been turned off all memory will have been effaced, including the instruction tables. We cannot use instruction tables to get the information back, because the instruction tables are not there. We are able to get over this difficulty as will be seen below.

(ii) Output Organ - The output will go on to 32 columns of some Hollerith cards. All the 12 rows may be used. On the receipt of a signal from the calculator a card will begin to pass through a punch or 'reproducer'. Shortly before each row comes into position for punching a signal is sent back to the calculator and trigger circuits controlling the punches are set up. After the punching another signal is sent to the calculator and the trigger circuits are cleared. The reproducer punch also gives a signal on the final exit of the card. The circuit is shown in connection with CA (Fig. 26).

(iii) Input Organ - Let us first describe the action of this without worrying about the difficulty concerning absence of instruction tables. It is very similar to the output organ in many ways. The input is from 32 columns and 12 rows of a Hollerith card. When the calculator is ready a card release signal goes out to the card reader and a card begins to pass through. As each row comes into position for reading a signal is sent back to the calculator, which then prepares to accept the output from the reader at the moment appropriate for sending it to its destination in the delay line. It is assumed that this destination is already decided by the calculator. A signal is sent back to the calculator on the final exit of the card.

Now let us consider what is done right at the beginning. Arrangements are made for setting into CI and CD a certain invariable initial order and IN. These state that the card is to be transferred into a particular delay line, and that the next order is to be taken from a particular spot, which will actually be in this same delay line. The information in this delay line can contain sufficient orders to 'get us started'. The first few orders obeyed will probably be to take in a few more cards. The information on these will later be sorted to its final destination. When the final instructions are in place it will be as well to 'read them back'.

Actually it has been arranged that the special initial order consists of 0 throughout so that there is no need to set it up.

(iv) Binary-decimal conversion. - It is proposed to do binary-decimal and decimal-binary conversion as ITC.* This will be appreciably assisted by the fact that short multiplication is a CAO.*

(v) Instruction-table cards. - It was explained in connection with the input organ that the instructions would be on cards, of whose columns all but 32 were available for external use. A proposed use of the 80 columns is suggested below, without proper explanation; the explanation comes later.

	Columns
Genuine input	41-72
Repeat of destination	26-40
Popular name of group	1-8
Detail figure (popular)	9-11
Instruction (popular)	12-25
Job number	73-77
Spare	78-80

Of these the genuine input has already been spoken of to some extent, and will be spoken of again further. The job number and the spare columns do not require explanation. The popular data describe the instruction in letters and figures in a manner appropriate for the operator to appreciate quickly if for instance the cards are listed. In this respect we might say that the popular data is like a telephone number Mol 1380 whereas the genuine input is like the pulses used in dialling: indeed we shall probably carry the analogy further and really only distinguish 10 different letters, as is done on automatic exchanges. The popular data have also another important function, which only appears when we consider that the same instructions will be used on quite different jobs. If we were just to number the instructions serially throughout all the instructions ever used on any job, then, in the set of instructions actually used in any particular job there would be large gaps in the numbering. Suppose now that these instructions were stored in the DS with positions according to their numbers there would be a lot of wasted space, and we should need elaborate arrangements for making use of this space. Instead, when a new job appears we take the complete set of cards involved and make a new copy of each of them; these we sort into the order of popular group name and detail figure. We then renumber them consecutively in the binary scale. This number goes into the columns described as 'repeat of destination'. The renumbering may be done either with a relay counter attached to a collator, or by interleaving a set of master cards with the binary numbers in serial order. To complete the process we have to fill in other instruction numbers in binary form into the genuine input, e.g. if an instruction in popular form

were/

* ITC = Instruction Table Operation. CAO = Central Arithmetic Operation.

were "... and carry out instruction Potpan 15" the genuine input will have to be of form "... and carry out instruction 001101...1" where 001101...1 is the new number given to Potpan 15 in this particular job. This is a straightforward sorting and collating process.

It would be theoretically possible to do this rearrangement of orders within the machine. It is thought however that this would be unwise in the earlier stages of the use of the machine, as it would not be easy to identify the orders in machine form and popular form. In effect it would be necessary to take an output from the calculator of every order in both forms.

8. Scope of the Machine.

The class of problems capable of solution by the machine can be defined fairly specifically. They are those problems which can be solved by human clerical labour, working to fixed rules, and without understanding, provided that

(a) The amount of written material which need be kept at any one stage is limited to the equivalent of 5,000 real numbers (say), i. e. about what can conveniently be written on 50 sheets of paper.

(b) That the human operator, doing his arithmetic without mechanical aid, would not take more than a hundred thousand times the time available on the calculator, this figure representing the ratio of the speeds of calculation by the two methods.

(c) It should be possible to describe the instructions to the operator in ordinary language within the space of an ordinary novel. These instructions will not be quite the same as the instructions which are normally given to a computer, and which give him credit for intelligence. The instructions must cover every possible eventuality.

Let us now give real examples of problems that do and problems that do not satisfy these conditions.

Problem 1. - Construction of range tables. The complete process of range-table construction could be carried out as a single job. This would involve calculation of trajectories by small arcs, for various different quadrant elevations and muzzle velocities. The results at this stage would be checked by differencing with respect to other parameters than time. The figures actually required would then be obtained by interpolation and these would finally be rearranged in the most convenient form. All of this could in theory be done as a single job. In practice we should probably be wiser to do it in several parts in order to throw less responsibility on to the checking arrangements. When we have acquired more practical experience with the machine we will be bolder.

It is estimated that the first job of this kind might take one or two months, most of which would be spent in designing instruction tables. A second job could be run off in a few days.

Problem 2. - To find the potential distribution outside a charged conducting cube. This is a problem which could easily be tackled by the machine by a method of successive approximations; a relaxation process would probably be used. In relaxation processes the action to be taken at each major step depends essentially on the results of the steps that have gone before. This would normally be considered a serious hindrance to the mechanisation of a process, but the logical control of the proposed calculator has been designed largely with such cases in view, and will have no difficulty on this score. The

problem proposed is one which is well within the scope of the machine, and could be run off in a few minutes, assuming it was done as one of a sequence of similar problems. It is quite outside the scope of hand methods.

Problem 3. - The solution of simultaneous linear equations. In this problem we are likely to be limited by the storage capacity of the machine. If the coefficients in the equations are essentially random we shall need to be able to store the whole matrix of coefficients and probably also at least one subsidiary matrix. If we have a storage capacity of 6400 numbers we cannot expect to be able to solve equations in more than about 50 unknowns. In practice, however, the majority of problems have very degenerate matrices and we do not need to store anything like as much. For instance problem (2) above can be transformed into one requiring the solution of linear simultaneous equations if we replace the continuum by a lattice. The coefficients in these equations are very systematic and mostly zero. In this problem we should be limited not by the storage required for the matrix of coefficients, but by that required for the solution or for the approximate solutions.

Problem 4. - To calculate the radiation from the open end of a rectangular wave-guide. The complete polar diagram for the radiation could be calculated, together with the reflection coefficient for the end of the guide and interaction coefficients for the various modes; this would be done for any given wavelength and guide dimensions.

Problem 5. Given two matrices of degree less than 30 whose coefficients are polynomials of degree less than 10, the machine could multiply the matrices together, giving a result which is another matrix also having polynomial coefficients. This has important applications in the design of optical instruments.

Problem 6. - Given a complicated electrical circuit and the characteristics of its components, the response to given input signals could be calculated. A standard code for the description of the components could easily be devised for this purpose, and also a code for describing connections. There is no need for the characteristics to be linear.

Problem 7. - It would not be possible to integrate the area under a curve, as the machine will have no appropriate input.

Problem 8. - To count the number of butchers due to be demobilised in June 1946 from cards prepared from the army records. The machine would be quite capable of doing this, but it would not be a suitable job for it. The speed at which it could be done would be limited by the rate at which cards can be read, and the high speed and other valuable characteristics of the calculator would never be brought into play. Such a job can and should be done with standard Hollerith equipment.

Problem 9. - A jig-saw puzzle is made up by cutting up a halma-board into pieces each consisting of a number of whole squares. The calculator could be made to find a solution of the jig-saw, and, if they were not too numerous, to list all solutions.

This particular problem is of no great importance, but it is typical of a very large class of non-numerical problems that can be treated by the calculator. Some of these have great military importance, and others are of immense interest to mathematicians.

Problem 10. - Given a position in chess the machine could be made to list all the 'winning combinations' to a depth of about three moves on either side. This is not unlike the previous problem, but raises the question 'Can the machine play chess?' It could fairly easily be made to play a rather bad game. It would be bad because chess requires intelligence. We stated at the beginning of this section that the machine should be treated as entirely without intelligence. There are indications however that it is possible to make the machine display intelligence at the risk of its making occasional serious mistakes. By following up this aspect the machine could probably be made to play very good chess.

9. Checking.

It will be almost our most serious problem to make sure that the calculator is doing what it should. We may perhaps distinguish between three kinds of error.

(1) Permanent faults that have developed in the wiring or components, e.g. condensers that have become open circuit.

(2) Temporary errors due to interference, noise reaching unexpected levels, unusual combinations of voltages at some point in the circuit, etc.

(3) Errors due to the use of incorrect instruction tables, or even due to mistaken views as to what the circuit should do.

It will be our intention to install monitoring circuits to detect errors of form (1) fairly soon. The ideal to aim at should be that each conceivable form of failure would give a different indication on the monitor. In practice we should probably simply localise the error to some part, e.g. an adder, which could be changed and then examined at leisure.

Errors of type (2) should not occur when the apparatus is in proper working order, however when a component is beginning to age its deficiencies will often show themselves first in this sort of way. For instance, if the omission of a valve in a Kipp relay circuit is beginning to fail it will eventually not pass on any of the pulses it should, but this will begin with some occasional failures to react. The worst of this can probably be eliminated by frequent test runs in which the conditions of H.T. volts, interference, etc., are all modified in a way calculated to accentuate the deficiencies of the components. Those which are rather down at heel may then be removed, and when the conditions are restored to normal there should be a good margin of safety. We cannot of course rely on this 100%. We need a second string. This will be provided by a variety of checks of the types normally employed in computing, i.e. wherever we can find a simple identity which should be satisfied by the results of our calculations we shall verify it. For instance, if we were multiplying polynomials algebraically we should check by taking a particular value for the variable. If we were calculating the values of an analytic function at equal intervals we should check by differencing. Most of these checks will have to be set up as part of the instruction tables, and the appropriate action to be taken will also be put into them. A few checks will be made part of the circuit. For instance, all multiplications and additions will be checked by repeating them modulo 255.

Incorrect instruction tables (3) will often be shown up by the checks which have been put into these same instruction tables. We may also apply a special check whenever we have made up a new instruction table, by comparing the results with the same job done by

means of a different table, probably a more straightforward but slower one. This should eliminate all errors on the part of the mathematicians, but would leave the possibility of lost cards, etc., when the table is being used a second time. This may perhaps be corrected by running a test job as soon as the cards have been put into the machine.

There are three chief functions to be performed by the checking. It must eliminate the possibility of error, help to diagnose faults, and inspire confidence. We have not yet spoken at all of this last requirement. It would clearly not be satisfactory if the checking system in fact prevented all errors, but nobody had any confidence in the results. The device would come to no better end than Cassandra. In order to inspire confidence the checking must have some visible manifestations. Certainly whenever a check fails to work out the matter must be reported by the machine. There would not be time for all checks which do work out to be reported, but there could be a facility by which this could be laid on temporarily at moments of shaken confidence. Another facility which should have a good effect on morale is that of the artificial error. By some means the behaviour of the machine is disturbed from outside, and one waits for some error to be reported. This could be managed quite easily. One could arrange to introduce an unwanted pulse at any point in the circuit. In fact of course we cannot do very much about checking until the machine is made. We cannot really tell what troubles of this kind are in store for us, although one can feel confident that none of them will be insurmountable. We can only prepare against the difficulties we can foresee and hope that they will represent a large percentage of the whole.

10. Time-table, Cost, Nature of Work, Etc.

The work to be done in connection with the machine consists of the following parts:

- (1) Development and production of delay lines.
- (2) Development and production of other forms of storage.
- (3) Design of valve-elements.
- (4) Final schematic circuit design of LC and CA.
- (5) Production of the electronic part, i.e. LC and CA.
- (6) Making up of instruction tables.
- (7) External organs.
- (8) Building, power supply cables, etc.

(1) Delay lines have been developed for R.D.F. purposes to a degree considerably beyond our requirements in many respects. Designs are available to us, and one such is well suited to mass production. An estimate of £20 per delay line would seem quite high enough.

(2) The present report has only considered the forms of storage which are almost immediately available. It must be recognized however that other forms of storage are possible, and have important advantages over the delay line type. We should be wise to occupy time which falls free due to any kind of hold-up by researching into these possibilities. As soon as any really hopeful scheme emerges some more systematic arrangement must be made.

We must be ready to make a change over from one kind of storage to another, or to use two kinds at once. The possibility of developing a new and better type of storage is a very real one, but is too uncertain, especially as regards time, for us to wait for it; we must make a start with delay lines.

(3) Work on valve element design might occupy four months or more. In view of the fact that some more work needs to be done on schematic circuits such a delay will be tolerable, but it would be as well to start at the earliest possible moment.

(4) Although complete and workable circuits for LC and CA have been described in this report these represent only one of a considerable number of alternatives. It would be advisable to investigate some of these before making a final decision on the circuits. Too much time should not however be spent on this. We shall learn much more quickly how we want to modify the circuits by actually using the machine. Moreover if the electronic part is made of standard units our decisions will not be irrevocable. We should merely have to connect the units up differently if we wanted to try out a new type of LC and CA.

(5) In view of the comparatively small number of valves involved the actual production of LC and CA would not take long; six months would be a generous estimate.

(6) Instruction tables will have to be made up by mathematicians with computing experience and perhaps a certain puzzle-solving ability. There will probably be a great deal of work of this kind to be done, for every known process has got to be translated into instruction table form at some stage. This work will go on whilst the machine is being built, in order to avoid some of the delay between the delivery of the machine and the production of results. Delay there must be, due to the virtually inevitable snags, for up to a point it is better to let the snags be there than to spend such time in design that there are none (how many decades would this course take?). This process of constructing instruction tables should be very fascinating. There need be no real danger of it ever becoming a drudge, for any processes that are quite mechanical may be turned over to the machine itself.

The earlier stages of the making of instruction tables will have serious repercussions on the design of LC and CA. Work on instruction tables will therefore start almost immediately.

(7) Very little need be done about the external organs. They will be essentially standard Hollerith equipment with special mounting.

(8) It is difficult to make suggestions about buildings owing to the great likelihood of the whole scheme expanding greatly in scope. There have been many possibilities that could helpfully have been incorporated, but which have been omitted owing to the necessity of drawing a line somewhere. In a few years time however, when the machine has proved its worth, we shall certainly want to expand and include these other facilities, or more probably to include better ideas which will have been suggested in the working of the first model. This suggests that whatever size of building is decided on we should leave room for building-on to it. The immediate requirements are:

Room for 200 delay lines. These each require about 6 inches of wall space if they are to be individually accessible, and if this is partly provided by cubicle construction 300 square feet is probably a minimum. To this we might add another 100 square feet for the temperature correction arrangements.

Space for IC and CA. This is difficult to estimate, but 5 eight foot racks might be a reasonable guess and would require another 200 square feet or more. In the same room we would put the input and output organs which might occupy 40 square feet. We should also provide another 100 square feet for operators tables, etc. 400 square feet would not be unreasonable for this room.

Card storage room. We would probably keep a stock of about 100,000 cards, a very insignificant number by normal Hollerith standards. 200 square feet would be quite adequate.

Maintenance workshop. We would do well to be liberal here. 400 square feet.

This total of 1400 square feet does not allow for the planning of operations, which would probably be done in an office building elsewhere, nor for the processing of Hollerith cards which will probably be done on machinery already available to us.

Cost. It appears that the cost of the equipment will not be very great. An estimate of \$20 per delay line would be liberal, so that 200 of these would cost us \$4000. The valve equipment at \$25 per inch of rack space might total \$5000. The power supply might cost \$200. The Hollerith equipment would be hired, which would be advantageous because of the danger of it going out of date. The capital cost of such Hollerith equipment even if bought would not exceed \$2000. With this included the total is \$11,200.

PART II.

TECHNICAL PROPOSALS.

11. Details of Logical Control.

In this section we shall describe circuits for the logical control in terms of the circuit elements introduced in § 5. It is assumed that § 5, 6 are well understood.

The main components of LC are as follows:-

- (1) A short storage (like a TS) called current data CD. This contains nothing but the appropriate instruction number IN, i. e. the position of the next instruction to be carried out.
- (2) A short storage called current instructions CI. This contains the instruction being or about to be carried out.
- (3) A tree for the selection of a particular delay line, with a view to finding a particular instruction.
- (4) Timing system for the selection of a particular minor cycle from a delay line.
- (5) Timing system for the selection of particular pulses from within a minor cycle.
- (6) Arrangements for controlling CA, i. e. for passing instructions on to CA.
- (7) Arrangements for the continual change of the contents of CD, CI.
- (8) Timing arrangements for LC itself.
- (9) Starting device.

Let us first describe the starting device. This merely emits pulses synchronously with the clock from a certain point onwards, on the closing of a switch manually. The switch causes a voltage to rise and this eventually operates a trigger limiter. This starting mechanism sets a pulse running round a ring of valve elements providing the timing within a minor cycle. (Fig. 12, 13).

In order to check that this circuit is behaving we compare P32 with a signal which should coincide with it and which is obtained in another way, stimulating an SOS signal when there is failure. This forms one of the monitoring devices. We are not showing many of them in the present circuits. (Fig. 14).

The timing system for the selection of minor cycles is quite simple, consisting chiefly of a 'slow counter' SCA, which counts up to 255 in the scale of 2, keeping the total in a delay line of length 8. The pulses counted are restricted to appearing at intervals which are multiples of eight. As shown (Fig. 15) it is counting the pulses P 10. The suppression of the outputs at P 9 prevents undesirable carries from the most significant digit to the least.

The information in CD and CI being in dynamic (time) form is not very convenient for control purposes. We therefore convert this information into static form, i. e. we transfer it on to trigger circuits. (Fig. 16).

It will be convenient to make use of a symbolic notation in connection with the valve circuits. We write $A \& B$ (or manuscript $A + B$) to mean 'A and B'. If A and B are thought of as numbers 0 or 1 then $A \& B$ is just AB . We write $A \vee B$ for 'A or B'. With numbers $A \vee B$ is $1 - (1 - A)(1 - B)$. We also write $\sim A$ (manuscript $\sim A$) for 'not A' or $1 - A$. Other logical symbols will not be used. Where a whole sequence of pulses is involved, it is to be understood that these operations are to be carried out separately pulse by pulse. We shall combine these symbols with the symbol $+$ which refers to the operations of the adder. Thus for example $(A + (P3 \vee P4)) \& \sim P5$ means that we take the signal A and add to it a signal consisting of pulses in positions 3 and 4 and nowhere else, (addition in the sense of the adder circuit), and that we then suppress any pulses in position 5, as in Fig. 17. We will also abbreviate such expressions as $P5 \vee P6 \vee P7 \vee \dots P19$ to $P5-19$, and expressions such as $A \& P14-18$ to $A14-18$.

In circuit diagrams we have the alternatives of showing the logical combinations by formulae or by circuits. There is little to choose but there may be something to be said for an arrangement by which purely logical combination is not shown in circuit form, in order that the circuits may bring out more clearly the time effects.

We have agreed that there shall be two kinds of instructions, A and B. These are distinguished by CI 5. The standard forms for the two types of instructions are:

Type A. Carry out the CA operations given by digits CI 5-32, and construct a new CD according to the equation $CD = (CD' + P19) \& \sim P17$.

Type B. Construct a new CD according to the equation $CD = CI17-32$. Pass the old CD into TS 13.

CD' here represents the old CD. The significance of the formula for CD in case A is this. Normally it is intended that after an operation of type A the next instruction to be followed will be that with the next number, and it might be supposed therefore that the formula $CD = CD' + P17$ would apply. Actually we deviate from this simple arrangement in two ways. Firstly we find it convenient to have a facility by which an instruction may be taken from a TS, viz. TS 6; this has considerable time saving effects. The convention is that a digit 1 in column 17 indicates that the next instruction is to be taken from TS 6. This will involve our having only the digits CI 18-32 available to indicate normal positions for instructions and would suggest that the formula should be $CD = CD' + P18$. However if we did this we should always be obliged to have orders of type B in TS 6, for if we had an order of type A we should find that we had to go on repeating that order. If however we have the formula $CD = (CD' + P18) \& \sim P17$ we can obey an instruction in TS 6 and then revert to the instruction given by CI 18-32; a much more convenient arrangement. It remains to explain why we have P19 rather than P18. This is due to the fact that we wish to avoid the necessity of waiting a long time for our instructions. If the equation were the one with P18 it would mean that the next instruction to be obeyed, after one of type A, is always adjacent to it in time. This would mean that even with the shortest CA operations the next instruction would have gone by before we were ready to apply it; we should always just miss the boat. By putting P19 instead of P18 we give ourselves an extra minor cycle of time which is normally just what we need. In order that the consecutive instructions may be consecutively numbered in spite of this it is best to adopt a slightly unconventional numbering system for the minor cycles (see Fig. 19).

A number of trigger circuits are employed to keep track of the stages which the various processes have reached at any moment. The most important of these are listed below with a short description of the functions of each.

OKCI. This is stimulated when the new instruction has been found and is available at the input of CI, and the CA operations belonging to the last instruction have been carried out. Stimulation begins simultaneously with stimulation of P1, and ends on a P32. The end of OKCI has to wait for the gating of CD, indicating that the new CD is available at its input.

OKCA. Only applies in case A and indicates that the CA operations have been finished.

OKCX. Indicates that we may now begin to look for the next instruction with a view to putting it into CI. It is stimulated when OKCI is extinguished, and is itself extinguished when the new CI has been found.

We may now describe the time cycle of IO. Let us begin at the point where OKLK is stimulated indicating that the search for the new CI may now begin, because we have finished with the old one and information for finding the new one is now available in CD. The new CI is determined by digits 17-32 of CD. Of these digits 23-32 determine the delay line and 18-22 determine the minor cycle within the delay line. A digit 1 in column 17 indicates that the order is to be taken from TS 6 instead of from the longer delay lines. This digit is erased whenever we obey an instruction of type A. Digits 23-32 are set up on trigger circuits and operate via trees as described below. Digits 18-22 determine the time at which we must take the output of the delay line. We compare these digits with the output of the slow counter SOA (Fig.15) and when they agree we know that the right moment has come. It is convenient to arrange that the slow counter is always one minor cycle ahead of time, so as to give us time to organise ourselves before taking the required output. As has been mentioned the order of the digits in CD is arranged rather unconventionally in order to put consecutively numbered minor cycles in alternate positions; this has time saving effects. The required minor cycle now passes into CI and the signal OKSS is given; OKLK is suppressed. When the CA operations belonging to the last instruction have been finished OKCA is stimulated and with it OKCI. We are now able to initiate any new CA operations (case A) and to set up the new CD. When this has been done we have finished with CI and suppress OKCI, which automatically stimulates OKLK beginning the cycle over again. (Figs.22,22a).

The digits 23-32 determine the delay line required. This amounts to 10 digits and will certainly be adequate for our present programme. Treeing is done in two stages, going first through trees for three or four digits only. These are TRA 000 ... TRA 111, TRB 000 ... TRB 111, TRC 0000, ..., TRC 1111. These number 32 valve elements. At the second stage there are 1024 valve elements TREECI 0000000000, ... TREECI 1111111111. The connections are shown for TREECI 1011101101. The connection from CI 17 prevents any of the TREECI elements being stimulated when CI 17 is stimulated. This is required to deal with the case where the next order is taken from TS 6 and not from the delay lines. (Fig.20).

It is very probable that some other form of tree circuit, not capable of being drawn in terms of our valve elements, will be used, and the same will apply to many parts of the circuit. It is thought worth while however to draw these circuits, if only to clarify what it is intended the circuits should do.

We have a similar tree system for the selection of temporary storages.

12. Detailed description of the arithmetic part (CA).

We shall divide the CA operations into a number of types. We shall make provision for 16 types, but for the present will only use nine. The types are distinguished by digits CI 5-8.

Type K. Pass the content of TS 6 into a given minor cycle.

Type L. Pass the content of a given minor cycle into TS 6.

Type M. Pass the content of a given TS into TS 6.

Type N. Pass the content of TS 6 into a given TS other than TS 4 or TS 5, or TS 8 or TS 1.

Type O. Pass the content of the first 12 minor cycles of a given DL out onto a card via the reproducer.

Type P. Pass the content of the card at present in the card reader on to a given DL.

Type Q. Pass CI 17-32 into TS 6.

Type R. Various logical operations and others yielding results forming one minor cycle, to be performed on the contents of TS 9 and TS 10 and transferred to TS 8.

Type S. Arithmetical operations yielding a result requiring more than one minor cycle for its retention. Results go into TS 4 and TS 5.

Type T. Stimulate a given valve element.

A trigger circuit is associated with each type. With the exception of Q these are all excited for a period consisting of a number of complete minor cycles beginning with a P1 and ending with a P32.

The main components of CA are the 32 temporary storages TS 1-32. Of these TS 1-12 have some special duties.

TS 1 is used to carry the retiring data, i.e. the CD which applied just before the last instruction of type B.

TS 2 and TS 3 contain the arguments for the purely arithmetical operations, or most of them, and for the logical operations.

TS 4 and 5 contain the results of the arithmetical operations. They are frequently connected up in series to form a DL 64. This is because the results of most of the arithmetical operations are sequences of more than 32 but not more than 64 digits.

TS 6 is used as a shunting station for the transfer of information from place to place.

TS 7 is used to carry the digits of a number n when it is proposed to multiply by 2^n .

TS 8 is used to carry the result of logical operations and other operations not requiring more than one minor cycle.

TS 9 and TS 10 are the inputs for the logical operations.

TS 11 will usually be used in connection with error calculations, and accordingly has a special role in the production of multipliers.

TS 12 is used for the timing in 'automatic' multiplication and for the selection of unusual combinations of digits in the multiplier. The word 'automatic' is used because of an analogy from desk machines.

To decide between types K to T we use CI 5-8. Digits 5,6,7 are tread out to the valve elements TRG 000, ... TRG 111, as in Fig. 23. These three elements are each associated with two types, which are distinguished by CI 8. Thus TRG 000 would be identical with KVL if it were not for timing. For this timing we introduce CATIM which is to be stimulated during the appropriate time in CA operations. KVL is identical with TRG 000 & CATIM (Fig. 24).

In case K we pass the output of TS 6 to COMMEN and hence to the inputs of all the delay lines. We gate the appropriate one of these at the appropriate time, given by TIMCA by comparison of the output of the slow counter SCA with CI.

In case L we do somewhat similarly, passing the appropriate output to COMMOUT and thence to the input of TS 6 at the appropriate time given by TIMCA.

In case M we gate the appropriate output and pass into TS 6.

In case N we pass the output of TS 6 to the inputs of the other TS, only gating the one required.

In case O the first effect is to set the mechanism in motion to pass a card through the reproducer. By means of a commutator arrangement or otherwise the reproducer sends back a series of pulses which indicate the times when the reproducer punches are ready to accept current. In the circuit diagram (Fig. 25) two sets of pulses are shown which are intended to mark the beginnings and ends of these periods. They may be separately provided by the reproducer, or one may be derived from the other by delaying or otherwise. The two sets of pulses each control trigger limiters connected up so as to extinguish one another. (Do not confuse this with the two mutually extinguishing triodes that will normally form part of a trigger circuit or trigger limiter). One of the trigger limiters TIMOUTCARD stimulates the trigger circuit OUTIM on the first admissible P 10. A pulse on the stimulation of OUTIM goes into a slow counter SCB and enables us to keep track of the number of rows of the card that have been punched. The content of SCB is compared with that of SCA and when they agree we know that the minor cycle which we wish to pass out is now available, and TIMCA is accordingly stimulated. TIMCA and OUTIM together permit COMMOUT to pass out to the trigger circuits OUT 1 ... OUT 32 on which it is set up statically and controls the punches.

On the final exit of the card the reproducer sends back a signal to the calculator, which, in combination with O operates a trigger limiter CARDEXOUT. This suppresses CATIM and hence O. CARDEXOUT has feedback to suppress itself, and this will be successful because O will have been suppressed by the time it comes to act.

The behaviour in case P (input) is very similar. The chief difference is that whereas OUTIM was used to gate the output from the calculator INTIM is used to gate the input.

It should be noticed that a completely blank instruction has a definite meaning, viz. to pass the material on the card in the reader into DL 0000000000.

In Fig. 27 TS 01101 typifies any of the TS as regards output connections shown on other diagrams. It is also typical as regards input connections, except as regards TS 4,5,8,1, which have no input connections except those shown on other diagrams.

In the case of operations of type R we shall calculate all of the expressions involved and select them by means of tree elements, digits 18 to 23 being used. The operations so far are:

Digits 000000 TS 8 = TS 9 & TS 10.

Digits 001000 TS 8 = TS 9 v TS 10.

Digits 010000 TS 8 = -TS 10.

Digits 011000 TS 8 = (TS 9 & TS 10) v (-TS 9 & -TS 10).

Digits 100000 TS 8 = 0.

As we shall have very much to say about type S we shall make a few remarks first about type F. In order to be able to obtain a rather direct access from the instructions to the valves we shall introduce a number of valve elements which can be stimulated to order. We may have 64 of these, say FLEX 000000 to FLEX 111111. The circuit will be simply as shown in Fig. 31. It is intended that the outputs of these valve elements should be connected in various ways into the circuit when it is desired to try out new circuit arrangements. It is thought that they may often provide means for doing things simply which could be done lengthily as an IFO. To an extent this represents a compromise between the new system of 'control by paper' and the old plugboard and soldering-iron techniques.

We shall also describe the timing arrangements before passing on to type S. We have already mentioned CATIM which determines the timing but we have still to mention what controls CATIM. CATIM is stimulated as soon as the first P1 appears after the signal A, or, in case Q, the first P17. It is extinguished by a variety of means. In cases K and L it is extinguished by the ending of TIMCA indicating that the required minor cycle has just passed through. In cases M, N, R, T, it is only permitted to last for one minor cycle. In case Q it is also only allowed to last for half a minor cycle. In cases O, P the extinguishing signal is CARDEX, which is given by the card reproducer of reader on the final exit of the card, via a trigger-limiter. In case S the signal comes from FLNARITH.

The facilities provided under type S are not easily enumerated, because they do not consist of a number of different operations stimulated by different tree valve elements, as for instance applies in the case of the logical processes. Rather they are to be thought of as one process which can be modified in various ways. The standard process always involves converting the content of TS 4 and TS 5 into 'series form', i. e. instead of connecting the outputs of TS 4 and TS 5 to their own inputs they are connected to each others. When they are so connected their content will be described as the 'partial sum'. Some quantities are then added to or subtracted from the partial sum. If the quantity is to be added then FOS is stimulated, otherwise they are subtracted. We may if we wish cancel the original partial sum

before/

before adding in which case we must stimulate CANCEL for a period of two minor cycles. The quantity to be added or subtracted is expressible as the product of a quantity known as the 'multiplicand' and an integer which may be taken to lie in the range -7 to 15, positive values being the more normal. The multiplicand may be taken from TS 3 or from the partial sums register itself. This latter case is convenient for the purpose of multiplying the partial sum by a small integer without a complicated series of previous transfers; if the multiplicand is taken from the partial sums register then SELF is stimulated. The multiplier may also be taken from a variety of sources. It may be taken from TS 2 or from CI or from TS 11, and we accordingly stimulate NOR, GIV or ERR. The multiplier consists of four consecutive digits from whichever source is chosen. The choice of the digits is made by means of a choice of one of the pulses P1 to P32 to enter on a certain line (DIGIT). At present it is suggested that in case NOR this should be P1, resulting in the use of digits 1,2,3,4, in case IV it should be P 23 resulting in the use of digits 23,24,25,26, in case ERR1 it should be P 10, and in case ERR2 it should be P 14. In case DIFF these arrangements are to be overridden and the pulse will be stored in TS 12 and taken from there.

In case AUTO the above fundamental process is repeated eight times. In each repetition the multiplicand is taken from TS 3, but it is modified each time by multiplication by 2^4 , this effect being obtained by allowing it to circulate in a DL34 during AUTO. We also wish to take different digits of the multiplier at each repetition of the process; this is done by taking our pulse from TS 12 but allowing it to circulate in a DL 34 also. Facilities are also provided for multiplying the partial sum by a power of 2. Although the circuits are arranged so that this could be combined with other operations, it is not intended that this should be done. The facility consists in enabling the partial sums to be delayed by any time up to 63 and passed through for a period of 2 or 3 minor cycles as desired. The amount of delay is taken from digits 1-5 of TS 7. We stimulate ROTATE 2 or ROTATE 3 according as we wish the rotation to last for 2 or 3 minor cycles.

It may be as well to describe how some rather definite operations are done.

Addition. We do not have a facility for addition of two given numbers so much as for the addition of a given number into the partial sum. To add the content of TS 3 into the partial sum we must stimulate S, POS, GIV, and must also set up the number 1 in columns 24-27. The multiplicand is then TS 3 and the multiplier is 1.

Subtraction. As addition but we do not stimulate POS.

Short multiplication (A). To multiply TS 3 by 6 (say) proceed as for addition with 0110 in columns 24-27 instead of 1000. We shall very likely also want to cancel the original content of the partial sums register and therefore stimulate CANCEL.

Short multiplication (B). To multiply the partial sum by 6 we must stimulate S, POS, CANCEL, SELF, GIV, and set up 0110 in CI 24-27.

Short multiplication (C). As B but do not cancel and put 1010 in CI 24-27.

Short multiplication with addition. We wish to multiply TS 3 by TS 2 and add into the partial sum. We stimulate POS, NOR, AUTO, DIFF.

Long multiplication with subtraction. If we wish to subtract from the partial sum we do not stimulate POS.

Division is an ITO and will probably be carried out by means of the recurrence relation $u_0 = \frac{3}{4}$, $u_{n+1} = u_n (2 - au_n)$. The limit of the sequence u_n is a^{-1} provided $1 < a < 2$.

The appropriate instructions for these operations will be found in Fig. 37.

The content of TS 2 or TS 3 is best considered to be a binary integer, i.e. that the least significant digit is in the units position. We must also consider that the most significant digit has reversed sign. The least significant digit appears at time P1 and the most significant at P32. In the partial sums register similarly the least significant digit is to be considered to be in the units position and the most significant to have reversed sign and to appear 63 pulses later. In order to keep track of which part of the partial sum is available at any moment we have a signal ODD which is stimulated during the first minor cycle of the stimulation of S, and thereafter in alternate minor cycles so long as S is stimulated. When the multiplicand is taken from TS 3 we have to make some slight modifications to it before it is in suitable condition for adding into the partial sum. We have to convert the periodic signal with period 32 or 34 into a sequence of 64 digits of which 32 form the original content of TS 3, and the rest is a sort of padding. We may call the 32 digits the genuine digits. Those digits of padding which are less significant than the genuine digits are to be all zero, those which are more significant are to be the same as the most significant genuine digit. It will be seen that this modified multiplicand MUCAND 2 has the same meaning as the original multiplier, but expressed in the code which is appropriate to the partial sum, and multiplied by the power of 2 which is required at the time. It may be necessary to change the sign of this multiplicand, if POS was not stimulated. A simple circuit will do this (Fig. 34).

Owing to the fact that the partial sums register is a closed cycle of 64 there is a danger of carries from the most significant digit on to the least significant. This has to be prevented, and it is done by suppressing the carry in the appropriate adder at the time P32 & -ODD. This is shown by an inhibiting connection on to the adder.

The detailed correctness of the circuits is best verified by working through various particular cases. It is necessary to work several different ones in order to bring out the various different special points involved. In Fig. 35 the preliminaries to a long multiplication have been worked. This shows the setting up of the new CI and the transfer of digits to the valve elements Z1, Z2, Z3, Z4. It brings out the point of adding 2 rather than 1 to the CD in cases A, B, for we are just in time to catch the next instruction. The final stages of the multiplication are shown in Fig. 36. Here it has been assumed that the minor cycle is of length 16, in order to reduce the space occupied by the working.

13. Examples of Instruction Tables.

In this chapter a short account of the paper technique of using the machine will be given. I shall try to give some idea of what the instruction tables for a job will be like and how they are related to the job and to the machine. This account must necessarily be very incomplete and crude because the whole project as yet exists only in imagination.

Each instruction will appear in a number of different forms, probably three or four.

Machine form. - When the instruction is expressed in full so as to be understood by the machine it will occupy one minor cycle. This we call machine form.

Permanent form. - The same instruction will appear in different machine forms in different jobs, on account of the renumbering technique as described in pp. 13, 14. Each of these machine form instructions arises from the permanent form of the instruction. These permanent forms are on Hollerith cards and are kept in a sort of library.

Popular form. - Besides the cards we need some form of the table which can be easily read, i. e. is in the form of print on paper rather than punching. This will be the popular form of the table. It will be much more abbreviated than the machine form or the permanent form, at any rate as regards the descriptions of the CAO. The names of the instructions used will probably be the same as those in the permanent form.

In addition to these we must recognise the 'general description' of a table. This will contain a full description of the process carried out by the machine acting under orders from this table. It will tell us where the quantities or expressions to be operated on are to be stored before the operation begins, where the results are to be found when it is over and what is the relation between them. It will also tell us other important information of a rather dryer kind, such as the storages that must be left vacant before the operation begins, those that will get cleared or otherwise altered in the process, what checks will be made, and how various possible different outcomes of the process are to be distinguished. It is intended that when we are trying to understand a table all the information that is needed about the subsidiaries to it should be obtainable from their general descriptions.

The majority of actual instruction tables will consist almost entirely of the initiation of subsidiary operations and transfers of material. It should be recognised however that the time spent will be in quite different proportions. The three most time consuming operations are multiplication, waiting for material in long delay lines, and transfers of material. In some jobs the input and output of material may also be very time-consuming.

In order to give a fairly complete picture of what the tables are like I am giving examples of two tables, of which one is elementary and does not involve subsidiaries; the other is a more advanced table and consists largely of such orders. Besides these I have added a number of general descriptions of tables.

The fundamental table chosen is INDEXIN, used for finding a minor cycle whose position has been written down in a particular place.

In these tables DL m, n will denote the n th minor cycle of DL m .

INDEXIN (General Description). The minor cycle whose position is described in digits 17-32 of TS 27 is transferred to TS 28. The contents of TS 2, 3, 4, 5, 6, 8, 9, 10 get altered in the process.

Now follows the popular form of the table.

INDEXIN.

1	Q, 0000,0100,0000,0000	2
2	TS 6 - TS 2	3
3	ADD 'A'	4
4	ROTATE 16	5
5	TS 4 - TS 6	6
6	TS 6 - TS 9	7
7	TS 27 - TS 6	8
8	TS 6 - TS 10	9
9	OR	10
10	TS 8 - TS 6	11
11	B,1, INDEXIN 11	
12	TS 6 - TS 28	13
13	B, BURY	

The first column gives the popular form of the name of the instruction, and the last column that of the next instruction to be followed. In most cases this could in theory be omitted because of the instructions being of type A. When the instructions are of type A the middle column describes them in abbreviated form. For instance TS 6 - TS 3 describes the operation of transferring the content of TS 6 into TS 3. Expressions of form Q, ... mean an instruction of type Q, and the expression after the comma describes what is in columns 17-32. ADD 'A' is to mean 'Add TS 2 into TS 4 cancelling the partial sums', ROTATE 16 means 'Rotate the content of TS 4, TS 5 forwards 16 places', OR is a logical operation.

The expression B,1,INDEXIN 11 is intended to stand for B in column 3, 1 in column 17 and INDEXIN 11 in columns 17-32.

Outline of operation (INDEXIN). From 1 to 10 we are constructing the instruction which tells us to make the appropriate transfer and putting that instruction into TS 6. The instruction B,1, INDEXIN 11 requires us to carry out the instruction in TS 6. The new IN formed will be O, INDEXIN 12 so that we then continue with instruction INDEXIN 12.

The table for INDEXIN is shown in full in Fig.38.

We use the convention that no digit is shown if the value of the digit is not significant. Both 0 and 1 are shown if either value is possible, and significant.

DISCRIM (General description). If TS 8 contains any digit 1 then TS 15 is passed into TS 24, otherwise TS 16 is passed into TS 24. The contents of TS 2, TS 3, TS 4, TS 5, TS 8 are altered.

Outline of operation. TS 8 is transferred to TS 2 and then subtracted from zero, passing into the partial sums register TS 4, TS 5. By taking out TS 5 we obtain a minor cycle full of digits 1 or of digits 0 according as there was or was not a digit 1 in TS 8 originally. We then form (TS 5 & TS 15) v (\sim TS 5 & TS 16) by logical operations and pass it on to TS 24.

This table provides the main means of deciding between two alternative procedures, by setting up one or the other of two instructions, contained in TS 15 or TS 16.

PLUSIND (General Description). 1 is added to the position reference in TS 27, e.g. DL 7, 9 becomes DL 7, 10, but DL 7, 32 becomes DL 8, 1.

TRANS 45 (General Description). The following set of transfers is made

TS 22 - TS 20, TS 23 - TS 21.

BURY (General Description). The content of TS 1 with 1 added is transferred to the position indicated in TS 31, and 1 is added to the reference in TS 31. We then proceed to carry out the instruction in TS 1.

UNBURY (General Description). The minor cycle whose position is given in TS 31 is taken to be position of the next instruction.

MULTIP (General Description). The number in TS 18, 19 is multiplied by the number in TS 20, 21; the result is brought to standard form by shift of decimal point. An error is obtained for the product by using the errors in the given numbers and allowing for rounding off. The result is stored in TS 22, 23.

ADD is analogous to MULTIP.

As an example of a more complicated process, I have chosen the calculation of the value of a polynomial.

CALPOL (General Description). The minor cycles of DL 3 taken in pairs contain the coefficients of a polynomial in descending order. Evidently we are restricted to degrees not exceeding 15, and we assume the degree always to be 15, filling up with appropriate zero coefficients. The value of this polynomial will be calculated for the argument in TS 13, TS 14 and the result will be transferred to TS 25, 26. Before starting we require special contents in DL 1, 14 and DL 1, 15. There are

DL 1, 14 0000,0101,0000,0000,0100,0110,0000,0000

DL 1, 15 0000,0000,0000,0000,0000,0100,0000,0000

the expression in DL 1, 14 representing the order to transfer DL 3,1 to TS 6.

CALPOL:

CALPOL 1. Clear TS 22, 23; DL 1, 14 - TS 27;

DL 1,15 - TS 29. CALPOL 8

CALPOL 8. B, BURY; B, INDEXIN; TS 28 - TS 18; B, BURY; B, PLUSIND;

B, BURY; B, INDEXIN; TS 28 - TS 19; B, BURY; B, ADD; B, BURY;

B, PLUSIND; TS 27 - TS 2; TS 29 - TS 3; AND; Q, CALPOL 40;

TS 6 - TS 15; Q, CALPOL 37; TS 6 - TS 16; B, BURY; E, DISCRIM; B, 1

CALPOL 37. TS 13 - TS 18; TS 14 - TS 19; B, BURY; B, TRANS 45;

B, BURY; B, MULTIP; B, BURY; B, TRANS 45. CALPOL 49.

CALPOL 49. B, CALPOL 8.

CALPOL 50. TS 22 - TS 25; TS 23 - TS 26; B, UNBURY.

The above table for CALPOL has been expressed in a more abbreviated form than the one we gave for INDEXIN, several operations being listed at a time. AND is of course the logical operation and B,1 indicates B with a 1 in column 17.

Outline of operation (CALPOL). If we denote the polynomial by $a_1 x^{15} + a_2 x^{14} + \dots$ the calculation proceeds by the equations $b_1 = a_1, c_1 = b_1 x, b_2 = c_1 + a_2, c_2 = b_2 x, \dots$ After the calculation of each b_r we have to determine whether this is the one required, viz. b_{16} or not. This is done by examining the content of TS 27 which includes the number r and is also, one might say principally, used to describe the position of the next coefficient a_{r+1} . If it is the one required we find ourselves at CALPOL 40 and have to pass b_r out to TS 25,26. Otherwise we go to CALPOL 31, and after multiplying b_r by x to give c_r we find ourselves back at CALPOL 8 and repeating processes we have done before.

It will be evident that the table CALPOL is somewhat wasteful of space. Each time a subsidiary operation is required we have to repeat B, BURY, and each time we make a transfer we have to do it in two stages, each of which uses a whole minor cycle of which most is wasted. It is possible to avoid this waste of space by keeping the instruction tables in some abbreviated form, and expanding each table whenever we want it. This will require a table EXPAND, and will require each table to include appropriate references to the table EXPAND. These references will however be put in by EXPAND itself (when working under contract to a higher authority), just as EXPAND will put in the references to BURY and UNBURY.

BINDEC (General Description). The number in TS 13, 14 is translated into decimal form of the type $\alpha \times 10^m$ where $1 \leq \alpha < 10$, and is transferred into DL 10. The notation of the decimal form is such that the content of DL 10 can be passed out onto a card in the usual way and if the card is then listed the digits of the numbers α, m will then appear on the listing paper in the usual way. Or in other words only the first 10 minor cycles of DL 10 are used, and a decimal digit is represented by the minor cycle in which a pulse occurs, and its significance by the position of it within the minor cycle.

(This account is incomplete as regards signs and some other details).

14. The Design of Delay Lines.

(i) General. - A considerable amount of work has been done on delay lines for R.D.F. purposes. On the whole our problems coincide with the R.D.F. problems but there are a few differences.

(a) Owing to the fact that there will be more than one tank used in the calculator the stability of the delay is of importance. In R.D.F. the delay is allowed to determine the recurrence frequency and the effects of variations in it are thereby eliminated.

(b) In R.D.F. it is required that the delayed signal should not differ from the undelayed by an error signal which is less than 60 dB (say) down on the signal proper. We are less difficult to please in this respect. We only require to be able to distinguish mark from space with a very high probability (e.g. at least $1-10^{-32}$). This requires a high signal to noise ratio, so far as the true random noise and the interference are concerned, but it does not require much as regards hum, frequency distortion and other factors producing unwanted signals of fairly constant amplitude.

Our main concerns then in designing a delay line will be:

(1) To ensure sufficient signal strength that noise does not cause serious effects.

(2) To eliminate or correct frequency and phase distortion sufficiently that we may correctly distinguish mark and space.

(3) To stabilise the delay to within say 0.2 pulse periods.

(4) To eliminate interference.

(5) To provide considerable storage capacity at small cost.

(6) To provide means for setting the crystals sufficiently nearly parallel.

The questions of noise and signal strength are treated in some detail in the following pages. It is found that there is plenty of power available unless either very long lines or very high frequencies are used. The elimination of interference is mainly a matter of shielding and is a very standard radio problem, which in our case is much less serious than usual. Various means have been found by the R.D.F. workers for setting the crystals. Some prefer to machine the whole delay line very accurately, others to provide means for moving the crystals through small angles, e.g. by bending the tank. All are satisfactory.

I list below a number of questions which must be answered in our design of delay lines. In order to fix ideas I have added the most probable answers in brackets after each question.

(1) What liquid should be used in the line? (Either mercury or a water-alcohol mixture).

(2) Should we use a carrier? If so, of what frequency? (Yes, certainly use a carrier. Frequency should be about 10 Mc/s with water-alcohol mixture, but may be higher if desired when mercury is used).

(3) What should be the clock-pulse frequency? (1 Mc/s).

(4) What should be the dimensions of the crystals? (Diameter might be half that of the tank, e.g. 1 cm. Thickness should be such that the first resonances of the two crystals are two or three megacycles on either side of the carrier, if water-alcohol is used. With mercury the thickness is less critical and may be either as with water-alcohol or may have resonance equal to carrier.

(5) Should the inside of the tank be rough or smooth? (Smooth).

(6) What should be the dimensions of the tank? (Standard tanks to give a delay of 1 ms. should be about 5' long whether water-alcohol or mercury. Diameter $\frac{1}{2}$ ').

(Keep all the tanks within one degree Fahrenheit in temperature. Correct systematic temperature changes by altering the pulse frequency.)

In order to be able to answer these questions various mathematical problems connected with the delay lines will have to be solved.

(ii) Electromagnetic conversion efficiency. - The delay line may best be considered as forming an electrical network of the kind usually (rather misleadingly) described as 'four-pole', i.e. a network which has one input current and one input voltage which together determine an output voltage and current. Such a network is described by three complex numbers at each frequency. In the case where there is little coupling between the output and input, which will apply to our problem, we may take these quantities to be the input and output admittances and the 'transfer admittance'. Strictly speaking we should specify whether the output is open circuit or short circuit when stating the input impedance, but with weak coupling these are effectively the same; similarly for the output impedance. The transfer admittance is the current produced at one end due to unit voltage at the other, and does not depend on which end has the voltage applied to it. In the case of the delay lines the input and output admittances will be effectively the capacities between the crystal electrodes. We need only determine the transfer admittance.

We shall consider the following idealised case. Two crystals of thicknesses d and d' are immersed in a liquid, with their faces perpendicular to the x -axis. The liquid extends to infinity in both the positive and the negative x -directions, and both liquid and crystals extend to infinity in the y and z directions (Fig. 40). The distance between the near side faces of the crystals is ℓ . It is assumed that there is considerable attenuation of sound waves over a distance of the order of ℓ but hardly any over a distance of the order of d or d' .

These assumptions are introduced largely with a view to eliminating the possibility of reflections. In practice the reflections would be eliminated by other means. For instance, the infinite liquid on the extreme right and left would be replaced by a short length of liquid in a stub of not very regular shape, so that the reflected waves would not be parallel to the face of the crystal. More likely still, of course, we should have some entirely different medium there.

The physical quantities involved are:

(a) The density ρ . We write ρ for the density of the crystal and ρ_1 for that of the liquid. Likewise a suffix 1 will indicate liquid values throughout.

(b)/

(b) The pressure p . In the case of the crystal this is understood to mean the xx -component of stress.

(c) The displacement ξ in the x -direction.

(d) The velocity v in the x -direction.

(e) The radian frequency ω .

(f) The elasticity η . This is the rate of change of pressure per unit decrease of logarithm of volume due to compression.

(g) The velocity of propagation c .

(h) The mechanical characteristic impedance Σ .

(i) The reciprocal radian wave length β .

(j) The piezo-electric constant ξ . This gives the induced pressure due to an electric field strength of unity. This field strength should normally be thought of as in the x -direction, but we shall have to consider the case of a field in the y or z direction briefly also.

These quantities are related by the equations

$$c = \sqrt{\eta/\rho}, \quad \Sigma = \sqrt{\eta\rho}, \quad \beta = \frac{\omega}{c}, \quad v = i\omega\xi, \quad i\omega\rho v = -\frac{dp}{dx}, \quad p = -\eta \frac{d\xi}{dx} + E\xi$$

In what follows we assume that all quantities such as p , v , ξ depend on time according to a factor $e^{i\omega t}$, which we omit.

We now consider the 'transmitting crystal', which we suppose extends from $x = -a$ to $x = a$ where $d = 2a$. The solution of the equations will be of form

$$p = E\xi + B \cos \beta x$$

within the crystal, i.e. for $|x| < a$. Since the pressure is continuous we shall have

$$p = (E\xi + B \cos \beta a) e^{i\beta l(a - |x|)} \text{ if } |x| > a.$$

This gives for the velocity

$$v = \frac{1}{\omega\rho} \cdot -B\beta \sin \beta x = -iB \Sigma^{-1} \sin \beta x \text{ if } |x| < a$$

$$v = \Sigma^{-1} (E\xi + B \cos \beta a) e^{i\beta l(a - |x|)} \sin \beta x \text{ if } |x| > a.$$

Continuity of velocity now gives

$$B \left(\cos \beta a + \frac{i\Sigma}{\beta} \sin \beta a \right) = -E\xi$$

and/

and therefore the velocity at a is

$$\frac{-iB \sin \beta a}{\gamma} = \frac{iE \xi \sin \beta a}{\gamma \cos \beta a + i \gamma_1 \sin \beta a}$$

i.e. the velocity at the inside edge of the crystal is

$$\frac{iE \xi}{\gamma} \cdot \frac{1}{\cot \frac{dw}{2c} + iu}$$

where $u = \gamma_1 / \gamma$.

Assuming that the exciting voltage is longitudinal we may say that

$$\frac{\text{Velocity}}{\text{Exciting voltage}} = \frac{i \xi}{\gamma a} \cdot \frac{1}{\cot \frac{dw}{2c} + iu}$$

The effect of the medium between the two crystals we will not consider just yet. Let us simply assume that

$$\frac{\text{Velocity at inside edge of receiving crystal}}{\text{Velocity at inside edge of transmitting crystal}} = \gamma$$

We have now to consider the effect of the receiving crystal. Fortunately we can deal with this by the principle of reciprocity. When applied to a mixed electrical and mechanical system this states that the velocity produced at the mechanical end by unit voltage at the electrical end is equal to the current produced at the electrical end by unit force at the mechanical end. Hence

$$\frac{\text{Current at receiving end}}{\text{Force on receiving crystal}} = \frac{i \xi}{d' \gamma} \cdot \frac{1}{\cot \frac{d'w}{2c} + iu}$$

To these equations we may add that the ratio of force to pressure is the area A' of the receiving crystal, and that the ratio of pressure to velocity is the mechanical characteristic impedance γ_1 . Combining we obtain

$$Y = \text{Transfer admittance} = \frac{A' \xi^2 \gamma_1}{d d' \gamma^2} \cdot \frac{1}{\left(\cot \frac{dw}{2c} + iu \right) \left(\cot \frac{d'w}{2c} + iu \right)}$$

Let us now assume that the input to the valve from the receiving crystal consists of a tuned circuit with a fairly low 'Q' as in Fig. 41.

Then

$$\text{Voltage attenuation and phase change factor} = \frac{\text{Grid voltage}}{\text{Input voltage}}$$

$$= \frac{Y}{\frac{1}{Li\omega} + Ci\omega + \frac{1}{R}}$$

$$= \frac{Y}{Ci\omega} \frac{ww_0}{\left(w + w_0 + \frac{iw_0}{2Q}\right) \left(w - w_0 + \frac{iw_0}{2Q}\right)}$$

where $LCw_0^2 \left(1 + \frac{1}{4Q^2}\right) = 1, C = C_s + C_x$

$$Q = RCw_0$$

$$= \gamma \frac{C_x}{C_x + C_s} \cdot \frac{2\pi\xi^2}{k\eta} \cdot R(w)$$

where $k =$ Dielectric constant of crystal.

$\gamma =$ Attenuation due to viscosity of medium and geometrical causes.

$$R(w) = \frac{u}{\frac{dw_0}{2c} \left(\cot \frac{dw}{2c} + iu\right) \left(\cot \frac{d'w}{2c} + iu\right)} \cdot \frac{ww_0}{\left(w + w_0 + \frac{iw_0}{2Q}\right) \left(w - w_0 + \frac{iw_0}{2Q}\right)}$$

The quantity $\frac{2\pi\xi^2}{k\eta}$ depends only on the crystal, i.e. on the

material of which it is made and its cut and form of excitation. Both ξ^2 and η are of the dimensions of a pressure. $4\pi\xi$ is of the dimensions of an electric field, and may be thought of as a constant electric field which has to be added to the varying field in order that the combination should produce the correct pressure variations, somewhat like the permanent magnet field in a telephone receiver. A typical

value for $\frac{2\pi\xi^2}{k\eta}$ is 0.004.

Let us now consider the frequency-dependent factor, $R(w)$. The parameter u entering here is the ratio of the characteristic impedances of the crystal and the liquid. It is equal to

$$\frac{\text{Velocity of sound in liquid} \times \text{density of liquid}}{\text{Velocity of sound in crystal} \times \text{density of crystal}}$$

The velocity of sound in the crystal (X-cut quartz) is 5.72 km/sec. and its density is 2.7. The velocity in water is 1.44 km/sec., and the density 1, hence

$$u(\text{water}) = 0.1 \text{ abt.}$$

The velocity in mercury is much the same but the density is 13.5. Hence

$$u(\text{mercury}) = 1.3 \text{ abt.}$$

These figures suggest that we consider the two cases where u is small and where u is 1. The latter case may be described by saying that the liquid matches the crystal.

It may be assumed for the moment that our object is to make the minimum value of $|R(w)|$ in a certain given band of frequencies as large as possible. If the width of the band is $2\frac{\omega}{Q}$ and it is centred on ω_0 and if we ignore the variations in $\frac{\omega}{\omega_0}$ we shall find that

the optimum value of u is of the form $N\frac{\omega}{\omega_0}$ where N is some

numerical constant probably not too far from 1. The value of Q should be as large as possible. With $Q = 1 \text{ Mc/s}$, $\omega_0 = 10 \text{ Mc/s}$ this seems to suggest that water ($u = 0.1$) is very suitable. In practice the differences due to the value of $\frac{\omega}{\omega_0}$ are more serious than those due to u , and there is in any case plenty of power. We would not in practice take Q as large as we could but would rather try to arrange that $|R(w)|$ was fairly constant throughout the band concerned and $\arg R(w)$ fairly linear when plotted against w . If water were used one would probably choose the thicknesses of the crystals and the value of Q to give poles of $|R(w)|$ somewhat as shown in Fig. 41. With this arrangement of the poles the gain corresponding to $|R(w)|$ is 9 dB throughout the range 8 Mc/s and the phase characteristic lies within 5° of the straight line within this range.

With mercury where u is nearly 1 we should put

$$\frac{d\omega_0}{2c} = \frac{\pi}{2}, \quad \frac{d^2\omega_0}{2c} = \frac{\pi}{2},$$

and then

$$|R(w)| = \frac{2}{\pi} \left(\sin \frac{\pi}{2} \frac{w}{\omega_0} \right)^2 \left| \frac{w \omega_0}{\left(w + \omega_0 + \frac{i\omega_0}{2Q} \right) \left(w - \omega_0 + \frac{i\omega_0}{2Q} \right)} \right|$$

We should probably find it desirable to omit the tuned circuit, in which case $R(w)$ would represent a fairly constant loss of 4 dB. One could use a Q of 2 if one wished, giving a gain of 2 dB instead.

We have assumed above that the crystal is longitudinally excited. If it were transversely excited the figures would be much less satisfactory. At the transmitting end a far larger voltage would have to be applied in order to obtain the same field strength, and at the receiving end the stray capacities will have a more serious effect with transverse electrodes, although if the stray capacity were zero transverse electrodes at the receiving end would actually be more efficient.

(iii) Geometrical attenuation. - If a rectangular crystal is crookedly placed in a plane parallel beam, the tilt being such that the one edge of the crystal is advanced in phase by an angle ψ then the attenuation due to the tilt is $\frac{\sin \frac{1}{2} \psi}{\frac{1}{2} \psi}$. With a square crystal whose side is 1 cm. and a frequency of 15 Mc/s this would mean that we get the first zero in the response for a tilt of about 16'. The setting is probably not really as critical as this, owing to curvature of the wave fronts. If the crystals are operating in a free medium without the tube this effect is easily estimable and we find that, for crystals sufficiently far apart the allowable angles of tilt are of the order of the angle subtended at one crystal by the other. It has been found experimentally with tubes operating at 15 Mc/s that tilts of the order of half a degree are admissible.

Now let us consider the loss due to boundary effects. We assume a wave inside the tank of form $p = J_0(\beta r) e^{-i\beta z + i\omega t}$ and assume a boundary condition of form $\frac{1}{p} \frac{dp}{dn} = \gamma$ where we do not know γ nor even whether it is real or complex. The radius of the tank is a , so that the boundary condition becomes $\frac{J_1(\beta a)}{J_0(\beta a)} = \gamma a$. Let the solution of $\frac{u J_1(u)}{J_0(u)} = \gamma a$ be $u(\gamma a)$. Then we have $\beta^2 + \left(\frac{u(\gamma a)}{a} \right)^2 = \frac{\omega^2}{c^2}$ and therefore $\beta \frac{J_1(\beta a)}{J_0(\beta a)} + \frac{1}{a^2} \Re u \int u = 0$. But since $\frac{u(\gamma a)}{\beta a}$ is small this means approximately $\beta \frac{J_1(\beta a)}{J_0(\beta a)} = -\frac{\Re u \int u}{a^2}$, and the loss in a length l of the tank is $\frac{l_0}{a^2 \omega} \Re u \int u$ nepers. For a given value of γ there are many solutions of $\frac{u J_1(u)}{J_0(u)} = \gamma a$ but there is a bounded region of the u plane in which there is always a solution whatever value γa may have. This means to say that for any boundary condition there is always a mode in which the attenuation does not exceed $\tau \frac{c}{a^2 \omega}$ where τ is some numerical constant.

The value of τ is about 1.9. It is the largest value of xy such that $(x + iy) J_1(x + iy) / J_0(x + iy)$ is pure imaginary and $y > 0, 0 < x < 2.4$.

Taking $l_0/c/a^2 \omega_0 = 0.31$ (as p.41) the maximum loss in this mode is 6 dB. We should however probably add a certain amount to allow for the fact that not all of the energy will be in this mode. A total loss of 10 dB would probably not be too small.

(iv) Attenuation in the medium. - The attenuation coefficient is given by $\frac{2\omega^2 \nu}{3c^3}$ where ν is the dynamic coefficient of viscosity, i.e., the ratio of viscosity to density. With water ($\nu = .013$ c = 1.44 Km/sec.) at a frequency of 10 megacycles and a delay of 1 ms we have a loss of 12 dB. With mercury under the same circumstances the loss is only 1 dB.

These figures suggest that if water is used the frequency should not be much above 10 Mc/s, but that we can go considerably higher with mercury.

(v) Noise.

Before leaving the subject of attenuation we should verify how much can be tolerated. The limiting factor is the noise, due to thermal agitation and to shot effect in the first amplifying valve. The effect of these is equivalent to an unwanted signal on the grid of the first valve, whose component in a narrow band of width f cycles has an R.M.S. value of

$$V_N = 4 k T f (R + R_e)$$

where T is the absolute temperature, k is Boltzmann's constant and R is the resistive component of the impedance of the circuit working into the first valve, including the valve capacities. R_e is a constant for the valve and describes the shot effect for the valve. In the case that we use mercury and do not tune the input the value of R will be quite negligible in comparison with R_e , which might typically be 1000 ohms. For a pulse frequency of 1 megacycle we must take $f = 10^6$ (the theoretical figure is $\frac{1}{2}10^6$ but this is only attainable with rather peculiar circuits). At normal temperatures $4 k T = 1.6 \times 10^{-20}$ and therefore $V_N = 4 \mu V$. In the case that we

use water and tune the input, we have $R = \frac{Q}{w(C_x + C_s)}$ at the worst frequency. Assuming $\frac{w}{2\pi Q} = 2$ Mc/s (see Fig. 41) and $C_x + C_s = 20$ pf and ignoring the fact that the effect will not be so bad at other frequencies, we have $V_N = 9 \mu V$.

Now suppose that we wish to make sure that the probability of error is less than p , and that the difference in signal voltage between a digit 0 and a digit 1 is V . Then we shall need

$$2 \int_{V/2V_N}^{\infty} e^{-\frac{1}{2}x^2} dx < p.$$

(This follows from the fact that a random noise voltage is normally distributed in all its coordinates). If we put $p = 10^{-32}$ we find

$$\frac{V}{V_N} \geq 24, \quad V \geq 0.1 \text{ mV.}$$

(vi) Summary of output power results. - Summarising the voltage attenuation and noise questions we have:-

(a) There is an attenuation factor depending on the material of the crystal and its cut and for quartz typically giving a loss of 48 dB.

(b) There is a factor R depending on the ratio of band width required to carrier frequency, and the matching factor u between crystal and liquid. In practical cases this amounts to gains of 10 dB with water and 2 dB with mercury.

(c) There is a loss factor $C_N/C_x + C_s$ due to stray capacity C_s across the receiving crystal. This might represent a loss of 6 dB.

(a)/

(d) There is a loss due to the viscosity of the medium. For a water tank with a delay of 1 ms. and a carrier of 10 Mc/s the loss may be 12 dB: with mercury and a carrier of 20 Mc/s it may be 4 dB.

(e) Losses in the walls of the tank. Apparently this should not exceed 10 dB.

(f) The noise voltage may be 4×10^{-6} volts RMS (mercury) or 9×10^{-6} volts RMS (water).

(g) The signal voltage (peak to peak) should exceed the noise voltage (RMS) by a factor of 24 for safety.

These figures require input voltages (peak to peak) of 0.2 volts or 4.5 volts with mercury and water respectively. We could quite conveniently put 200 volts on, so that we have 60 dB (or 53 dB) to spare. There is no danger of breaking the crystals when they are operated with so much damping.

(vii) Phase distortion due to reflections from the walls. - We cannot easily treat this problem quantitatively because of lack of information about the boundary conditions and because the ratio of diameter of crystal to diameter of tank is significant. Let us however try to estimate the order of magnitude by assuming the pressure zero on the boundary and considering the gravest mode. In this case

the pressure is of form $J_0\left(\frac{k_1 r}{a}\right) e^{-i(\beta z + i\omega t)}$ where $2a$ is the diameter of the tank and $k_1 = 2.4$ is the smallest zero of J_0 , and

$\beta^2 + \frac{k_1^2}{a^2} = \frac{\omega^2}{c^2}$. In this case the change of phase down the length ℓ

of tank is $\varphi = \ell \frac{\omega^2}{c^2} - \frac{k_1^2}{a^2}$. If we are using carrier working we are

chiefly interested in $\frac{d^2 \varphi}{d\omega^2}$ which turns out to be $-\frac{k_1^2 c \ell}{\omega_0^3 a^2}$ where ω_0

is the carrier frequency. If we suppose that the band width involved is $2Q$, then the greatest phase error which is introduced is

$\frac{k_1^2 Q^2 c \ell}{2 \omega_0^3 a^2}$. Let us suppose that the greatest admissible error is

0.2 radians, then we must have

$$\frac{\ell_0}{a^2 \omega_0} \leq \frac{0.4}{k_1^2} \left(\frac{\omega_0}{Q}\right)^2$$

Taking $\omega_0 = 10$ Mc/s

$Q = 1$ Mc/s

$c = 1.4 \times 10^5$ cm/sec.

$\ell = 1.4 \times 10^2$ cm.

$a = 1$ cm.

Then/

Then
$$\frac{c}{w_0} = 2.2 \times 10^{-3} \text{ cm.}$$

$$\frac{L_0}{a^2 w_0} = 0.31$$

$$\frac{0.4}{k_1^2} \left(\frac{w_0}{\Omega} \right)^2 = 6.95$$

The situation is thus entirely satisfactory. The carrier frequency could even be halved.

(viii) The choice of medium. - In choosing the medium we have to take into account

(a) That a medium with a small characteristic impedance such as water has a slight advantage as regards the factor $R(w)$.

(b) That water is more attenuative than mercury.

(c) That mercury gives wide band widths more easily than water because of closer matching, but that adequate band widths are nevertheless possible with water.

(d) That a water-alcohol mixture can be made to have a zero temperature coefficient of velocity at ordinary temperatures.

On the whole the advantages seem to be slightly on the side of mercury.

(ix) Long lines. - The idea of using delay lines with a long delay, e.g. of the order of 0.1 second, is attractive because of the very large storage capacity that such a line would have. Although the long delay would make these unsuitable for general purposes they would be very suitable for cases where very large amounts of information were to be stored: in the majority of such cases the material is used in a fairly definite order and the long delay does not matter.

However such long lines do not really seem to be very hopeful. In order to reduce the attenuation to reasonable proportions it would be necessary to abandon carrier working, or else to use mercury. In either case we should probably be obliged to make the tank in the form of a bath rather than a tube; in the former case in order to avoid the phase distortion arising from reflections from the walls, and in the latter to economise mercury, using a system of mirrors in the bath. In any case the technique would involve much development work.

We propose therefore to use only tanks with a delay of 1 ms.

(x) Choice of parameters. - Considerations affecting the carrier frequency are:

(a) The higher the carrier frequency the greater the possible band width.

(b) The difficulty of cutting thin crystals, somewhat modified by the absence of necessity of frequency stability.

(c)/

(c) The attenuation at high frequencies of the sound wave in the liquid.

(d) The difficulty of setting the crystals up sufficiently nearly parallel if the wavelength is short.

(e) The difficulty of amplification at high frequencies.

Of these (a) and (c) are the most important. A reasonable arrangement seems to be to choose a frequency at which the attenuation in the medium is about 15 db.

With the comparatively low frequencies and with wide tanks the setting up difficulty will not be serious. With long lines we should probably not attempt to do temperature correction, but would rephase the output.

Considerations affecting the pulse frequency are:

(a) The limitation of the pulse frequency to a comparatively small fraction of the carrier frequency if water is the transmission medium, and the limitation of this carrier frequency.

(b) The finite reaction times of the valves.

(c) The greater capacity of a line if the frequency is high.

(d) Greater speed of operation of the whole machine if the pulse frequency is high.

(e) Cowardly and irrational doubts as to the feasibility of high frequency working.

If we can ignore (c) the other considerations appear to point to a pulse frequency of about 3 megacycles or even higher. We are however somewhat alarmed by the prospect of even working at 1 megacycle since the difficulty (b) might turn out to be more serious than anticipated.

Considerations affecting the diameter of the tank are:

(a) That the crystals are most conveniently adjusted to be parallel by bending the tanks and that the diameter should therefore not be too large.

(b) That the diameter should be at least large enough to accommodate the crystal.

(c) That small diameters give phase distortion (p.40).

(d) That with mercury small diameters are economical. At a price of £1 sterling per 1 lb. avoirdupois of mercury a 1 ms. tank of diameter 1" would contain mercury to the value of about £2-2-6.

A diameter of 1" or rather less is usual in R.D.F. tanks and appears reasonable in view of these conditions.

(xi) Temperature control system. - The temperature coefficient of the velocity of propagation in mercury is quite small at 15 Mc/s, being only 0.0003/degree centigrade. This means that if the length of a 1 ms. line is to be correct to within 0.2 ms. then the temperature must be correct to within two-thirds of a degree centigrade.

15. The Design of Valve-elements.

(i) Outline of the problem. - To design valve-elements with properties as described in § 5 and to work at a frequency of say 30 or 100 kilocycles would be very straightforward. When the pulse recurrence frequency is as high as a megacycle we shall have to be more careful about the design, but we need not fear any real difficulties of principle about working at these frequencies, and with such band widths. The successful working of television equipment gives us every encouragement in this respect. A word of warning might perhaps be in order at this point. One is tempted to try and carry the argument further and try to infer something from the success of R.D.F. at frequencies of several thousands of megacycles. Such an analogy would however not be in order for although these very high frequencies are used the bandwidth of intelligence which can be transmitted is still comparatively small, and it is not easy to see how the band width could be greatly increased.

In this chapter I shall discuss the limitations inherent in the problem, and shall also show very tentative circuit diagrams by way of illustration. These circuits have not yet been tried out, and I have too much experience of electronic circuits to believe that they will work well just as they stand. (This does not represent a superstitious belief in the cussedness of circuits and the inapplicability of mathematics thereto. Rather it means that normally the amount of mathematical argument required to get a reliable prophecy of the behaviour of a circuit is out of proportion to the small trouble required to try it out, at any rate if one is in an electrical laboratory. In practice one compromises with a rough mathematical argument and then follows up with experiment. The apparent "cussedness" of electronic circuits is due to the fact that it is necessary to make rather a lot of simplifying assumptions in these arguments, and that one is very liable to make the wrong ones, by false analogy with other circuits one has dealt with on previous occasions. The cussedness lies more in the minds dealing with the problem than in the electronic circuits themselves.)

(ii) Sources of delay. - There are two main reasons why vacuum tubes should cause delays, viz. the input capacity and the transit time. Of these perhaps the first is in practice the more serious, the second the more theoretically unavoidable.

The delay due to the input capacity, when the valves are driven to saturation or some other limiting arrangement is used, is of the order of C/g_m , where C is the input capacity and g_m is the mutual conductance of the valve. We may, for instance consider the idealised circuit Fig. 44. (Coupling with a battery is of course not practical politics, but it produces essentially the same effects as more practical circuits, and is more easily understood). If I is the saturation current then the grid swing required to produce it is I/g_m and the charge which must flow into the grid to produce this voltage is CI/g_m . If the whole saturation current is available the time required is C/g_m . This argument is only approximate, and omits some small purely numerical factors. However it illustrates the more important points. In particular we can see that Miller effect is not a very serious matter because of the limiting, which reduces the effective amplification factor to 1. On the other hand, if one valve is used to serve several inputs the delay will be correspondingly increased because the capacity has become multiplied by the number of grids served.

This connecting of several grids to one anode, and a number of other practical points will tend to make the actual delay due to input capacity several times greater than C/g_m , e.g. $10 C/g_m$.

The delay due to transit time may be calculated, in the case of a plane structure, to be $3d(m/2eV)^{1/2}$ where m , e are respectively the mass and charge of the electron, V is the voltage of the grid referred to cut-off and d is the grid-cathode spacing. In other words the transit time may be calculated on the assumption that the average velocity of the electrons between cathode and grid is one-third of the velocity when passing the grid. This time may be compared with C/g_m which, if C

is calculated statically, has the value $\frac{3}{2} d(m/2eV)^{1/2}$, i. e. half of the

transit time. That there should be some such relation between C/g_m and transit time can be seen by calculating $C/(g \times \text{Transit time})$, where C is the grid-cathode capacity and g is the actual conductance, i. e., the ratio of current to V .

$$\begin{aligned} \frac{C}{g \times \text{Transit time}} &= \frac{CV}{I \times \text{Transit time}} \\ &= \frac{\text{Charge on grid}}{\text{Charge in transit}} \end{aligned}$$

Let us now calculate actual values. The voltage V by which the grid exceeds cut-off might be 10 volts which corresponds to a velocity about 1/300 of velocity of light (Note: annihilation energy of electron is half a million volts) or one metre per microsecond. If d is 0.2 cm the transit time is 0.006μ s. A typical value for C/g_m is 0.002μ s.

The relation between C/g_m and transit time brings up an important point, viz. that these two phenomena of time delay are really inseparable. The input capacity of the tube when 'hot' really consists largely of a capacity to the electrons. When the motion of the electrons is taken into account the capacity is found to become largely resistive (Ferris effect).

Before proceeding further I should try to explain the way I am using the word 'delay'. When I say that there is a delay of so many microseconds in a circuit I do not mean to say that the output differs from the input only in appearing that much later. I wish I did. What I mean is something much less definite, and also less agreeable. Strictly speaking I should specify very much more than a single time. I should specify the waveform of the output for every input waveform, and even this would be incomplete unless it referred both to voltages and currents. We have not space to consider these questions, nor is it really necessary. I should however give some idea of what kind of distortion of output these 'delays' really involve. In the case of the input capacity the distortion may be taken to be of the form that an ideal input pulse of unit area is converted into a pulse of unit area with sharp leading edge and exponentially decaying trailing edge, the time constant of the delay being the 'delay', thus Fig. 44c. In the case of the transit time the curve is probably more nearly of the 'ideal' form (Fig. 44b).

To give the word 'delay' a definite meaning, at any rate for networks, I shall understand it to mean the delay for low frequency sine waves. This is equal to the displacement in time of the centre of gravity in the case of pulses.

In order to give an idea of the effect of these delays we have shown in Fig. 45 a pulse of width $0.2 \mu\text{s}$ and the same pulse delayed, after the manner of Fig. 44a, by $0.03 \mu\text{s}$, this representing our calculated value of 0.003 multiplied by 10 to allow for numerous grids, etc. etc. It will be seen that the effect is by no means to be ignored, but nevertheless of a controllable magnitude.

(iii) Use of cathode followers. - In order to try and separate stages from one another as far as possible we shall make considerable use of cathode followers. This is a form of circuit which gives no amplification, and indeed a small attenuation (e.g. 0.5 dB); but has a very large input impedance and a very low output impedance. This means chiefly that we can load a valve with many connections into cathode followers without its output being seriously affected.

Fig. 46 shows a design of cathode follower in which the input capacity effect has been reduced by arranging that the anode is screened from the grid and that the screen voltage as well as that of the cathode moves with the grid. If one could ignore transit time effects this would have virtually zero input capacity.

(iv) The 'limiting amplifier' circuit. - When low frequencies are used the limiter circuit can conveniently be nothing more nor less than an amplifier, the limiting effect appearing at cut-off and when grid and cathode voltages are equal. At high frequencies we cannot get a very effective limiting effect at cathode voltage, owing to the fact that the grid must be supplied from a comparatively low impedance source to avoid a large delay arising from input capacity, but on the other hand, in order to get a limiting effect we need a high impedance, high compared with the grid conduction impedance (about 2000 ohms probably).

At high frequencies it is probably better to use a 'Kipp relay' circuit. This is nothing more than a multivibrator in which one leg has been made infinitely long (and then some), i.e. one of the two semi-stable states has been made really stable. An impulse will however make the system occupy the other state for a time and then return, producing a pulse during the period in which it occupies the less stable state. This pulse can be taken in either polarity. It is fairly square in shape and its amplitude is sensibly independent of the amplitude of the tripping pulse, although its time may depend on it slightly. These are all definite advantages.

A suggested circuit is shown in Fig. 47, and the waveforms associated with it at various points in Fig. 48.

(v) Trigger circuit. - The trigger circuit need only differ very little from the limiter or Kipp relay. It needs to have two quite stable states, and we therefore return both of the grids of the $6\text{SN}7$ to -15 volts instead of returning one to ground. Secondly the inhibitory connection is different. In the case of the limiter it simply consists of an opposing or negative voltage on the cathode follower; in the case of the trigger circuit it must trip the valve back, and therefore we need a second cathode follower input connected to the other grid of the $6\text{SN}7$.

(vi) Unit delay. - The essential part of the unit delay is a network, designed to work out of a low impedance and into a high one. The response at the output to a pulse at the input should preferably be of the form indicated in Fig. 50, i.e. there should be a maximum response at time $1 \mu\text{s}$ after the initiating pulse, and the response should be zero by a time $2 \mu\text{s}$ after it, and should remain there. It is particularly important that the response should be near to zero at the integral multiples of $1 \mu\text{s}$ after the initiating pulse (other than $1 \mu\text{s}$ after it).

A simple circuit to obtain this effect is shown in Fig. 51a. The response is shown in Fig. It differs from the ideal mainly in having its maximum too early. It can be improved at the expense of a less good zero at $2 \mu\text{s}$ by using less damping, i. e. reducing the 500 ohm resistor. It is also possible to obtain altogether better curves with more elaborate circuits.

The 1000 ohm resistors at input and output may of course be partly or wholly absorbed into the input and output circuits. Further the whole impedance scale may be altered at will.

The fact that the pulse has become greatly widened in passing through the delay network does not signify. It will only be used to gate a clock pulse or to assist in tripping a Kipp relay, and therefore will give rise to a properly shaped pulse again.

(vii) Trigger limiter. - We can build up a trigger limiter out of the other elements, although we cannot replace it by such a combination in the circuit diagrams because we are not putting a legitimate form of input into all of them. The circuit is (Fig. 52).

The valve P is merely a frequency divider. It can be used to supply all the trigger limiters. The trigger circuit Q should be tripped by the combination of pulse from P and continuous input, and will itself trip R. The arrangement of two trigger circuits prevents any danger of half-pulse outputs, which we are most anxious to avoid. In order that there might be a half-pulse output the trigger circuit Q would have to remain near its unstable state of equilibrium for a period of time of $1 \mu\text{s}$. In order that this may happen the magnitude of the continuous input voltage has to be exceedingly finely adjusted; the admissible range is of the form $Ae^{-t} g_m/C$ where A might be say 100 volts (it doesn't matter really) and t is the time between pulses, C and g_m the input capacity and mutual conductance of the valves used in the trigger circuit; C/g_m might be $0.002 \mu\text{s}$ (we do not need to allow for Miller effect), so that the admissible voltage range is about 10-200 volts which is adequately small.

16. Alternative Forms of Storage.

(i) Desiderate for storage systems. - A storage system should have a high monetary economy, i. e. we wish to be able to store a large number of digits per pound sterling of outlay: it should also have a high spacial economy. For the majority of purposes we like a form of storage to be erasable, although there are a number of purposes, such as function tables and the greater part of the instruction tables, for which this is not necessary. For the majority of purposes we also like to have a short accessibility time, defining the accessibility time to be the average time which one has to wait in order to find out the value of a stored digit. Normally we shall be interested in the values of a group of digits which are all stored close together, and very often it does not take much longer to obtain the information about the whole group than about the single digit. Let us say that the additional time necessary per digit required is the digit time (reading). We may also define the accessibility and digit times for recording in the obvious analogous way, though they are usually either equal to the reading time or else exceedingly long.

(ii) Survey of available storage methods. - The accompanying table gives very rough figures for the various available types of storage and the quantities defined above. This table must not be taken too seriously. Many of the figures are based on definite numerical data, but most are guesses. In spite of the roughness of the figures the table brings out a number of points quite clearly.

	Monetary economy (digits/£)	Spatial economy (digits/litre)	Access. time (reading)	Digit time (reading)	Access. time (recording)	Digit time (recording)	Remarks
<u>Inerasible systems.</u>							
Punched Paper Tape	10^7	$5 \cdot 10^6$	≥ 1 min.	.03 sec.	(= reading)	.03 sec.	
Hollerith Cards	10^6	$3 \cdot 10^5$	≥ 1 min.	1 ms.	(= reading)	1 ms.	Permutable
Print on paper	10^8	10^8	30 secs.	10 ms.			Human use. Not very convenient for mechanical or electrical reading.
Film (a) Displayed stationary (b) Wound on reels	10^4 10^9	10^4 $3 \cdot 10^{10}$	5 μ s ≥ 1 min.	1 μ s 1 μ s		1 μ s 1 μ s	
Soldered Connections	1000	200	$< 1 \mu$ s	$< 1 \mu$ s	15 mins.	1 min.	
<u>Erasible Systems.</u>							
Plugboards	50	50	$< 1 \mu$ s	$< 1 \mu$ s	30 secs.	10 secs.	
Wheels, etc.	20	$2 \cdot 10^3$	30 ms.	30 ms.	30 ms.	30 ms.	(Mechanically read).
Relays	2	2	$< 1 \mu$ s	$< 1 \mu$ s	10 ms.	10 ms.	
Thyratrons	2	2	$< 1 \mu$ s	$< 1 \mu$ s	10 μ s	10 μ s	
Neons	20	50	$< 1 \mu$ s	$< 1 \mu$ s	30 μ s	30 μ s	
Trigger Circuits	3	3	$< 1 \mu$ s	$< 1 \mu$ s	1 μ s	$< 1 \mu$ s	
Cerebral Cortex	10^5	10^9	5 sec.	30 ms.	30 sec.	5 sec.	Man at £300 p.a. capitalised.
Acoustic delay lines	200	50	1 ms.	1 μ s	1 ms.	1 μ s	More optimistic estimate than in §10.
Electric delay lines	100	200	100 μ s	$< 1 \mu$ s	100 μ s	$< 1 \mu$ s	Circular wave guide with 1 cm. waves. Numerous carriers.
Storage tubes	10^4	10^4	5 μ s	1 μ s	5 μ s	1 μ s	Described as 'Iconoscope' by J. V. Neumann.
Magnetic tape	10^8	$3 \cdot 10^8$	1 min.	10^{-4} sec.	1 min.	10^{-4} sec.	

(1) All the well established forms of storage (excepting the cerebral cortex) are either very expensive and bulky, or else have a very high accessibility time.

(2) The really economical systems consist of layers packed into the form of a solid. They are read by exposing the layer wanted.

(3) The systems which are both economical and fairly fast have the information arranged in two dimensions. This apparently applies even to the cerebral cortex.

(4) Much the most hopeful scheme, for economy combined with speed, seems to be the 'storage tube' or 'iconoscope' (in J. v. Neumann's terminology).

(5) Some use could probably also be made of magnetic tape and of film for cases where the accessibility time is not very critical.

(iii) Storage tubes. - In an iconoscope as used in television a picture of a scene is stored as a charge pattern on a mosaic, and is subsequently read by scanning the pattern with an electron beam. The electron beam brings the charge density back to a standard value and the charge lost by the mosaic registers itself through its capacity to a 'signal plate' behind the mosaic. The information stored in this way on an iconoscope, using a 500 line system, corresponds to a quarter of a million digits.

One might possibly use an actual iconoscope as a method of storage, but there are better arrangements. Instead of putting the charge pattern on to the 'mosaic' with light we can put it on with an electron beam. The density of the charge pattern left by the beam can be varied by modulating either the voltage of the signal plate or the current in the beam. The advantages of this are:

(a) The charge pattern can be set up more quickly with an electron beam than with light.

(b) Less apparatus is required.

(c) The same beam can be used for reading and recording, so that distortion of the pattern does not matter.

It seems probable that a suitable storage system can be developed without involving any new types of tube, using in fact an ordinary cathode ray tube with tin-foil over the screen to act as a signal plate. It will be necessary to refurbish up the charge pattern from time to time, as it will tend to become dissipated. The pattern is said to last for days when there is no electron beam, but if we have a beam scanning one part of the target it will send out secondary electrons which will tend to destroy the remainder of the pattern. If we were always scanning the pattern in a regular manner as in television this would raise no serious problems. As it is we shall have to provide fairly elaborate switching arrangements to be applied when we wish to take off a particular piece of information. It will be necessary to stop the beam from scanning in the refurbishing cycle, switch to the point from which the information required is to be taken, do some scanning there, replace the information removed by the scanning, and return to refurbishing from the point left off. Arrangements must also be made to make sure that refurbishing does not get neglected for too long because of more pressing duties. None of this involves any fundamental difficulty, but no doubt it will take time to develop.